

Content

for

Research on the a priori of Physics

*Geometric Critique of
Pure Mathematical Reasoning.*

by

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Volume I – 2020, Edition 2 – 2022,

Revision 6 – November 2022

Research on the a priori of Physics
Geometric Critique of Pure Mathematical Reasoning
Jens Erfurt Andresen
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¹ I was introduced to semiotics by physicist Peter Voetmann Christiansen by first following a graduate course in Eco-Physics for my

a priori of physics	Content
	Content
	Preface
	Prologue
	I. The Time in the Natural Space
	1. The Idea of Time
	1.1. Primary Quality
	1.2. Quantity
	1.3. The Causal Action
	1.3.1. Logic and Numbers
	1.3.1.2. The Number Sequence
	1.3.2. Time, Action, and Sequence
	1.3.2.1. Extension of Time
	1.3.3. Quantity in Time
	1.3.3.1. The Passage of Time
	1.3.4. Speed of Times, the Quantum of Time, and the Frequency
	1.3.4.2. The Frequency in Action
	1.3.4.3. Associations with the Known Physics
	1.3.5. Continuous Time and Action
	1.3.5.1. Continuous Timing
	1.4. The Cyclic Time
	1.4.1.1. The Period
	1.4.1.2. The Circle Plan
	1.5. The Complex Numbers
	1.5.2. The Complex Exponential Function
	1.5.3. The Imaginary Approach to the Cyclic Circle of Rotation
	1.6. The Complex Oscillation - the Circular Movement
	1.6.2. The Cyclic Circle Clock
	1.6.2.2. The Cyclic Rotation Oscillation
	1.6.2.3. The Time Concept as a Running Wheel
	1.6.2.4. Euler Circle as the A Priori Clock
	1.6.3. The Continuous Measure for the Concept of Time
	1.7. The Cyclic Rotation
	1.7.1.1. A Entity in Physics and its Quantitative Functions
	1.7.2. The Derivative Function
	1.7.3. The Parameter Derived Quantity
	1.7.4. The Circular Rotation and the Unitary Group $U(1)$
	1.7.5. The Circular Rotating Oscillator Synchronometry
	1.7.5.2. The Real Rotation
	1.7.5.3. The Internal Oscillation
	1.7.6. The Oscillator Rotation in Physics
	1.7.7. Fourier Transformation
	1.7.8. The Local Internal Time
	1.7.8.2. The Orthogonal Frequencies
	1.7.8.3. The local Homogeneous Parameter and the Constant Oscillator Frequencies

a priori of physics	Content	
1.7.8.4. Orthogonality and Dependency in the Information Problem	47	
2. The Parameter Dependent Mechanics	48	
2.1. The Lagrange Formalism	48	
2.1.1. The Lagrange Function	48	
2.1.2. Action	49	
2.1.3. The Conservative Energy	49	
2.2. The Hamilton Function	50	
2.2.1.2. Generalised Canonical <i>Quantities</i>	50	
2.2.1.3. The Poisson Bracket	51	
2.2.2. The Operator Quantised Circle Oscillator	52	
2.2.2.2. The <i>Spectrum</i> of Oscillators	52	
2.2.3. Quantised Probability	53	
2.2.3.2. Heisenberg Picture	53	
2.2.3.3. Schrödinger Picture	54	
2.2.4. Stationary Eigenstates and Eigenvalues	54	
2.2.5. Commutator Relations	55	
2.2.6. The Frequency-energy Quantised Evolution Parameter	56	
2.2.7. Momentum	57	
2.2.7.1. The Canonical Quantum Operators	57	
2.2.7.2. The Measurable Expectation Values of Quantum Mechanics	58	
2.3. A classical Formulation of the Cyclic Rotation Oscillation	58	
2.3.1. Hamilton Formulation for the Harmonic Oscillator	58	
2.3.1.2. The Energy of an Oscillator	59	
2.3.1.3. The Lagrange Function for the Cyclical Rotating Oscillator	59	
3. The Quantum Harmonic Oscillator	61	
3.1.2. The <i>Quantum Real Scalar Field</i> for the Linear Harmonic Oscillator	61	
3.1.3. Ladder Operators of the Quantum Harmonic Oscillator	62	
3.1.4. Eigenstates in the Real Field <i>Linear Quantum Harmonic Oscillator</i>	62	
3.1.5. The Quantum Number Operator	63	
3.1.6. The Ground State	64	
3.1.7. The Traditional Rotational Movement Seen as a Cyclical Object, a Circle Oscillator	65	
3.1.7.2. The Transversal Plane	66	
3.1.7.3. The Rotating Direction with Orientation	67	
3.1.7.4. An Idea of a Primary Quantum Operator	67	
3.1.8. Classical Angular Momentum	68	
3.1.9. Quantising the Angular Momentum	68	
3.1.9.2. Differential Operator in 3 Dimensions	69	
3.2. The Two-Dimensional Quantum Harmonic Oscillator	70	
3.2.1. The Plane Super-positioned Hamilton Operator	70	
3.2.2. The Angular Momentum Operator	70	
3.2.3. Ladder Operators of the Plane Quantum Mechanical Harmonic Circle Oscillator	71	
3.2.4. The Circular Rotating Oscillator Eigenvalues	72	
3.2.4.2. The Rotating Circle Oscillator in Polar Coordinates	73	
3.2.4.3. Annihilation and Creation Operators in a Polar Plane	74	
3.2.5. Ground State of the Circle Oscillator	75	
3.3. Excitation of the Plane Harmonic Circle Oscillator	78	
3.3.1. The First Excited States of the Circle Oscillator	78	
3.3.2. The Higher Excited States of the Circle Oscillator	80	

