This multivector we can use as an operator, which we then call a *spinor* for $\mathfrak{D}$-space of physics. To make an observable we construct a measure by the bilinear Clifford quadratic form $\psi\widetilde{\psi}$. This product of even elements is even and all resulting bivectors cancel in pairs using (7.38). Therefore, only parts of scalars and pseudoscalars contribute to $\psi\widetilde{\psi} = \alpha_s + \upsilon_i \imath$, with $\alpha_s, \upsilon_i \in \mathbb{R}$. These two real components $\alpha_s = \alpha^2 - \upsilon^2 - F^2$ and $\upsilon_i = 2\alpha\upsilon$ we can transform to a *probability-density-amplitude*[411] $\rho = \alpha_s^2 + \upsilon_i^2 \in \mathbb{R}$, together with a trigonometric *abstract angle*

(7.58) $$\beta = \cos^{-1}(\alpha_s/\rho) = \sin^{-1}(\upsilon_i/\rho) \quad \in \mathbb{R}.$$

Then the observable measure is written in geometric algebraic multivector form[412] for $\mathfrak{D}$-space

(7.59) $$\psi\widetilde{\psi} = \rho e^{\imath\beta} = \rho(\cos\beta + \imath\sin\beta) \qquad \in \mathcal{G}_{1,3}^{+}(\mathbb{R}),$$

where $\rho \geq 0$ is a real scalar dilation factor. Special when $\beta = 0$, or just $\beta = n\pi$, $n \in \mathbb{Z}$ (7.59) is a real scalar $\langle\psi\widetilde{\psi}\rangle_0$, but else it is a scalar + pseudoscalar multivector

(7.60) $$\psi\widetilde{\psi} = \langle\psi\widetilde{\psi}\rangle_0 + \langle\psi\widetilde{\psi}\rangle_4.$$

A naïve interpretation is the scalar part has the potential to be a traditional observable real ***quantity***, and the pseudoscalar part has the opportunity to carry a helicity $\mathfrak{D}$-volume information ***quantity***.[413]
Anyway, $\widetilde{\imath} = \imath$ and (7.59) are reversing invariant as (7.38) which is $\widetilde{\psi\widetilde{\psi}} = \psi\widetilde{\psi} = \widetilde{\rho e^{\imath\beta}} = \rho e^{\imath\beta}$.[414]
Hereby we can write $(\psi\widetilde{\psi})^{½} = (\rho^{½}e^{\imath½\beta})^{\sim} = \rho^{½}e^{\imath½\beta}$, and therefore divide (7.57) by this giving

(7.61) $$\left.\begin{aligned} R &= \psi\,(\psi\widetilde{\psi})^{-½} \\ \widetilde{R} &= \widetilde{\psi}\,(\psi\widetilde{\psi})^{-½} \end{aligned}\right\} \in \mathcal{G}_{1,3}^{+}(\mathbb{R}) \subset \mathcal{G}_{1,3}(\mathbb{R}),$$

that is a general rotor fulfilling the *unitary* spinor condition

(7.62) $$R\widetilde{R} = \widetilde{R}R = 1.$$

Hereby we now can write the spinor (7.57) in the Hestenes canonical form

(7.63) $$\left.\begin{aligned} \psi &= \rho^{½}e^{\imath½\beta}R = \left(\rho e^{\imath\beta}\right)^{½}R \\ \widetilde{\psi} &= \rho^{½}e^{\imath½\beta}\widetilde{R} = \left(\rho e^{\imath\beta}\right)^{½}\widetilde{R} \end{aligned}\right\} \in \mathcal{G}_{1,3}^{+}(\mathbb{R}) \subset \mathcal{G}_{1,3}(\mathbb{R}).$$

We will comment on the reversion invariant even multivector factor in $\mathcal{G}_{1,3}^{+}(\mathbb{R})$ of $\mathfrak{D}$-space

(7.64) $$\mathcal{D} = (\psi\widetilde{\psi})^{½} = \left(\rho e^{\imath\beta}\right)^{½} = \rho^{½}(\cos ½\beta + \imath\sin ½\beta)$$

