The information event point in $\mathfrak{D}$-space expressed as a 1-vector (7.10) spanned from $\{\gamma_0,\gamma_1,\gamma_2,\gamma_3\}$

(7.25) $\quad \boxed{x = x^{\mu}\gamma_{\mu}} \quad \in V_{1,3}(\mathbb{R}) \subset \mathcal{G}_{1,3}(\mathbb{R}),$

with the contravariant relation coordinate calculation

(7.26) $\quad \boxed{x^{\mu} = \gamma^{\mu}\cdot x} \quad \in \mathbb{R}.$

We see that we use the reciprocal frame idea $\{\gamma^{\mu}\}$ to determine the coordinates of the information events point x in $\mathfrak{D}$-space from the geometric algebra *standard basis* $\{\gamma_0,\gamma_1,\gamma_2,\gamma_3\} \subset \mathcal{G}_{1,3}(\mathbb{R})$.

7.1.3.2. The Multivector Decomposition in a Sum of Grades for $\mathfrak{D}$-space of Physics

For the general multivector element X in STA, the separation in the different ***direction grades*** is

(7.27) $\quad X = \langle X\rangle_0 + \langle X\rangle_1 + \langle X\rangle_2 + \langle X\rangle_3 + \langle X\rangle_4 \quad \in \mathcal{G}_{1,3}(\mathbb{R}),$ (7.9)

(7.28) $\quad X = \alpha + x + F + \mathit{i}y + \upsilon\mathit{i} \quad \in \mathcal{G}_{1,3}(\mathbb{R}).$

We have separated the multivector concept into five different ***primary quality grades***

(7.29) $\quad \langle X\rangle_0 = \alpha \quad \in \mathbb{R},$ ***pqg***-0, scalar, $\dim(\mathbb{R}) = 1,$

(7.30) $\quad \langle X\rangle_1 = x \; = (x_0\gamma_0 + x_1\gamma_1 + \gamma_2 + x_3\gamma_3),$ ***pqg***-1, 1-vector, $\dim({}^1V_{1,3}) = 4,$

(7.31) $\quad \langle X\rangle_2 = F = (F^{10}\gamma_1\gamma_0 + F^{20}\gamma_2\gamma_0 + F^{30}\gamma_3\gamma_0) + (F^{32}\gamma_2\gamma_3 + F^{13}\gamma_3\gamma_1 + F^{21}\gamma_1\gamma_2),$ ***pqg***-2, bivector, $\dim({}^2V_{3,3}) = 6,$

(7.32) $\quad \langle X\rangle_3 = \mathit{i}y = \mathit{i}(y_0\gamma_0 + y_1\gamma_1 + y_2\gamma_2 + y_3\gamma_3),$ ***pqg***-3, trivector, $\dim({}^3V_{1,3}) = 4,$

(7.33) $\quad \langle X\rangle_4 = \upsilon\mathit{i}, \quad \upsilon \in \mathbb{R}$ ***pqg***-4, pseudoscalar, $\dim(\mathbb{R}) = 1.$

All coordinate coefficients are real scalars $x_{\mu}, y_{\mu} \in \mathbb{R}$, $\mu = 0,1,2,3$, and $\beta_k, \phi_k \in \mathbb{R}$, $k = 1,2,3$ of $\mathcal{G}_{1,3}(\mathbb{R})$.
For this ***grade four*** geometric algebra, we have $2^4 = 16$ different dimensions in STA that all describe ***qualitative*** impact in $\mathfrak{D}$-space of physics.
We have a mixed basis structure of STA with 16 ***direction*** generators

(7.34) $\quad \left\{ 1,\; \{\gamma_{\mu}\},\; \{\gamma_{\mu}\wedge\gamma_{\nu}\},\; \{\mathit{i}\gamma_{\mu}\},\; \mathit{i} \right\},\quad \mu,\nu,\kappa = 0,1,2,3.$ or $\left\{ 1,\; \{\gamma_{\mu}\},\; \{\gamma_{\mu\nu}\},\; \{\gamma_{\mu\nu\kappa}\},\; \mathit{i} \right\}$