Research on the a priori of Physics *Geometric Critique of Pure Mathematical Reasoning*

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a priori of physics	Content	
3.3.2.1. The Possibility of the Second Excited States of the Circle Oscillator	80	
3.3.2.2. The Possibility of a Third and Higher Excited States of the Circle Oscillator	81	
3.3.3. The Plane Excited Circle Oscillator	81	
3.3.4. The Possible Excitation of a Circular Oscillator with \pm Signed Orientation.	82	
3.3.4.2. The Oscillation Freedom from Portable Energy as the concept of Rest Mass	83	
3.3.4.3. The Qualitative Unit of the Circle Oscillator <i>Entity</i>	84	
3.3.4.4. Polar Radial Distribution of the Angular Momentum Over the Circular Oscillator Plan	86	
3.3.4.5. The Area Distribution of the Action Over the Transversal Plane	86	
3.3.4.6. The Energy Intensity Momentum	87	
3.3.5. Frequency Scaling of the Circle Oscillator	88	
3.3.5.2. Examples of Commonly Used Reference Clocks Seen as One Circle Oscillator	88	
3.3.5.3. The Relative Reference for the Circle Oscillator and the Autonomous Norm	88	
3.3.5.4. Scaling of the Frequency Energy in The Propagation	89	
3.4. The Quantum Excited Direction	90	
3.4.1. The Direction of the First Excitation Described in Cylindrical Coordinates.	90	
3.4.1.2. Annihilation of an Excited Circle Oscillator	92	
3.4.1.3. Change of Direction of an Excited Circle Oscillator	92	
3.4.1.4. The Fundamental Substance for an <i>Entity</i> and the Extensive Difference	93	
3.4.1.5. The Substance of the Concept of a Photon	93	
3.4.2. The Linear Movement	94	
3.4.2.1. The Concept of the Straight Line, through what?	94	
3.4.2.2. The Unitary Direction as an Abstract Interpretation	95	
3.4.2.3. An Interpretation of the Angular Excited <i>Quantum</i>	96	
3.4.2.4. An Interpretation of the Excited <i>Direction</i>	96	
3.4.2.5. The Past Versus the Depth	97	
3.4.3. The Phase Angle and the Parameter Dependent States	99	
3.4.4. The Principle of Superposition	100	
3.4.4.1. The Simple Superposition of the Two Degenerated <i>subtons</i>	100	
3.4.4.2. The General Authentic Superposition in One <i>Direction</i>	101	
3.4.4.3. The Monochromatic Transversal Plane Wave	101	
3.4.4.4. The Amplitude for a Monochromatic Transversal Plane Wave	102	
3.4.5. Linearly Polarization	103	
3.4.5.1. Is the Idea of a Linearly Polarized ‘Photon’ an elementary particle?	103	
3.4.5.2. One <i>double±subton</i> Interpreted as a Progressive Wave	104	
3.4.5.3. Superposition of Linear Polarized <i>double±subtons</i>	105	
3.5. Modulation of a Quantum Mechanical Field	107	
3.5.1. The Macroscopic Modulation	107	
3.5.2. The Carrier	107	
3.5.2.2. Macroscopic Coherent Amplitude Modulation	110	
3.5.2.3. Phase Modulation	110	
3.5.2.4. Amplitude Modulation at Mutual Frequencies	112	
3.5.2.5. QAM Modulation	112	
3.5.2.6. The Two Helicities	113	
3.5.3. Elliptical Polarisation	113	
3.5.3.2. The Elliptical Polarized Monochromatic Transversal Plane Wave	113	
3.5.4. One <i>double±subton</i> as an Information q-bit Real	115	
3.6. The Cyclic Quantum Oscillator Idea	116	
3.6.1.2. The Direction	117	
3.6.1.3. The Spectrum in One <i>Direction</i>	118	
3.6.2. The Development of Entities in Physics	119	
3.6.3. Conclusion in Traditional Terms for the Concept of Time and Energy	121	
3.6.3.1. The Fundamental Quality of the Fundamental <i>Quantum</i>	121	

