For $4 < Z \leq 10$ we have *shell* $n = 2$, $\ell_5 = \ell_6 = \ell_7 = \ell_8 = \ell_9 = \ell_{10} = 1$, $m_l = -1,0,1$, $m_{½,i} = \pm ½$, six possible electron stats in the first subshell, with spin compensation $m_{½,i} + m_{½,i+1} = 0$.

6.7.4. The Spatial Wavefunction Probability Distribution Structure of atoms

In the quantum mechanics tradition, the solution of the Schrödinger equation for a hydrogen-like atom (Bohr-Rutherford) for one electron in spherical coordinates is

(6.533) $$\psi_{n,\ell,m_\ell}(r_i,\theta,\varphi) = R_{n,\ell}(r_i)\cdot Y_\ell^{m_\ell}(\theta,\varphi),$$

where for each electron $\Psi_{½,i}$

(6.534) $R_{n,\ell}(r_i)$ is the *radial probability distribution function*, (Figure 6.26)

and

(6.535) $Y_\ell^{m_\ell}(\theta,\varphi)$ is the *spherical harmonic function for the probability distribution*, (Figure 6.27)

around the nucleus with the atomic number Z.
The index $i = i(n,\ell,m_\ell) = 1,2,3\ldots Z$ we dedicate for each electron $\Psi_{½,i}$, in the idea of sequential filling $Z = 1,2,3,\ldots$ of the shell structure (n,ℓ,m_ℓ) following Madelung rule indicated in Figure 6.28, and Hund's rule filling the free subshell energy level, first without spin pairing.

(6.536) $$\text{The } \textit{subshells} \text{ are } \begin{cases} \ell = \ 0,\ 1,\ 2,\ 3,\ 4,\ \ldots \\ \text{named: } \ s,\ p,\ d,\ f,\ g,\ \ldots \end{cases}$$

In principle, each electron has its autonomous chronometric reference, and we may demand that its spatial distribution over the development of its interaction with the surroundings governed by the atomic nucleus is normalized to unity

(6.537) $$\int_3 \left|\psi_{n,\ell,m_\ell}(r_i,\theta,\varphi)\right|^2 d\tau_i = 1, \qquad \text{where } d\tau_i = dr_i d\theta d\varphi.$$

We may presume that each subshell has the same r_i external reference for the same energy level of the binding in the specified atom Z.

We will not in this book go further into the theories for the practise atomic structure and address the reader to the general rich literature on the subject.

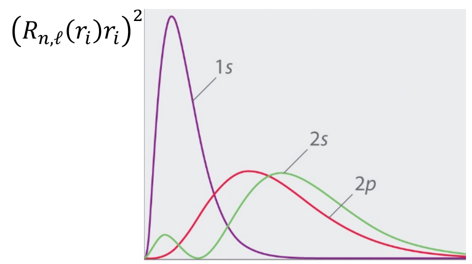

Figure 6.26 $r_i \to (R_{n,\ell}(r_i)r_i)^2$.
https://chem.libretexts.org/...

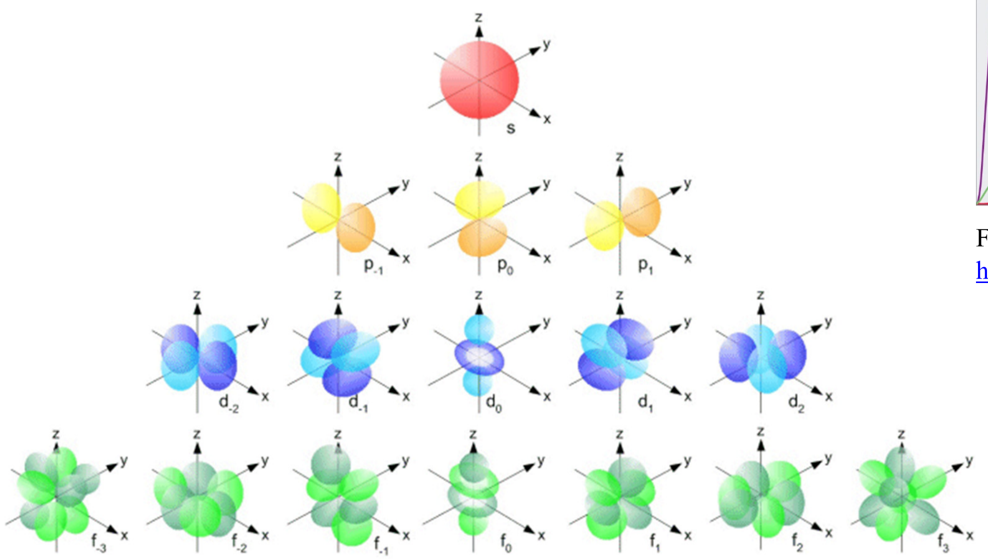

Figure 6.27 Angular distribution $Y_\ell^{m_\ell}(\theta,\varphi)$ from
https://chem.libretexts.org/..._Spherical_Harmonics

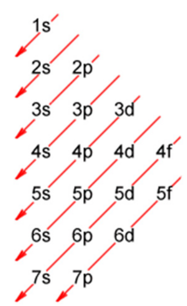

Figure 6.28 Madelung rule.

