III. Space-Time Relations in Physics

Research on the a priori of Physics

Geometric Critique of Pure Mathematical Reasoning

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7. Relation Space of Physics

7.1. Vision of Relations by the Development Concept of Physics in Nature

We start with the traditional Euclidean orthonormal basis for a Cartesian 1-vector space $V_3(\mathbb{R})$ $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3\}$ that represents three intuitive perpendicular line ***directions*** of physical $\mathfrak{Z}$-space in Nature. This is the foundation of the whole geometric algebra $\mathcal{G}_3(\mathbb{R})$ where the Clifford measure signature is positive $\boldsymbol{\sigma}_k^2 = 1$, and unit normalised. We form its unit pseudoscalar chiral ***direction*** $\boldsymbol{\sigma}_3\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_3\wedge\boldsymbol{\sigma}_2\wedge\boldsymbol{\sigma}_1$ as § 6.9.1.1:3. that carry the fundamental chiral volume structure of $\mathfrak{Z}$-space. To emphasise the need for an external measure norm we expand the basis with this real scalar ***non-directional*** unit $1 \in \mathbb{R}$ and write $\{1, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3\}$

In chapter 5.7 we introduce an extra *external* 1-vector for the development ***direction unit*** γ_0.

We now transform the space narrative of physics by right multiplying with the *orthogonal* γ_0, by making this the measure doing the multiplication transformation of the 1-vector basis for $\mathcal{G}_3(\mathbb{R})$

(7.1) $$\{1, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3\}\gamma_0 = \{\gamma_0, \boldsymbol{\sigma}_1\gamma_0, \boldsymbol{\sigma}_2\gamma_0, \boldsymbol{\sigma}_3\gamma_0\} = \{\gamma_0, \gamma_1, \gamma_2, \gamma_3\}.$$

This is an epistemology ideology change from $\mathfrak{Z}$-space to the idea of Space-Time-Algebra (STA)

(7.2) $$\mathcal{G}_3(\mathbb{R})\otimes\gamma_0 \leftrightarrow \mathcal{G}_{1,3}(\mathbb{R}).$$

In STA, we define all four unit ***directions*** γ_μ as the orthonormal basis $\{\gamma_0, \gamma_1, \gamma_2, \gamma_3\}$ of a four-dimensional linear 1-vectors space $V_{1,3}(\mathbb{R})$ that is the foundation of the full *geometric space-time algebra* $\mathcal{G}_{1,3}(\mathbb{R})$ of mixed Clifford signature $(+,-,-,-)$ for STA, as defined (5.300)-(5.301)

(7.3) $$\gamma_0^2 = +1, \quad \text{and} \quad \gamma_k^2 = -1, \quad \text{for } k = 1,2,3. \text{ Thus: } |\gamma_0| \coloneqq 1 \Rightarrow |\gamma_1| \coloneqq |\gamma_2| \coloneqq |\gamma_3| \coloneqq 1,$$

for the connected unit magnitude norm, and the orthogonality (5.303) and (5.377)

(7.4) $$\gamma_0\cdot\gamma_1 = \gamma_0\cdot\gamma_2 = \gamma_0\cdot\gamma_3 = 0, \text{ and } \gamma_1\cdot\gamma_2 = \gamma_2\cdot\gamma_3 = \gamma_3\cdot\gamma_1 = 0.,$$

In the epistemological narrative of STA, the founding basis for $\mathcal{G}_{1,3}(\mathbb{R})$, $\{\gamma_0, \gamma_1, \gamma_2, \gamma_3\}$ is interpreted as 1-vectors $\gamma_\mu = \langle\gamma_\mu\rangle_1 \in \mathcal{G}_{1,3}(\mathbb{R})$, for $\mu = 0,1,2,3$.

- Alternative in the middle term of (7.1) basis $\{\gamma_0, \boldsymbol{\sigma}_1\gamma_0, \boldsymbol{\sigma}_2\gamma_0, \boldsymbol{\sigma}_3\gamma_0\}$ of the of $\mathfrak{Z}$-space narrative form $\mathcal{G}_3(\mathbb{R})\otimes\gamma_0$ we interpret the outer chronometer ***direction*** basis-element: γ_0 as a 1-vector, and the rest three as basis bivectors $\boldsymbol{\sigma}_k\gamma_0 = \boldsymbol{\sigma}_k\wedge\gamma_0$, for k=1,2,3, having signatures $(\boldsymbol{\sigma}_k\gamma_0)^2 = -1$.

Nonetheless in the Space-Time Algebra narrative $\mathcal{G}_{1,3}(\mathbb{R})$, we start from the 1-vector basis $\{\gamma_\mu\}$. From the three orthonormal extensions operators γ_k, we form the bivectors as (5.341), (5.333)←(5.328)

(7.5) $$\left.\begin{array}{l} \gamma_1 \longrightarrow \boldsymbol{\sigma}_1 \overset{\leftarrow}{=} \mathcal{B}_1 \coloneqq \gamma_1\gamma_0 \in \mathcal{G}_{1,1} \\ \gamma_2 \longrightarrow \boldsymbol{\sigma}_2 \overset{\leftarrow}{=} \mathcal{B}_2 \coloneqq \gamma_2\gamma_0 \in \mathcal{G}_{1,1} \\ \gamma_3 \longrightarrow \boldsymbol{\sigma}_3 \overset{\leftarrow}{=} \mathcal{B}_3 \coloneqq \gamma_3\gamma_0 \in \mathcal{G}_{1,1} \end{array}\right\} \subset \mathcal{G}_{1,3}(\mathbb{R}), \text{ with positive signatures: } \begin{array}{l} \mathcal{B}_1^2 = \boldsymbol{\sigma}_1^2 = 1, \\ \mathcal{B}_2^2 = \boldsymbol{\sigma}_1^2 = 1, \\ \mathcal{B}_2^2 = \boldsymbol{\sigma}_1^2 = 1, \end{array}$$

by simply left operation multiply the extension units γ_k on the unique development unit measure γ_0. Further letting the extensions unit operators γ_k act mutually at each other we get the three bivectors

(7.6) $$\begin{array}{l} \boldsymbol{i}_1 \coloneqq \boldsymbol{\sigma}_3\boldsymbol{\sigma}_2 \overset{\leftarrow}{=} \mathcal{B}_3\mathcal{B}_2 = \gamma_3\gamma_0\gamma_2\gamma_0 = -\gamma_3\gamma_2 = \gamma_2\gamma_3, \\ \boldsymbol{i}_2 \coloneqq \boldsymbol{\sigma}_1\boldsymbol{\sigma}_3 \overset{\leftarrow}{=} \mathcal{B}_1\mathcal{B}_3 = \gamma_1\gamma_0\gamma_3\gamma_0 = -\gamma_1\gamma_3 = \gamma_3\gamma_1, \\ \boldsymbol{i}_3 \coloneqq \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 \overset{\leftarrow}{=} \mathcal{B}_2\mathcal{B}_1 = \gamma_2\gamma_0\gamma_1\gamma_0 = -\gamma_2\gamma_1 = \gamma_1\gamma_2, \end{array} \quad \text{with:} \quad \begin{array}{l} (\gamma_2\gamma_3)^2 = \boldsymbol{i}_1^2 = -1, \\ (\gamma_3\gamma_1)^2 = \boldsymbol{i}_2^2 = -1, \\ (\gamma_1\gamma_2)^2 = \boldsymbol{i}_1^2 = -1. \end{array}$$

This is just the well-known quaternion basis for the even algebra $\mathbb{H} \sim \mathcal{G}_{0,2}(\mathbb{R}) \subset \mathcal{G}_3(\mathbb{R})$,

(7.7) $$\{1, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3\} = \{1, \gamma_2\gamma_3, \gamma_3\gamma_1, \gamma_2\gamma_1\} \subset \mathcal{G}_{0,2}(\mathbb{R}) \subset \mathcal{G}_{1,3}(\mathbb{R}).$$

Similarly, the $\mathcal{G}_3(\mathbb{R})$ dextral pseudoscalar is transformed into the pseudoscalar of $\mathcal{G}_{1,3}(\mathbb{R})$

(7.8) $$\boldsymbol{i} \coloneqq \boldsymbol{\sigma}_3\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 \overset{\leftarrow}{=} \mathcal{B}_3\mathcal{B}_2\mathcal{B}_1 = \gamma_3\gamma_0\gamma_2\gamma_0\gamma_1\gamma_0 = -\gamma_3\gamma_2\gamma_1\gamma_0 = \gamma_0\gamma_3\gamma_2\gamma_1 = \gamma_1\gamma_2\gamma_3\gamma_0 \coloneqq \mathit{i}.$$