a priori of physics	Content
3.6.3.2. The Concept of Time as Complementarity to Frequency-Energy.	121
3.6.3.3. The Causal Action of Light Gives the Extension	121
3.6.3.4. Space	121
II. The Geometry of Physics	
4. The Linear Natural Space in Physics	123
4.1. The Linear Algebraic Space	124
4.1.1. The Abstract Linear Space, a Vector Space	124
4.1.1.1. Algebra of a Linear Spaces	124
4.1.1.2. The Dimensions of a Linear Algebra of a Linear Vector Space	124
4.1.1.3. Sum of Subspaces	125
4.1.1.4. The Simplest Linear Space for a <i>Quality</i> of Physics	125
4.1.1.5. Definition of One Dimension of First Grade	126
4.1.1.6. The Simplest Multidimensional Space	126
4.1.1.7. The Real Numbers as a Vector Space	126
4.1.1.8. The Reality of Time as a Vector Space	127
4.1.1.9. The Abstract 1-Dimensional Vector Space for Real Numbers	128
4.1.2. The Real Spatial Linear Vector Space	129
4.1.2.2. Multiple Linear Spatial Dimensions	129
4.1.3. The Vector Space of Complex Numbers	131
4.1.3.2. The Complex Scalar	132
4.1.3.3. A Multi-dimensional Complex Vector Space	132
4.1.4. Vector Spaces of Infinite Dimensions	133
4.1.4.2. The Vector Space of Fourier Integrals	134
4.1.5. Qualitative Substance of a Vector Space	136
4.1.5.1. Linear Relationship of Geometry	136
4.1.5.2. The Geometry	136
4.2. The Geometric Space \mathfrak{G}	137
4.2.2. The Dimensions and <i>Qualitative Grades</i> of Geometric Space	138
4.3. The Idea of a Point - in the Geometric Space \mathfrak{G}	139
4.3.1.1. No Extension. The Euclidean Elements:	139
4.3.1.2. The <i>Quality</i> of the Concept of Points	139
4.3.1.3. No <i>Quantity</i>	140
4.3.1.4. The Concept of the <i>Simplest Primary Quality</i>	140
4.3.1.5. Geometric Points are not objects	140
4.4. The Straight Line \mathfrak{L} in Geometry of Physical Space	141
4.4.1.1. Euclidean Elements for the Concept of a Straight Line	141
4.4.1.2. Additional Features –	141
4.4.2. The Concept of Geometric Vectors	144
4.4.2.2. Geometric Translation	144
4.4.2.3. The 1-vector Concept	144
4.4.2.4. Addition of Co-linear 1-vectors	145
4.4.2.5. Multiplication of 1-vector with a Real Scalar	145
4.4.2.6. The unit object for a linear <i>direction</i>	146
4.4.2.7. The Linear Extension from a 1-vector	146
4.4.2.8. The Parametric Development of a Straight Linear Ray	147
4.4.2.9. The Parametric Span of a Straight Line	147
4.4.2.10. Co-linear 1-vectors	147
4.4.2.11. The Spatial Line as a Real Linear Vector Space $\mathbb{R}e\mathbf{l}$	147
4.4.2.12. A 1-vector Intuited as a Translation	147
4.4.2.13. The Translation of a Geometric 1-vector	148

Research on the a priori of Physics *Geometric Critique of Pure Mathematical Reasoning*