The first part of this mixed basis is the scalar $\langle X\rangle_0$, the last part is the pseudoscalar $\langle X\rangle_4$ and the middle part is bivector part $\langle X\rangle_2$ of the geometric STA. These middle part bivector basis can be divided into two groups: The line-extension-like as (7.5) $\sigma_k \stackrel{\leftarrow}{=} \mathcal{B}_k = \gamma_k\gamma_0 = \gamma_k\wedge\gamma_0$, $k = 1,2,3$, and the dual angular-area-extension-like as (7.6) e.g. $\mathit{i}_3 \stackrel{\leftarrow}{=} \mathit{i}\mathcal{B}_3 = \gamma_1\gamma_2 = \gamma_1\wedge\gamma_2$.
With the use of this and (7.16) we can for the $\mathfrak{D}$-space structure write the mixed basis (7.34) as

(7.35) $\quad \left\{ 1,\; \{\gamma_{\mu}\},\; \{\mathcal{B}_k, \mathit{i}\mathcal{B}_k\},\; \{\mathit{i}\gamma_{\mu}\},\; \mathit{i} \right\},\; \mu = 0,1,2,3,\; k = 1,2,3,$

for the full geometric Space-Time Algebra $\mathcal{G}_{1,3}(\mathbb{R})$, which we can use for all types of information event ***entities*** in $\mathfrak{D}$-space of physics. (In [22]p.7(2.16) the bivectors are called $\{\sigma_k, \mathit{i}\sigma_k\} = \{\mathcal{B}_k, \mathit{i}\mathcal{B}_k\}$.)[408]

7.1.3.3. Conjugation in Space-Time Algebra

In STA, we use three different types of ***conjugation***

Table 7.1 ***Conjugation*** of		*scalar*	$\mathcal{G}_{1,3}(\mathbb{R})$, Space-Time Algebra for $\mathfrak{D}$-space.						$\mathcal{G}_3(\mathbb{R})$ for $\mathfrak{Z}$-space.		
Type	***conjugation***	$\mathbb{R}$	x	x	$F\leftarrow\mathcal{B}_k$	$F\leftarrow\mathit{i}\mathcal{B}_k$	$\mathit{i}y$	i	$\mathbf{x}$	$\mathbf{B}$	$\boldsymbol{i}$
Origin	e.g.: $\downarrow X \leftarrow$	α	γ_k	γ_0	$\gamma_k\gamma_0$	$\gamma_k\gamma_j$	$\mathit{i}y$	i	$\boldsymbol{\sigma}_k$	$\boldsymbol{\sigma}_j\boldsymbol{\sigma}_k$	$\boldsymbol{i}$
Reversion	$(X)^{\dagger}$	$+\alpha$	$+\gamma_k$	γ_0	$-\gamma_k\gamma_0$	$\gamma_j\gamma_k = -\gamma_k\gamma_j$	$y\mathit{i} = -\mathit{i}y$	i	$+\boldsymbol{\sigma}_k$	$\boldsymbol{\sigma}_k\boldsymbol{\sigma}_j = -\boldsymbol{\sigma}_j\boldsymbol{\sigma}_k$	$-\boldsymbol{i}$
Inversion	$\overline{(X)} = (X)^{-}$	$+\alpha$	$-\gamma_k$	γ_0	$-\gamma_k\gamma_0$	$+\gamma_k\gamma_j$	$-\mathit{i}y$	i	$-\boldsymbol{\sigma}_k$	$\boldsymbol{\sigma}_j\boldsymbol{\sigma}_k$	$-\boldsymbol{i}$
Clifford~	$\widetilde{(X)} = (X)^{\sim}$	$+\alpha$	$+\gamma_k$	γ_0	$-\gamma_k\gamma_0$	$\gamma_j\gamma_k = -\gamma_k\gamma_j$	$y\mathit{i} = -\mathit{i}y$	i	$-\boldsymbol{\sigma}_k$	$\boldsymbol{\sigma}_k\boldsymbol{\sigma}_j = -\boldsymbol{\sigma}_j\boldsymbol{\sigma}_k$	$+\boldsymbol{i}$
Even or odd parts:		even	odd		even		odd	even	odd	even	odd

$X = \langle X\rangle_0 + \langle X\rangle_1 + \langle X\rangle_2 + \langle X\rangle_3 + \langle X\rangle_4$ | $\langle A\rangle_1$, $A_+ = \langle A\rangle_0 + \langle A\rangle_2$, $\langle A\rangle_3$

[408] We use bold $\boldsymbol{\sigma}_k \in \mathcal{G}_3(\mathbb{R})$ to indicate 1-vector in $\mathfrak{Z}$-space algebra and $\sigma_k \in \mathcal{G}_{1,3}(\mathbb{R})$ indicate a bivector member in STA of $\mathfrak{D}$-space.

In the $\mathcal{G}_{1,3}(\mathbb{R})$ algebra the Clifford~conjugated is the same as the reversion in [23]p.5,(20)-(21),
Where the general multivector element form (7.28) has the $\mathfrak{D}$-space reversion conjugation

(7.36) $\quad \left.\begin{array}{l} X \;=\; \alpha + x + F + \mathit{i}y + \upsilon\mathit{i} \\ \widetilde{X} \;=\; \alpha + x - F - \mathit{i}y + \upsilon\mathit{i} \end{array}\right\} \in \mathcal{G}_{1,3}(\mathbb{R}).$

We take the product XA write another multivector as e.g., $A = \alpha_A + a + F_A + \mathit{i}b + \upsilon_A\mathit{i}$, and find

(7.37) $\quad \widetilde{(XA)} = (XA)^{\sim} = \widetilde{A}\,\widetilde{X}.$

We have the simple relation $\widetilde{(\widetilde{X})} = (\widetilde{X})^{\sim} = X$. Hence for the product we call the Clifford conjugation quadratic form $X\widetilde{X} = \widetilde{X}X$, we have the conjugation invariance in $\mathfrak{D}$-space

(7.38) $\quad (X\widetilde{X})^{\sim} = \widetilde{X}X = X\widetilde{X} \quad \in \mathcal{G}_{1,3}(\mathbb{R}).$

7.1.3.4. Reversion of the Odd Chirality Volume Pseudoscalar $\boldsymbol{i}$ in the $\mathcal{G}_3(\mathbb{R})$ Algebra for $\mathfrak{Z}$-space

In $\mathfrak{Z}$-space treated in chapter 6, we use the $\mathcal{G}_3(\mathbb{R})$ algebra with even subalgebra $\mathcal{G}_{0,2}(\mathbb{R}) \sim \mathcal{G}_3^{+}(\mathbb{R}) \subset \mathcal{G}_3(\mathbb{R})$. The two dual concepts connects; the odd 1-vectors $\mathbf{x} \in \mathcal{G}_3^{-}(\mathbb{R}) \subset \mathcal{G}_3(\mathbb{R})$ and the even bivectors $\mathbf{B} \in \mathcal{G}_3^{+}(\mathbb{R})$ by the dextral chiral unit pseudoscalar $\boldsymbol{i}$ that is endowed with the ***quality*** $\boldsymbol{i}^2 = -1$. In $\mathcal{G}_3(\mathbb{R})$. This unit pseudoscalar inherits ***conjugation qualities*** expressed in Table 7.1:
The *reversion* of this is $\boldsymbol{i}^{\dagger} = -\boldsymbol{i}$ due to the definition $\boldsymbol{i} := \boldsymbol{\sigma}_3\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 \Rightarrow (\boldsymbol{\sigma}_3\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1)^{\dagger} = \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\boldsymbol{\sigma}_3 = -\boldsymbol{i}$.
Further, the *extension parity inversion* gives $\boldsymbol{i}^{-} = -\boldsymbol{i} \Leftarrow (\boldsymbol{\sigma}_3\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1)^{-} = (-\boldsymbol{\sigma}_3)(-\boldsymbol{\sigma}_2)(-\boldsymbol{\sigma}_1) = -\boldsymbol{i}$.
The combination of these two *conjugations* gives the *Clifford~conjugation*

(7.39) $\quad (\boldsymbol{i}^{\dagger})^{-} = (\boldsymbol{i})^{\sim} = \widetilde{\boldsymbol{i}} = \boldsymbol{i} \quad \in \mathcal{G}_3(\mathbb{R}),$

that is a combined *inversion and reversion* as the *conjugation* $A \longrightarrow \widetilde{A} \in \mathcal{G}_3(\mathbb{R})$ in $\mathfrak{Z}$-space.

