

We call the *quality* of these symbolic instances: **a, b** for 1-vectors, because we demand by reason them to fulfil rules of a linear additional algebra, e.g. (5.11)-(5.21) p.160.

In this book we call 1-vectors for a *primary quality of first grade direction (pqg-1)* in physics.

What Hestenes contributed is taking the direct product of these instances $\mathbf{ab} = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \wedge \mathbf{b}$, following the algebraic multiplication rules (5.38)-(5.42), (Originally Clifford had merged these two products, the inner product and the external product of Grassmann and called it *geometric algebra*) and Hestenes make a new instance performing a new *grade quality* representing an orientated area in a geometric substance in Nature. This orientated area of the Grassmann external wedge product $\mathbf{a} \wedge \mathbf{b}$ is the instance we here call a bivector of *primary quality of second grade direction (pqg-2)* of physics. The inner product $\mathbf{a} \cdot \mathbf{b}$ result in a real scalar of *primary quality of zero grade* that does not possess *direction* but certainly has a *quantitative* impact on physics of Nature.

The force of this simple expression of the direct product **ab**, as a physics instance Hestenes call a *spinor*⁴⁰⁴ that possesses the *primary quality of even grade direction (pqg-0,2)* of physics.

By further multiplication operation performed by this instance (**ab**), make other more or less complicated geometric algebraic instances transform by rotation and dilation. (As in the above Chapters)

When we restrict the spinor to be normalized $|\mathbf{ab}| = 1 \Rightarrow U = \mathbf{ab}$ we call it a rotor. A rotor is unitary due to the expression $UU^\dagger = \mathbf{abba} = 1$. The rotor instance can perform all invariant rotational transformations of geometric instances in the physical space of nature. For an introduction to all this see chapters 5 and 6 special sections 5.4.5 and 6.3-6.3.5.

The interesting of the bivector idea from a Euclidean basis is that a multiplication algebra of bivector instances together with real scalars form an even closed algebra $\mathcal{G}_{0,2}(\mathbb{R})$ equivalent to the Hamilton quaternions idea $\mathbb{H} \sim \mathcal{G}_{0,2}(\mathbb{R})$.

⁴⁰⁴ The name *spinor* with an equivalent reference to a more general advanced (thus mysterious) mathematic concept called spinors.

6.9.1.1. Extension of Space by Grassmann Exterior Products

Further product of finite mutual independent 1-vectors *objects* make the Grassmann products $\mathbf{a} \wedge \mathbf{x}$, $\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{x}$, and $\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c} \wedge \mathbf{x}$. If we for all **x** in the geometric algebras $\mathcal{G}_n(\mathbb{R})$ have:

1. $\mathbf{a} \wedge \mathbf{x} = 0$, we have a line concept algebra $\mathcal{G}_1(\mathbb{R})$ with pseudoscalars **a**, Chapter 4.4.
2. $\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{x} = 0$, we have a plane concept algebra $\mathcal{G}_2(\mathbb{R})$ with pseudoscalars $\mathbf{a} \wedge \mathbf{b}$, Chap. 5.
3. $\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c} \wedge \mathbf{x} = 0$, we have a 3-space concept algebra $\mathcal{G}_3(\mathbb{R})$ with pseudoscalars $\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c}$, Chap. 6.

We have founded these geometric algebras $\mathcal{G}_n(\mathbb{R})$ on a Euclidean 1-vector basis $V_n(\mathbb{R})$ with a positive Clifford signature metric (+). In the case $\mathcal{G}_3(\mathbb{R})$ the vector space $V_3(\mathbb{R})$ is by 1-vector products lifted into an even closed algebra $\mathbb{H} \sim \mathcal{G}_{0,2}(\mathbb{R}) \subset \mathcal{G}_3(\mathbb{R})$ that internally has negative signature (–) and therefore closed internal anti-Euclidean. Here a rotor instance is also called a versor $U \in \mathbb{H} \sim \mathcal{G}_{0,2}(\mathbb{R})$ from Hamilton’s quaternion algebra. In mathematics, rotation is a passive ideal, but in physics, rotation is an active reality. Therefore, we know the rotation is emancipated as cyclic oscillations in Euclidean planes possessing angular momentum in agreement with Kepler’s second law. We have in this book seen that in the versor case there are at least two independent Euclidean planes of cyclic oscillations. We have also seen that the intersection of such two $\mathcal{G}_2(\mathbb{R})$ planes gives interconnected raise to a third plane. This interconnectivity is best described by the quaternion algebra $\mathcal{G}_{0,2}(\mathbb{R})$ for angular momentum interconnection (6.289)←(6.123).

This is founded on a full 3-space geometric algebra $\mathcal{G}_3(\mathbb{R})$ for a physical space in Nature. But this geometric algebra $\mathcal{G}_3(\mathbb{R})$ is not sufficient to describe physic relations in nature because it is based on the idea of an unchangeable *object* basis 1-vectors $\sigma_1, \sigma_2, \sigma_3$, that autonomously are eternal as in a Newtonian picture.

But we know that the causal information of development has a finite speed **c** of propagation. In Chapter 5.7 we have started to address this issue in one extended direction. Now will describe an algebra that concerns the development of all three dimensions of Cartesian extension *directions*. For this we then need an *a priori synthetic judgment* that says:

(6.539) *The causal speed of information is isotropic finite.*

From this statement we make an *a priori analytic judgment* saying:

(6.540) *The speed of information is the universal unit c. (practical universal measure $c = 1$)*

This new geometric algebra is first been introduced by David Hestenes 1966 [6] and will be founded on the principle of the Grassmann product of four independent 1-vector *subjects*

4. $\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c} \wedge \mathbf{t} \wedge \mathbf{x} = 0$, a \mathcal{D} -space concept algebra $\mathcal{G}_4(\mathbb{R})$, with pseudoscalars $\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c} \wedge \mathbf{t}$, Chapter 7.

The concept idea \mathcal{D} -space represents a *substance* in Nature that manage information of development action relations in a natural space describable by the science of physics.