Jens Erfurt Andresen
Edition 2, © 2020-22

a priori of physics	Content
4.4.3. The Straight Line Idealism	149
4.4.3.1. The Objective Reality of a Difference	149
4.4.3.2. The Concept of Simple Memory	149
4.4.3.3. The Simplest Subject	149
4.4.3.4. The Simplest Object	149
4.4.3.5. The 1-vector Concept as the <i>Primary Quality of First Grade</i> (<i>pqg-1</i>)	150
4.4.3.6. The Scalar as a Simple Pure <i>Quantity</i>	150
4.4.4. Relationship Between the Concepts of the 0-vector Scalar and the 1-vector	150
4.4.4.1. The scalar product between the co-linear 1-vectors	150
4.4.4.2. The Unitary Co-Linear Direction Vector and the Inverse Geometric 1-vector	151
4.4.4.3. The Zero-vector Representing All Points	151
4.4.4.4. The <i>First Grade Object</i> , a Geometric 1-vector → a Subject in a Substance as Idealism	151
4.4.4.5. The linear paper folding	152
5. The Geometric Plane Concept	153
5.1. The Geometric plane \mathfrak{P}	153
5.1.1.2. Additional A Priori Judgments to the Euclidean Plane Geometry	155
5.1.1.3. The Concept of an Angle	155
5.1.1.4. The Concept of Different Angles	156
5.1.1.5. The Concept Circular Arc of Angle	156
5.1.1.6. The Concept of the Primary Quality of Second Grade (<i>pqg-2</i>)	156
5.1.1.7. The Quantity of an Angle	157
5.1.1.8. Addition of angular quantities	158
5.1.1.9. The Angular Quantity as a Sector Area	158
5.1.1.10. The perpendicular tangent to the circle in the plan	159
5.2. The Plane Geometric Algebra	160
5.2.1. Addition of 1-vectors in the Plane	160
5.2.1.2. The Additive Algebra for Vector Spaces of Geometric Substance	160
5.2.1.3. The Linear Span of the Geometrical Plane from 1-vectors	161
5.2.1.4. Bilinear Forms	161
5.2.1.5. Clifford Algebra	162
5.2.1.6. The Combined Linear Space	162
5.2.2. The Geometric Algebra with Direct Product	162
5.2.2.2. The Inner Symmetric Product of Geometric Vectors	163
5.2.2.3. The Scalar Product	163
5.2.3. The Inner Product Algebra	165
5.2.4. The Geometric Product	165
5.2.5. The Outer Product of Geometric Vectors	166
5.2.5.2. The Bivector Orientation as a Sequential Operation	166
5.2.5.3. Bivector <i>Quantity</i> and Form Structure <i>Quality</i> Direction	167
5.2.5.4. A Bivector Multiplied by a 1-vector	168
5.2.5.5. The Category a Bivector	168
5.2.6. The Orthonormal Bivector Object as a Unit for the Circular Rotation in a Plane	169
5.2.6.2. The Hodge Coordinate for the Pseudoscalar Span in the \mathfrak{P} plane Concept	169
5.2.6.3. Operations with the Unit Bivector Pseudoscalar for a Plane	169
5.2.6.4. The Form Structure of the Plane Subject i has Arbitrary Shaped Objects	170
5.2.6.5. The Unit-bivector i Multiplied by a 1-vector	171
5.2.7. The Unitary Rotor Operator as a Concept	171
5.2.7.1. The Geometric Rotor in the Euclidean Plane	171
5.2.7.2. Intuition of the Bivector and the Scalar for the Interpretation of a Rotation	172
5.2.7.3. The Plane-segment Unit	172
5.2.7.4. Rotor Independence of any <i>pqg-1</i> <i>Direction</i> in a Plane <i>Quality</i>	173

a priori of physics	Content
5.2.8. The Exponential Function with one plane <i>direction</i> Bivector as Argument	173
5.2.8.2. The Product of Rotors	174
5.2.8.3. The Rotor Product With a 1-vector	174
5.2.8.4. A Bivector is Self-identic by a Rotation in its own Plan	174
5.2.8.5. The Simple 1-rotor Algebra	174
5.2.9. A Complex <i>Quantity</i> in Space Called a Plane Spinor	175
5.2.9.1. The Complex <i>Quantity</i> as a Geometric Product of Two 1-vectors	175
5.2.9.2. The 1-Spinor as a Generator Radius-vector Multiplied to a Basis 1-vector	176
5.2.10. The Circle Oscillating 1-rotor in Development Action as One Plane Substance	176
5.3. The Rotor Concept as the Primary Quality of Even Grades (pqg-0-2).	177
5.3.1.2. Two Points Define the Primary Quality of First Grade (pqg-1)	178
5.3.1.3. The Orthonormal Basis for Circular Plane Symmetry	179
5.3.2. The Geometric Algebraic Complex Plane	179
5.3.2.2. The Polar Coordinates of a Plane Idea	181
5.3.2.3. Primary Qualities of the plane Concept	181
5.3.2.4. The Area Concept as Generator for Circle Rotation.	181
5.3.3. The Cartesian Coordinates for the Plane idea	183
5.3.3.2. The Cartesian Coordinate System and the Plane Pseudoscalar Concept	184
5.3.3.3. The Parity Inversion of the 2-dimensional Descartes Extension Coordinate	184
5.3.3.4. The Extension Grade One Parity Inversion of Scalars and Bivectors	185
5.3.4. The Qualities of the Geometric Algebra of the Plane	186
5.3.4.1. The Rotation Symmetrical Plane Concept	186
5.3.4.2. The 2-dimensional Plane	186
5.3.4.3. The Orthonormal Reference for the 2-dimensional Plane	187
5.3.5. A Grade-2 Object, 2-blade Bivector → a Subject in a Substance as Idealism	188
5.3.6. The Inadequate <i>Cartesian x, y</i> Coordinate System	189
5.3.7. The 1-vector Product Complex Quantity and Polar Coordinates of Plane Concept	190
5.3.7.2. A plane Rotation and Dilation added to a Translation	191
5.3.8. The Magnitude of a Multivector	192
5.4. Transformation of Geometric 1-vectors in the Euclidean plane	193
5.4.1. Parallel Translation of a Vector	193
5.4.2. Reflections	193
5.4.2.1. Reflection in a Geometric 1-vector	193
5.4.2.2. Reflection Along a Geometric 1-vector	194
5.4.2.3. Reflection Through a Non-normalized 1-vector	194
5.4.3. The Projection Operator From one 1-vector to Another 1-vector	194
5.4.4. Reflection in a Plane Surface as a Physical Process	195
5.4.5. Rotation Inside the Same Plane <i>Direction</i>	196
5.4.5.2. The Half-Angle Rotor of an Euler Rotation	197
5.4.5.3. The Idea of an Active Rotation	197
5.4.5.4. The Invariant Direction of a Rotor	197
5.4.5.5. The Duality of <i>Direction</i>	197
5.5. Inherit Quantities of the Algebra for the Euclidean Geometric Plane Concept	199
5.5.2. The Auto Product Square in the Euclidean plane	199
5.5.3. The Nilpotent Operation	199
5.5.3.2. The Spanned Spaces of Nilpotence Zero Divisors	201
5.5.3.3. The Spanned Spaces of Mutual Annihilation Zero Divisors	201
5.5.4. The Idempotent Operation	202
5.5.4.2. The Paravector Concept	203
5.5.4.3. The Projection of a Paravector on Its Idempotent Basis	203
5.5.4.4. Non Measurable Fictive Magnitude of Paravectors	204