Right multiplying the dextral chirality unit pseudoscalar $\boldsymbol{i} \in \mathcal{G}_3(\mathbb{R})$ with the information development unit γ_0 we achieve the positive helicity unit pseudoscalar $\boldsymbol{i}\gamma_0 \stackrel{\leftarrow}{=} \mathit{i} \in \mathcal{G}_{1,3}(\mathbb{R})$ *of* STA for the dextral development volume in $\mathfrak{D}$-space of information in physics.

7.1.3.5. Reversion of the Even Helicity Pseudoscalar i in the Algebra $\mathcal{G}_{1,3}(\mathbb{R})$ for $\mathfrak{D}$-space

In $\mathfrak{D}$-space *the unit dextral helicity pseudoscalar* (7.11) $\mathit{i} := \gamma_1\gamma_2\gamma_3\gamma_0$ is *conjugation invariant.*

(7.40) $\quad \mathit{i}^{\dagger} = \mathit{i}, \qquad \mathit{i}^{-} = \overline{\mathit{i}} = \mathit{i}, \qquad$ and $\qquad \mathit{i}^{\sim} = \widetilde{\mathit{i}} = \mathit{i}, \qquad$ because of $\gamma_1\gamma_2\gamma_3\gamma_0 = \gamma_0\gamma_3\gamma_2\gamma_1$.

For every X in $\mathcal{G}_{1,3}(\mathbb{R})$, the *Clifford conjugation* $X \longrightarrow \widetilde{X} \in \mathcal{G}_{1,3}(\mathbb{R})$ is the conjugation that is founded on the reversion of the product order of 1-vectors $x \in \mathcal{G}_{1,3}(\mathbb{R})$

(7.41) $\quad \widetilde{x} = x, \quad \widetilde{xy} = yx = -xy, \quad$ and $\quad \widetilde{axy} = yxa = -axy, \quad$ but $\widetilde{\mathit{i}} = \mathit{i}$ and scalars $\widetilde{\alpha} = \alpha$.

The Clifford conjugation in $\mathcal{G}_{1,3}(\mathbb{R})$ is just operation reversion and does not comprise inversion.
Just the same applies to dagger conjugation defined as the straight reversion $(XA)^{\dagger} = AX \in \mathcal{G}_{1,3}(\mathbb{R})$.
To avoid ambiguity,[409] we shall avoid the dagger symbol when we mean reversion inside $\mathcal{G}_{1,3}(\mathbb{R})$.

The straight *extension parity inversion* in $\mathcal{G}_{1,3}(\mathbb{R})$ is just the change of the founding standard basis $\{\gamma_0,\gamma_1,\gamma_2,\gamma_3\}$ to the reciprocal basis given by (7.22) as $\gamma_k^{-1} = -\gamma_k$, $\gamma_0^{-1} = \gamma_0$, and (7.24)

(7.42) $\quad \gamma^{\nu} = g^{\mu\nu}\gamma_{\mu} \;=\; \gamma_{\mu}^{-1} \;=\; \overline{\gamma_{\mu}} \;=\; -\gamma_{\mu},$

used to find the contravariant coordinates $x^{\mu} = \gamma^{\mu}\cdot x$ (7.26) for information relations between event points in $\mathfrak{D}$-space as each 1-vector information direction possibly expressed as (7.25) $x = x^{\mu}\gamma_{\mu} \in \mathcal{G}_{1,3}(\mathbb{R})$ from arbitrary orthonormal supporting basis $\{\gamma_{\mu}\}$ given coordinates $x^{\mu} = \gamma^{\mu}\cdot x$ measured by the reciprocal basis $\{\gamma^{\mu}\} = \{\gamma_{\mu}^{-1}\}$.
Every $X \in \mathcal{G}_{1,3}(\mathbb{R})$ can be constructed by a linear combination of (7.41). Making operational STA

[409] $\widetilde{\sigma_k} = -\sigma_k$ as $\widetilde{\gamma_1\gamma_0} = \gamma_0\gamma_1 = -\gamma_1\gamma_0$ in $\mathcal{G}_{1,3}(\mathbb{R})$, while $\boldsymbol{\sigma}_k^{\dagger} = \boldsymbol{\sigma}_k$ in $\mathcal{G}_3(\mathbb{R})$. And further parity inversion $\overline{\boldsymbol{\sigma}_k} = -\boldsymbol{\sigma}_k$.