For this combined ***quantity*** of real-scalar and pseudoscalar, we have the ***quality pqg***-0 and ***pqg***-4

(7.65) $$\widetilde{\mathcal{D}} = \mathcal{D}$$

of reversion invariance of both the ***direction-free*** scalar and the *helicity* $\mathfrak{D}$*-volume pseudoscalar* of primary quality of ***fourth grade direction***. It would be strange if this quantity structure does not emerge from the quality of Nature per se (itself).
This has to be investigated further! Later on?
We can now formulate any even multivector in $\mathcal{G}_{1,3}^{+}(\mathbb{R})$ as a product

(7.66) $$\psi = \mathcal{D}R, \quad \text{where} \quad \widetilde{\mathcal{D}} = \mathcal{D} \quad \text{and} \quad R\widetilde{R} = \widetilde{R}R = 1.$$

As Hestenes [24] call attention to this rotor R perform Lorentz invariant rotations of form

(7.67) $$\underline{R}: \quad X \longrightarrow X' = RX\widetilde{R}.$$

[411] Although the similarity to the form in (6.149) with oscillation amplitude, it is not the same, here rather mass-energy density.
[412] Here we refer to the generalised concept described in § 5.8.1.2 where the unit pseudoscalar is $I^2 = -1 \Rightarrow I \sim \sqrt{-1}$: $\imath \leftrightarrow I$.
Here we also remember that $e^{\imath\beta}$ as cos and sin has no connection to the complex number plane but acts along *the helicity volume* with ***primary quality of grade four direction*** $\imath$ and defined by exponential series as (5.388), (5.396) and (5.397).
Anyway, $\cos\beta$ and $\sin\beta$ are real scalar-valued $\in \mathbb{R}$, so the result (7.59) is a scalar and a pseudoscalar.
[413] This must be investigated further in the future.
[414] This is essentially different from reversing complex scalar numbers $\widetilde{e^{i\beta}} = (e^{i\beta})^{\dagger} = (e^{i\beta})^{*} = e^{-i\beta} \in \mathbb{C}$, where $\widetilde{i} = -i \in \mathbb{C}$, $\beta \in \mathbb{R}$.

7.1.4.5. The Composite Rotor Structure in $\mathfrak{D}$-space

We will take a closer look at the rotor part R of (7.66) by the knowledge from Table 5.3, where we have both the Lorentz hyperbolic rotor in a Minkowski $\mathcal{B}$-plane ($\mathcal{B}^2 = 1$)

(7.68) $$U_{\zeta,k} = e^{½\zeta\mathcal{B}_k},$$ or just in one chosen information $\mathcal{B}$-plane ***direction*** $U_{\zeta,\mathcal{B}} = e^{½\zeta\mathcal{B}}$,

and the Euclidean Euler angle rotors, e.g.,

(7.69) $$U_{\phi} = e^{\gamma_1\gamma_2½\phi} = e^{\boldsymbol{i}_3½\phi}.$$

That idea concerns three linear independent (orthogonal) oscillating rotation planes of locality

(7.70) $$U_{\phi_k,k} = e^{\boldsymbol{i}_k½\phi_k}, \quad k=1,2,3;\ \not\Sigma.$$

Thought interconnected as we best represent by the versor quaternion principal rotor (6.145) etc. $U = u_0 + u_1\boldsymbol{i}_1 + u_2\boldsymbol{i}_2 + u_3\boldsymbol{i}_3$. That as fermions spin½ particles with spin in one ***direction***, e.g., a spin bivector can be given $S_{½} = \frac{1}{2}\hbar\boldsymbol{i}_3$ from a chosen unit bivector $\boldsymbol{i}_3$ for the spin ***direction***.(see II. 6.5)
For spin one information subtons we can use (7.69) where the spin bivector is chosen $S_1 = \hbar\boldsymbol{i}_3$.

7.1.5. Stop this volume.

While waiting on the next volume of this book the inpatient reader is encouraged to study David Hestenes 2010 [24] further 2019 [25], [26] and to background [6], [23], [10], etc. and Doran & Lasenby [18], [22], etc.

For further inspiration see the figures on the next page: