calculations we try to avoid specifying such a basis until we need an objectification first by γ_0 then some extension direction. Anyway, the STA idea is founded on the orthonormal $\{\gamma_0, \gamma_1, \gamma_2, \gamma_3\}$.

We see that the Minkowski $\mathcal{B}$-bivector for the line-extension-like information (7.5) $\sigma_k \overset{\leftarrow}{=} \gamma_k \wedge \gamma_0$ has (space) Descartes *extension parity inversion* $\overline{\sigma_k} = -\sigma_k \overset{\leftarrow}{=} \overline{\gamma_k} \wedge \overline{\gamma_0} \in \mathcal{G}_{1,3}(\mathbb{R})$, and $\widetilde{\sigma_k} = -\sigma_k$. While the angular-area-extension-like information bivector (7.6) $\boldsymbol{i}_k \overset{\leftarrow}{=} \gamma_i \wedge \gamma_j$ is *parity inversion invariant* $\overline{\boldsymbol{i}_k} = \boldsymbol{i}_k \overset{\leftarrow}{=} \overline{\gamma_i} \wedge \overline{\gamma_j} = \gamma_i \wedge \gamma_j$, of course, not reversion invariant $\widetilde{\boldsymbol{i}}_k = -\boldsymbol{i}_k \overset{\leftarrow}{=} \gamma_j \wedge \gamma_i$.[410]

Here it is urgent to note $\overline{\gamma_k \wedge \gamma_0} = -\gamma_k \wedge \gamma_0 = \gamma^k \wedge \gamma^0$.

In general, we write the reciprocal bivector basis as $\gamma^{\mu\nu} = \gamma^\mu \gamma^\nu = \gamma^\mu \wedge \gamma^\nu$, $for\ \nu \neq \mu = 0,1,2,3$

(7.43) $$\{\gamma^{10}, \gamma^{20}, \gamma^{30}, \gamma^{23}, \gamma^{31}, \gamma^{12}\} = \{\gamma_{01}, \gamma_{02}, \gamma_{03}, \gamma_{23}, \gamma_{31}, \gamma_{12}\} = \{\overline{\gamma_\mu \wedge \gamma_\nu}\}$$

$$= \{\gamma_{01}, \gamma_{02}, \gamma_{03}, \gamma_{23}, \gamma_{31}, \gamma_{12}\} = \{-\gamma_{10}, -\gamma_{20}, -\gamma_{30}, -\gamma_{32}, -\gamma_{13}, -\gamma_{21}\}$$

The trivector basis is just $\{\boldsymbol{i}\gamma_\mu\}$ and its reciprocal $\{\boldsymbol{i}\gamma^\mu\}$ due to (7.15) and (7.40).

7.1.3.6. The STA Bivector Field *F* of STA in the Information Development $\mathfrak{D}$-space of Physics

We recall the stationary Pauli Basis (6.119) for the geometric algebra $\mathcal{G}_3(\mathbb{R})$ for $\mathfrak{Z}$-space

(7.44) $$\{1, \sigma_1, \sigma_2, \sigma_3, \ \boldsymbol{i}_1 = \boldsymbol{i}\sigma_1 = \sigma_3\sigma_2, \ \boldsymbol{i}_2 = \boldsymbol{i}\sigma_2 = \sigma_1\sigma_3, \ \boldsymbol{i}_3 = \boldsymbol{i}\sigma_3 = \sigma_2\sigma_1, \ \boldsymbol{i} := \sigma_3\sigma_2\sigma_1\}, \ \gamma_0\,\gamma_0$$

Right multiplying this with the information development measure unit γ_0 we get the odd *basis* $\mathcal{G}^-_{1,3}(\mathbb{R}) \subset \mathcal{G}_{1,3}(\mathbb{R})$ for STA in $\mathfrak{D}$-space concerning 1-vectors and trivectors

(7.45) $$\{\gamma_0, \ \gamma_1, \ \gamma_2, \ \gamma_3, \quad \gamma_3\gamma_0\gamma_2, \quad \gamma_1\gamma_0\gamma_3, \quad \gamma_2\gamma_0\gamma_1, \quad -\gamma_3\gamma_2\gamma_1\} \ ,\gamma_0$$

Right multiplying once again with the unit measure γ_0 we get an even basis $\mathcal{G}^+_{1,3}(\mathbb{R})$ for STA

(7.46) $$\{1, \ \gamma_1\gamma_0, \ \gamma_2\gamma_0, \ \gamma_3\gamma_0, \quad \gamma_2\gamma_3, \quad \gamma_3\gamma_1, \quad \gamma_1\gamma_2, \quad \gamma_0\gamma_3\gamma_2\gamma_1\}$$

To explicit this six element basis with bivectors in the even algebra $\gamma_0 \subset \mathcal{G}_{1,3}(\mathbb{R})$ we write

(7.47) $$\{1, \ \gamma_{10} = \gamma_1 \wedge \gamma_0, \ \gamma_{20} = \gamma_2 \wedge \gamma_0, \ \gamma_{30} = \gamma_3 \wedge \gamma_0, \ \gamma_{23} = \gamma_2 \wedge \gamma_3, \ \gamma_{31} = \gamma_3 \wedge \gamma_1, \ \gamma_{12} = \gamma_1 \wedge \gamma_2, \ \boldsymbol{i}\}$$

We note that $\gamma_{\mu\nu} = \gamma_\mu\gamma_\nu = \gamma_\mu \wedge \gamma_\nu$, for $\nu \neq \mu$ are the bivector basis $\{\gamma_\mu \wedge \gamma_\nu\}$ for $F = \langle X \rangle_2 \in \mathcal{G}_{1,3}(\mathbb{R})$.

For a general bivector field $F \in \mathcal{G}_{1,3}(\mathbb{R})$, we write the expansion

(7.48) $$F = \tfrac{1}{2}F^{\mu\nu}(\gamma_\mu \wedge \gamma_\nu) = \tfrac{1}{2}F^{\mu\nu}\gamma_\mu\gamma_\nu = \tfrac{1}{2}F^{\mu\nu}\gamma_{\mu\nu} \in \mathcal{G}^+_{1,3}(\mathbb{R}),$$

with six real scalar coordinate *components*

(7.49) $$F^{\mu\nu} = (\gamma^\nu \wedge \gamma^\mu)\cdot F = (\gamma^\nu\gamma^\mu)\cdot F = \gamma^{\nu\mu}\cdot F \quad \in \mathbb{R}, \qquad \text{as [24]p.2(13),}$$

with reason, coordinates after 1-vector ***directions***: first γ^μ, then γ^ν, as $\gamma^\nu\cdot(\gamma^\mu\cdot F) = (\gamma^\nu \wedge \gamma^\mu)\cdot F$.

In the matrix tradition $F^{\mu\nu}$ form an 4×4 antisymmetric contravariant matrix as a rank-2 tensor.
In Space-Time Algebra it is a bivector field that can be resolved in six independent components.
The $\frac{1}{2}$ in (7.48) comes from the anti-symmetry in the sum $\frac{1}{2}\sum_{\mu,\nu=0}^{3} F^{\mu\nu}\gamma_{\mu\nu}$, (12 terms and 4 zeros).