6.8. Conclusion on Topological Structure of 𝔍-space Founded in Physics

Traditionally we have founded our knowledge of the natural space structure on the Cartesian coordinate system with three line axis dimensions x, y, z endowed with real numbers representing the Descartes extending magnitudes of the Aristotelian *length*, *breadth*, and *depth*.
Gottfried Leibniz used relations to make a structure through his concept *analysis situs*.
Immanuel Kant reasoned that Newton's square law implies just three dimensions of 𝔍-space 1744 and introduced the necessary concept of ***direction*** 1768. (that we first learned as an arrow on a line segment)
Hermann Grassmann introduced the exterior (outer) product in 1844. Independent of this William Rowan Hamilton introduced the group product of unit elements in an interconnected structure of quaternions 1843-. William Kingdon Clifford united these with the inner product and called it geometric algebra. It was revitalised by Marcel Riesz in his lecture book1958 [21], which inspired David Hestenes [6] [10]etc. to develop a new founding structure of geometric operational product algebra that is easy to use and understand as an analogy for natural space.
This made it possible to form a new topological foundation structure by the even ***grade*** versor quaternion bivectors and scalars in a lifted Pauli algebra $\mathcal{G}_{0,2}(\mathbb{R}) \subset \mathcal{G}_3(\mathbb{R})$ with an internal anti-Euclidean signature $(-)$ for the anticommuting bivectors in analogy with the interconnected ***directional*** angular momentum ***quanta*** components from interacting complements of oscillations of an ***entity*** possessing the ***quality*** ability to give local measurable ***direction*** and ***quantities*** of fermionic ***entities*** in natural space of physics.

This topological structure is an enrichment on top of the traditional Euclidean space of 1-vectors with signature $(+)$, where we just use the Pythagorean-Cartesian-extensive measure $\sqrt{x^2+y^2+z^2}$.

The algebraic multiplicative operand, that gives the connection between these two dual views is the chiral ***directional grade three*** unit $\boldsymbol{i} \coloneqq \boldsymbol{\sigma}_3\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ which we have defined as dextral orientated. This $\boldsymbol{i} \in \mathcal{G}_3(\mathbb{R}), \boldsymbol{i} \notin \mathcal{G}_{0,2}(\mathbb{R}) \subset \mathcal{G}_3(\mathbb{R})$, commute with all elements of ***grade*** ≤ 3 in the algebra $\mathcal{G}_3(\mathbb{R})$, and its signature ***quality*** is $\boldsymbol{i}^2 = -1 \Rightarrow \boldsymbol{i} \sim \sqrt{-1}$, which thus is called a pseudoscalar.[400]

6.8.1. Conclusion on the Local Situated Topological Structure of Natural 𝔍-space

The lifted Pauli algebra $\mathcal{G}_{0,2}(\mathbb{R}) \subset \mathcal{G}_3(\mathbb{R})$ is also called for the quaternion group $\mathbb{H}$ in tribute to Hamilton who abstractly invented its unit basis elements $\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -1$, $\boldsymbol{ij} = \boldsymbol{k} = -\boldsymbol{ji}$, as anticommuting algebra for the structure of physical space. We have in this book named it differently $\boldsymbol{i}_1 \coloneqq \boldsymbol{i}$, $\boldsymbol{i}_2 \coloneqq \boldsymbol{j}$, $\boldsymbol{i}_3 \coloneqq \boldsymbol{k}$, described in section 6.4.3. The force of this group for physics is that its unit orthonormal basis $\{1, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3\}$ direct represents three geometric perpendicular plane ***directions*** in physical 𝔍-space, and these three planes intersect in just one point of a locality center. This solves Leibniz's conceptual *geometria situs* problem, seen by Kant in 1768, [11]p.361.
We have from this idea tried the construction with four intersecting planes not perpendicular and not parallel that intersects as a tetrahedron with four vertexes for a circumscribed sphere with a center of locality forming a tetraon basis of four 1-vectors with four dual faces of the plane directions forming the tetrahedron. This circumscribes the center of *locus situs* for one ***entity*** of 𝔍-space, where the sphere discriminates the internal from the external of each ***entity***. (⇒ ***outwards***)
Despite the idea of all the planes of possible internal circular oscillators, the resulting wavefunction for one fermion $\Psi_{½}$ is just a versor (6.454) $U = u_0 + u_3\boldsymbol{i}_3 + u_2\boldsymbol{i}_2 + u_1\boldsymbol{i}_1$ that is unitary $UU^\dagger = 1$ auto-normed to magnitude one that is equivalent to the radius of its unit sphere of S^3 symmetry.

[400] This ***primary directional quality of grade three*** chiral pseudoscalar unit $\boldsymbol{i}^2 = -1 \Rightarrow \boldsymbol{i} \sim \sqrt{-1}$, is a ***quality*** that is fundamental different from the complex number field $\mathbb{C}$ imaginary unit $i \in \mathbb{C}$ that possesses a plane pseudoscalar quality $i^2 = 1 \Rightarrow i \sim \sqrt{-1}$. This reason is that the complex number plane $\mathbb{C}$ from its idea does not possess any geometric ***direction*** in physical space and thus commutate with any number $\forall z \in \mathbb{C}$. When we opposite take the ***directional*** geometric plane spanned by $\mathbf{x} = x_1\boldsymbol{\sigma}_1 + x_2\boldsymbol{\sigma}_2$ we have the unit bivector $\boldsymbol{i} \equiv \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ that also possesses the ***quality*** $\boldsymbol{i}^2 = -1 \Rightarrow \boldsymbol{i} \sim \sqrt{-1}$, that anticommute with $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2$ and thus $\forall \mathbf{x}$. Ontologically existence in a plane, the ***directive*** unit $\boldsymbol{i} \not\equiv i$. Sometimes $\boldsymbol{i}$ is called the anticommuting pseudoscalar for the plane.

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