Research on the a priori of Physics Geometric Critique of Pure Mathematical Reasoning

Edition 2, © 2020-22

a priori of physics	Content
5.6. The Real Matrix Representation for the Plane Concept	205
5.6.1. The Fundamentals of Matrices in a Plane Algebra \mathcal{G}_2	205
5.6.1.1. Matrices for a Cartesian 1-vector Concept for a Euclidean plane \mathbb{R}^2	205
5.6.1.2. Examples of Matrices of Geometric Multivectors	205
5.6.1.3. The Matrices of the Geometric Algebra $\mathcal{G}_2(\mathbb{R})$	206
5.6.1.4. An Example of a Matrix in $\mathcal{G}_2(\mathbb{R})$	208
5.7. Plane Concept Idea of a Non-Euclidean Clifford Algebra	209
5.7.1. Plane Geometric Clifford Algebra with Minkowski Signature for Measure Information	209
5.7.1.2. An Entity Seen from the External Far Distant as a Null Signal	211
5.7.1.3. The \mathcal{B} -bivector as an Information Signal	212
5.7.2. The Traditional Display of the Minkowski \mathcal{B} -plane	212
5.7.2.2. The Traditional Display of the Minkowski space	214
5.7.2.3. Minkowski Space with Display of Three Extension Dimensions	216
5.7.3. The Paravector Space and the Minkowski 1-vector in STA	217
5.7.4. Lorentz Rotation in the Minkowski \mathcal{B} -plane	218
5.7.4.2. The Lorentz Transformation of a paravector	219
5.7.4.3. The Lorentz boost	220
5.7.4.4. The Doppler Effect of the Lorentz Boost	221
5.7.4.5. The Space-Time Algebra STA from a 4-dimensional 1-vector basis	222
5.7.5. The planes of Space-Time Algebra and the Euclidean Cartesian plane	223
5.7.5.1. Founding Summary of Minkowski Space with Euler and Lorentz Rotations	223
5.7.5.2. Mapping Operation Between STA planes and the Euclidean Cartesian plane	223
5.7.5.3. Exponential Function in the Plane Concepts	225
5.8. The Exponential Function of Arbitrary Multivectors	226
5.8.1.1. The Hyperbolic Functions of Multivectors	226
5.8.1.2. The cosine and sine Functions of Arbitrary \mathcal{G}_n Multivectors	226
5.8.2. Exponential and Hyperbolic Functions in the Plane Concept	227
5.8.2.2. The 1-Spinor in the Euclidean Cartesian plane	227
5.8.2.3. The Lorentz 1-Spinor in the Line <i>Direction</i> Paravector Space	227
5.8.2.4. The Lorentz 1-Spinor in the Minkowski \mathcal{B} -plane	228
5.8.3. A Philosophical Conclusion on All This Exercise	228
5.9. Concluding Summary on the Algebra for the Geometric Plane Concept	229
5.9.1. The Euclidean Plane Concept	229
5.9.2. The Non-Euclidean Plane Concept	229
5.9.3. General Exponential Series	229
6. The Natural Space of Physics	231
6.1. The Classic Geometric Extension Space 3	231
6.1.2. Additional A Priori Judgments to the Euclidean Stereo Space Geometry	233
6.1.2.1. The concept of Spatial Angular Structure	234
6.1.3. The Euclidean 1-vector Space for Natural Space.	234
6.1.3.2. Covariant Cartesian Coordinates	234
6.1.3.3. Contravariant Coordinates	234
6.1.3.4. The Classical Cartesian Coordinate System for Position Points in 3-Space	235
6.1.4. A Curiosum, the Concept of a Tetraon in a Tetrahedron	235
6.1.4.2. The Six Bivector Angular Planes of the Regular Tetraon	236
6.2. The Geometric Algebra of Natural Space	237
6.2.1. Addition of Bivectors	237
6.2.2. The Trivector concept	237
6.2.2.2. The Magnitude of a Trivector	238