7.1.3.7. Difference Between the Pseudoscalar Concepts for $\mathfrak{D}$-space and $\mathfrak{Z}$-space

– The $\mathfrak{D}$-space concept we endow with the geometric algebra $\mathcal{G}_{1,3}(\mathbb{R})$ managed by the four-vector top ***grade four direction*** helicity pseudoscalar concept unit $\imath := \gamma_1\gamma_2\gamma_3\gamma_0 = \gamma_0\gamma_3\gamma_2\gamma_1$, which is invariant to *Clifford reversion conjugation* $\widetilde{\imath} = \imath$, and commutate with every even multivector

(7.50) $$X_+\imath = \imath X_+, \quad \text{of the form} \quad X_+ = \langle X \rangle_0 + \langle X \rangle_2 + \langle X \rangle_4 \quad \in \mathcal{G}^+_{1,3}(\mathbb{R}) \subset \mathcal{G}_{1,3}(\mathbb{R}),$$

but anticommute with odd elements of this algebra

(7.51) $$\imath X_- = -X_-\imath, \quad \text{of the form} \quad X_- = \langle X \rangle_1 + \langle X \rangle_3 \quad \in \mathcal{G}^-_{1,3}(\mathbb{R}) \subset \mathcal{G}_{1,3}(\mathbb{R}).$$

[410] Were $k \neq j \neq i \neq k$, all $\in\{3,2,1\}$ are permutations.

– Opposite in $\mathfrak{Z}$-space endowed by $\mathcal{G}_3(\mathbb{R})$ managed by the trivector of top ***grade three*** chiral volume pseudoscalar concept unit $\boldsymbol{i} := \sigma_3\sigma_2\sigma_1$ ***direction***, that reverses as $\boldsymbol{i}^\dagger = -\boldsymbol{i}$, and commutes with all elements $A \in \mathcal{G}_3(\mathbb{R}) : \boldsymbol{i}A = A\boldsymbol{i}$. Yet invariant by *Clifford conjugation* $\widetilde{\overline{\boldsymbol{i}}} = \boldsymbol{i} \in \mathcal{G}_3(\mathbb{R})$.

7.1.4. The Odd and Even Part of the Geometric Space-Time Algebra $\mathcal{G}_{1,3}(\mathbb{R})$

The general multivector form (7.27)-(7.33) can be decomposed into a sum of *even* and *odd* parts

(7.52) $$X = X_+ + X_- = \langle X \rangle_+ + \langle X \rangle_-, \qquad \text{where}$$

(7.53) $$\langle X \rangle_+ = \langle X \rangle_0 + \langle X \rangle_2 + \langle X \rangle_4 = X_+ = \alpha + F + \upsilon\imath \quad \in \mathcal{G}^+_{1,3}(\mathbb{R}) \subset \mathcal{G}_{1,3}(\mathbb{R}),$$

(7.54) $$\langle X \rangle_- = \langle X \rangle_1 + \langle X \rangle_3 = X_- = x + \imath y \quad \in \mathcal{G}^-_{1,3}(\mathbb{R}) \subset \mathcal{G}_{1,3}(\mathbb{R}).$$

This decomposed concept idea we write as $\mathcal{G}_{1,3}(\mathbb{R}) = \mathcal{G}^+_{1,3}(\mathbb{R}) \oplus \mathcal{G}^-_{1,3}(\mathbb{R})$.
Algebraically this can be formed by $X_\pm = \frac{1}{2}(X \mp \imath X \imath)$, an idea taken from Hestenes [23](26)p.6.

7.1.4.2. The Transcendental Ignorance of the Odd part of the Geometric Algebra for $\mathfrak{D}$-space

The founding *substance* we have introduced as (essential ideal) three *subjects* of Descartes' extensional unit ***directions*** $\gamma_1, \gamma_2, \gamma_3$, that in their idea are *eternal* unchangeable orthonormal autonomous for any ***entity*** in Nature. The extra local cyclic unit count with (essential ideal) ***direction*** γ_0 into the *FUTURE* is the idea of a *persistent* chronometric *subject* for development.
The founding basis $\{\gamma_0, \gamma_1, \gamma_2, \gamma_3\}$ for the $\mathcal{G}_{1,3}(\mathbb{R})$ is the concept idea of *subjects* of the concept of a $\mathfrak{D}$-space *substance* of physics. The linear construction (7.10), (7.30) $x = x^\mu\gamma_\mu$ 1-vector and its algebraic dual $\imath x = x^\mu \imath\gamma_\mu$ or (7.32) $\imath y = y^\mu \imath \gamma_\mu$ trivector is a pure substantial mathematical construction that has no direct object image for the intuition of natural space for physics.
The odd standard basis $\{\gamma_0, \gamma_1, \gamma_2, \gamma_3\}$ is the ***directional*** subject foundation for our algebra $\mathcal{G}_{1,3}(\mathbb{R})$ established from a traditional non-Euclidean linear additional vector space $V_{1,3}(\mathbb{R}) \sim \mathbb{R}^4$ by (7.10).
Mutual multiplication of the odd elements $X_- \in \mathcal{G}^-_{1,3}(\mathbb{R})$ are lifted into the even algebra $\mathcal{G}^+_{1,3}(\mathbb{R})$ and further multiplication will stay closed inside this even algebra. (Out of the transcendental odd?)
Now we will ignore the odd case and concentrate on the even closed algebra $\mathcal{G}^+_{1,3}(\mathbb{R})$.

7.1.4.3. The Even Closed Geometric Algebra of $\mathfrak{D}$-space

The even closed algebra $\mathcal{G}^+_{1,3}(\mathbb{R})$ consists of scalars

$$\langle X \rangle_0 = \alpha \in \mathbb{R}, \qquad (7.29)$$

and pseudoscalars

$$\langle X \rangle_4 = \upsilon\imath, \qquad (7.33)$$

together with the bivector field concept in $\mathfrak{D}$-space of physics

$$\langle X \rangle_2 = F = (F^{10}\gamma_1\gamma_0 + F^{20}\gamma_2\gamma_0 + F^{30}\gamma_3\gamma_0) + (F^{32}\gamma_2\gamma_3 + F^{13}\gamma_3\gamma_1 + F^{21}\gamma_1\gamma_2) \quad (7.31) \Leftrightarrow$$

(7.55) $$\langle X \rangle_2 = F = (F^{10}\mathcal{B}_1 + F^{20}\mathcal{B}_2 + F^{30}\mathcal{B}_3) + (F^{32}\imath\mathcal{B}_1 + F^{13}\imath\mathcal{B}_2 + F^{21}\imath\mathcal{B}_3),$$

that is (a Minkowski $\mathcal{B}$-plane bivector) + (a Cartesian extension plane bivector). We recall that these two bivector parts stand in dual ***quality*** to each other. We have the field bivector

(7.56) $$F = \sum_{k=1}^{3} F^{k0}\mathcal{B}_k + \tfrac{1}{2}\sum_{j,k=1}^{3} F^{kj}\gamma_k\gamma_j \ \overset{\leftarrow}{=} \ F^{k0}\sigma_k + \tfrac{1}{2}F^{kj}\sigma_j\sigma_k = \vec{E} + \boldsymbol{i}\vec{B}.$$

The left side sum is the bivector field in $\mathcal{G}^+_{1,3}(\mathbb{R})$ that mimic the electromagnetic tensor field.
The right side is e.g. the electric field 1-vector plus the magnetic field bivector in $\mathcal{G}_3(\mathbb{R})$.
The reader is urged to consider the connection idea $\mathcal{G}^+_{1,3}(\mathbb{R}) \sim \mathcal{G}_3(\mathbb{R})$ as a *subject~object* principle.

7.1.4.4. The Even Geometric Spinor Quality in the Algebra $\mathcal{G}_{1,3}{}^+(\mathbb{R})$ of $\mathfrak{D}$-space

The general even multivector element form (7.28) is then written as a *spinor* (wavefunction)

(7.57) $$\left.\begin{aligned} \psi &= \alpha + F + \upsilon\imath \\ \widetilde{\psi} &= \alpha - F + \upsilon\imath \end{aligned}\right\} \in \mathcal{G}^+_{1,3}(\mathbb{R}) \subset \mathcal{G}_{1,3}(\mathbb{R}).$$