a priori of physics	Content
6.2.3. The Trivector and the 3-space Chiral Pseudoscalar	238
6.2.3.2. The Cartesian Orthonormal Basis 1-vector Set of <i>Primary Quality of Third Grade</i> (p_{qg-3})	238
6.2.3.3. The Hodge Coordinate for the Pseudoscalar Span in 3 Space	240
6.2.4. The Geometric Algebraic Basis of a 3-space	240
6.2.4.1. The 1-vector Basis	240
6.2.4.2. The Transversal Bivector Basis as a Dual Basis of a 3-Space	240
6.2.5. The Geometric Algebra for Euclidean 3-space	241
6.2.5.2. The Commuting Pseudoscalar for 3-Space	241
6.2.5.3. The Mixed Product Between a 1-vector and a Bivector	241
6.2.5.4. The Simple Product of Three 1-vectors	242
6.2.5.5. Even and Odd Multivector in general	242
6.2.5.6. Product of Two Bivectors	242
6.2.5.7. Commutator Product of Multivectors in Geometric Algebra	243
6.3. The 3-space Structure <i>Quality</i> Described by Multivectors	245
6.3.2. The Even and the Odd Geometric Algebra	245
6.3.3. Operational Structure of the Trivector Chiral Volume Pseudoscalar \mathbf{i} of 3 Space	246
6.3.4. Rotation in 3-space	247
6.3.5. Multiplication Combination of Rotors	248
6.3.5.1. The Unitary group $U(1)$ for Plane Combination of Rotors	248
6.3.5.2. Multiplication Combination of <i>Direction</i> Different 1-rotors in 3-space	248
6.3.5.3. Comment on the Ontology of <i>Directions</i> and Possibility of Location	249
6.3.5.4. The Abstract Generalised Rotor Form	249
6.3.6. Rotation of Multivectors	250
6.3.7. Framing a Field for Geometric Algebra in 3-space	250
6.4. The Geometric Clifford Algebra	251
6.4.1.1. The Quadratic Form in general	251
6.4.1.2. The Clifford Algebra for Complex Numbers	251
6.4.1.3. The simple Euclidean Plane Geometric Clifford Algebra $\mathcal{G}_{2,0}$	251
6.4.1.4. Plane Subjects in Euclidean 3 Space Clifford Algebra $\mathcal{G}_{3,0}$	252
6.4.2. The Pauli Basis for combined direction structures of 3-space	253
6.4.2.2. The Pauli Matrices as generator operators	253
6.4.2.3. The Pauli Basis Generated from 1-vectors	253
6.4.3. The Quaternion Picture	254
6.4.3.1. An Anti-Euclidean Geometric Algebra $\mathcal{G}_{0,2}$	254
6.4.3.2. Quaternions \mathbb{H}	255
6.4.4. The Verson as a <i>Direction</i> 2-Rotor	256
6.4.4.2. The Traditional View on the Spatial Rotation Problem	256
6.4.4.3. The Four Real Scalar Coordinates for the Verson Quaternion <i>Direction</i>	257
6.4.5. The Two Parameter Quaternion 2-Spinor	260
6.4.5.2. Two State Observable of a Fundamental <i>Entity</i> in 3 Space	261
6.4.6. Euler Angles for a Rotor in 3-space	263
6.4.6.2. The Other Euler Angle Sequence	266
6.4.7. The Transversal Bivector Idea Dual to a 1-vecetor Foundation for Rotations	267
6.4.7.1. The two Orthogonal Rotors as Generators for a Local Entity	267
6.4.8. The Rotated Direction in 3-space	268
6.4.8.2. Rotation of a Chosen Direction in 3 Space	268
6.4.9. Oscillations in 3-space	270
6.4.9.1. Review of the Quantum Mechanical Circle Oscillator	270
6.4.9.2. Multi Excitations of Angular Momentum Internal in One <i>Entity</i>	271
6.4.9.3. Intuition of Two Perpendicular Exited Circle Oscillators Inside one Entity	271
6.4.9.4. Breaking the Spherical Symmetry into One <i>Direction</i>	273
6.4.9.5. External and Internal <i>Directions</i> of an <i>Entity</i> in 3 space	274

a
Research on the a priori of Physics
Geometric Crystals as Being Mathematical Processing
Jens Erfurt Andresen
Edition 2 © 2020 22

priori of physics	Content	
6.4.9.6. Complementarity	274	
6.5. The Angular Momentum in 3 Space	275	
6.5.1. One Quantum of Angular Momentum	275	
6.5.1.1. 1-vector Angular Momentum	275	
6.5.1.2. The Bivector Angular Momentum	275	
6.5.1.3. The Angular Momentum Operator	276	
6.5.1.4. Excitation of Angular Momentum in three directions of 3 Space	276	
6.5.1.5. Commutator Products of Angular Momentum Bivectors and their Dual 1-vectors	277	
6.5.2. The Commutator Relations in Geometric Algebra for Angular Momenta	278	
6.5.2.2. Orthogonal Chirality of Angular Momenta in the Even $\mathcal{G}_{0,2}$ Geometric Algebra	279	
6.5.3. Thoughts About Symmetry Braking and Quantisation of <i>Direction</i> in 3-space	280	
6.5.4. Chiral Orientation of Combined Angular Momentum Bivectors and Dual 1-vectors	282	
6.5.4.2. The Total Angular Momentum of a local <i>entity</i> in 3 space	282	
6.5.5. The Quantum Stats of the Locally Combined Angular Momentum	284	
6.5.5.2. The Quantum Ladder Step Operations in 3 space	285	
6.5.5.3. Excitation of Angular Momentum in 3 space	286	
6.5.6. The Spin½ of a <i>Directional Entity</i> of Locality in 3-space	287	
6.5.6.1. The fundamental first excitation of 3 space	287	
6.5.6.2. Symmetry Braking of the Half Spin $\Psi_{\frac{1}{2}}$ Entity,	287	
6.5.7. Synthesis of the Locality of <i>Entities</i> in 3-space	290	
6.5.8. The Idea of One Spin½ <i>Entity</i> in Physical 3-space	291	
6.5.8.1. The Extension Distribution of the Wavefunctions	291	
6.5.8.2. The Internal Oscillating Wavefunction Components	291	
6.5.8.3. The Oscillator Fluctuating Versor Wavefunction for the <i>Entity</i> $\Psi_{\frac{1}{2}}$ in 3 space	292	
6.5.8.4. Versor Eigenwave-Function as for the Stationary State Existence of an <i>Entity</i> $\Psi_{\frac{1}{2}}$	294	
6.5.8.5. The One Eigen-Versor Separated in 1-Spinor Angular-Momentum-Wavefunctions	294	
6.5.8.6. The Versor Quaternion Spin½ <i>entity</i> $\Psi_{\frac{1}{2}}$	295	
6.5.8.7. Eight Qualitative States of a Spin½ <i>Entity</i> $\Psi_{\frac{1}{2}}$ in 3 Space	297	
6.5.9. The Internal Auto Synchronisation of an Indivisible-Atomic-Elementary <i>Entity</i>	298	
6.5.10. A Hypothetic Thought Intuition of One Four-Angular-Momenta Function	300	
6.5.10.2. The Autonomous Regular Tetraon Basis for Four-Angular-Momenta Function	300	
6.5.10.3. An Autonomous Four-Angular-Momenta Function	301	
6.5.10.4. Information from one Tetraon Symmetric <i>entity</i> $\Psi_{\frac{1}{2}}$	301	
6.5.10.5. The Regular Tetraon Symmetry <i>Quantity Cargo</i> of One Indivisible Spin½ <i>entity quality</i>	303	
6.5.10.6. The Freedom of the Tetraon Angular Composition in one Fermion	304	
6.5.10.7. Multiple Circular Oscillators as Structure Form <i>Qualities</i> for Spin½ Fermions	304	
6.5.11. The Full Geometric Algebra $\mathcal{G}_3(\mathbb{R})$ for Spin½ Fermions	305	
6.5.11.2. The Non Quaternion <i>Grades</i> ≤ 3 for of Indivisible <i>Entities</i> $\Psi_{\frac{1}{2}}$ in 3 Space	305	
6.5.12. The strange Intuition of Locality in 3-space	307	
6.5.13. The Fundamental Concept of <i>Direction Locality</i> in Space	308	
6.6. Identical <i>Entities</i> in 3-Space	309	
6.6.1.1. Classification of Fermions in the Structure of 3-space	309	
6.6.1.2. The Simple Versor Form and the $SU(2)$ Algebra of the Complex 2×2 Matrix Group	309	
6.6.1.3. Three Linear Independent Internal Components of Angular Momenta of Fermion Structure	310	
6.6.2. Fermions have by Tetraon Symmetry from the Platonic Tetrahedron Idea	310	
6.6.3. The <i>Categorical Quality</i> of Spin½ Fermions in 3-space	311	
6.6.3.2. The Categorical Classification of Identical Spin½ Fermions in 3 Space	312	
6.6.4. The Idea of One Interaction <i>Direction</i>	312	
6.6.4.1. The One Whole <i>Quantity Charged</i> Fermion	312	
6.6.4.2. The Field of Information About one Charge	313	
6.7. Multiple Numbers of Spin½ Fermions	314	

6.7.1. External Qualities of Charged Fermions	314
6.7.1.1. Identical Charged Fermions	314
6.7.1.2. Opposite Charge Fermions	314
6.7.1.3. The Bohr-Rutherford Atomic Model	314
6.7.2. Mutual Exclusive Extension of Fermions	314
6.7.2.2. The Pauli Exclusion Principle	314
6.7.3. Orbital Angular Momentum	315
6.7.3.1. The Squared Perpendicular Part of Orbital Integer Quantum Number Excitation	315
6.7.3.2. The Orbital Angular Momentum of Multiple Spin <i>entities</i> $\Psi_{1/2}$	315
6.7.3.3. Atomic Shells and Subshells	316
6.7.3.4. Categories of Atomic Quantum Numbers	316
6.7.3.5. Atoms in Practise	317
6.7.4. The Spatial Wavefunction Probability Distribution Structure of atoms	318
6.8. Conclusion on Topological Structure of 3-space Founded in Physics	319
6.8.1. Conclusion on the Local Situated Topological Structure of Natural 3-space	319
6.8.1.2. The Rest Mass Problem of Fermions	320
6.9. External Relations Between Fermions in an Extended Space of Information	321
6.9.1. A New Break Through for Physics Foundation in Human Knowledge of Nature	321
6.9.1.1. Extension of Space by Grassmann Exterior Products	323
III. Space-Time Relations in Physics	324
7. Relation Space of Physics	325
7.1. Vision of Relations by the Development Concept of Physics in Nature	325
7.1.2. The Space-Time as Development Information of Extension Relation is Called \mathfrak{D} -space	326
7.1.3. The Full Geometric Space-Time Algebra $\mathcal{G}_{1,3}(\mathbb{R})$ for Physical Relations in \mathfrak{D} -space	326
7.1.3.2. The Multivector Decomposition in a Sum of Grades for \mathfrak{D} -space of Physics	328
7.1.3.3. Conjugation in Space-Time Algebra	328
7.1.3.4. Reversion of the Odd Chirality Volume Pseudoscalar i in the $\mathcal{G}_3(\mathbb{R})$ Algebra for 3-space	329
7.1.3.5. Reversion of the Even Helicity Pseudoscalar i in the Algebra $\mathcal{G}_{1,3}(\mathbb{R})$ for \mathfrak{D} -space	329
7.1.3.6. The STA Bivector Field F of STA in the Information Development \mathfrak{D} -space of Physics	330
7.1.3.7. Difference Between the Pseudoscalar Concepts for \mathfrak{D} -space and 3-space	330
7.1.4. The Odd and Even Part of the Geometric Space-Time Algebra $\mathcal{G}_{1,3}(\mathbb{R})$	331
7.1.4.2. The Transcendental Ignorance of the Odd part of the Geometric Algebra for \mathfrak{D} -space	331
7.1.4.3. The Even Closed Geometric Algebra of \mathfrak{D} -space	331
7.1.4.4. The Even Geometric Spinor Quality in the Algebra $\mathcal{G}_{1,3}^+(\mathbb{R})$ of \mathfrak{D} -space	331
7.1.4.5. The Composite Rotor Structure in \mathfrak{D} -space	333
7.1.5. Stop this volume.	333
7.2. For inspiration to further work	334
7.3. Further Work on these Issues is Postponed	335
7.3.1. The Foundation of Physics is Nature Itself	335
7.3.1.1. Nature per se as a priori of Physics	335
Epilogue	337
8. Problematisation of the Philosophical approach	337
References	339
List of figures	341
Lexical Index	345

Research on the a priori of Physics

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