- II. . The Geometry of Physics – 6. The Natural Space of Physics – 6.7. Multiple Numbers of Spin½ Fermions –

The energy change between these electron orbital levels results in a spectral energy exchange after Rydberg's formula for the Hydrogen spectrum

(6.529)
$$\hbar\omega \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right).$$

From this, we conclude to know that the binding energy level of the Hydrogen electron is

(6.530)
$$E_n \approx -\frac{13.6 \text{ eV}}{n_2^2},$$

in general, for atoms
$$E_n \circ$$

We distinguish the different atom nucleus charge quantities by their atomic number Z, which corresponds to the number of electrons in each atom.

6.7.3.3. Atomic Shells and Subshells

(6.531)

This (6.530) gives electrons $\Psi_{1/2}$ in energy levels of a *principal quantum number* $n \in \mathbb{N}$. These so-called energy *shells* do not imply any specific *direction* at all.

For the directional orbital angular momentum, we introduce a subshell quantum number $\ell \in \mathbb{N}$. $\ell < n$.

that is counted exclusively from spin^{1/2} quantum number $m_{1/2} = \pm 1/2$ for each electron $\Psi_{1/2}$.

For the unexcited Hydrogen first shell n = 1, the electron does not possess any orbital angular momentum $\ell = 0$ and its spin^{1/2} angular momentum $|\mathbf{L}_{\mathbf{L}_{\mathbf{L}}}| = \hbar^{\frac{1}{2}}$ is compensated by the nucleus or easier by a sibling Hydrogen atom in a H₂ molecule.

For the unexcited Helium, where there can be two electrons $\Psi_{1/2}$ with shell number $n = 1 \Rightarrow l = 0$. Both have no orbital angular momentum $\ell = 0$ (no *subshell*). The two-electron spin¹/₂ balance each other in a spherical symmetric atom.

In the Bohr-Rutherford atomic model, it is common to separate and ignore the inner spin $\frac{1}{2}$ angular momentum of each *entity* $\Psi_{1/2}$. The reason is that spin is neutralised in spin pairs e.g., as (6.527). The classical electron was viewed as a point particle. The atomic electrons are now just an oscillating *entity* in *shells* and *subshells* revolving around the nucleus.

For shell principal **quantum** number n > 1 we can have subshell numbers $\ell = 0, 1, 2, \dots, n-1$. For the *direction* projection of the orbital angular momentum, we have the quantum integer number m_l where $|m_l| \leq \ell \Rightarrow m_l = -\ell \dots - 1, 0, 1 \dots \ell$.

We make the resultant $m = m_{\ell} \pm \frac{1}{2}$ that is a non-integer, half-valued, for each single electron Ψ_{1} From the orbital angular momentum number ℓ of each electron $\Psi_{1/2}$ we find the total spin as

 $j = |\ell \pm \frac{1}{2}|$ $\Rightarrow \ell = j + \frac{1}{2}$ or $\ell = j - \frac{1}{2}$. (6.532)

6.7.3.4. Categories of Atomic Quantum Numbers

C Jens Erfurt Andresen, M.Sc. Physics, Denmark

For the overall wavefunction Ψ for the atomic electrons, we use the Casimir scalar operator I^2 in eigenvalue equation $J^2 \Psi = j(j+1)\Psi$, (6.335), (6.347). Ignoring the electron spin¹/₂ we get by $j \rightarrow \ell$ the overall eigenvalue $\ell(\ell + 1)$ for electron orbital *directional* states in atoms.

- The Bohr atom model gives *direction*-free scalar energy *principal shell numbers* $n \in \mathbb{N}$.
- The *directional* angular revolving *subshell quantum numbers* $\ell \in \mathbb{N}$, $\ell < n$.
- The *direction* projection of angular momentum with magnetic impact $m_{\ell} \in \mathbb{N}$, $|m_{\ell}| < n$.
- The *directional* angular spin¹/₂ of each electron with magnetic emergence $m_{1/2} = \pm \frac{1}{2} \rightarrow \pm \frac{1}{2} \hbar e/m_1$

These four number quantities are in combination exclusive unique for each electron in an atom. In Table 6.3 we have tried to show the connection between these *quantum numbers* for each electron participating in one atom.

Internal in one atom the extension exclusion principle demand only one state of each occupied.

-316

Research on the a priori of Physics

	R	- 6.7.3.	Orbital An	gular Mome	entum – 6.	7.3.5 Atom	s in Practise	e –
	Research on the a priori of Physics Geometric Critique of Pure Mathematical Reasoning	Table 6.3.4	tomic qua	ntum numbe	exclusive	ve scalar sh	ell direction	nal
	hr ric		shell	subshell		spin ¹ /2	j =	
	Cl C		n	l	m_{ℓ}	$m_{1/2}$	$\left \ell+m_{\frac{1}{2}}\right $	m
	1 rit		1	0	0	$+\frac{1}{2}$	$\frac{1}{2}$	
	\mathbf{O}		1	0	0	$-\frac{1}{2}$		
	n ue		2	0	0	$+\frac{1}{2}$	$\frac{1}{2}$	
	tł oj		2	0	0	$\frac{-\frac{1}{2}}{+\frac{1}{2}}$	$ \begin{array}{r} 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 1 \\ 2 \\ 3 \\ 2 \\ 3 \\ 2 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5$	
	ne rp		2	1	0	$+\frac{1}{2}$	$\frac{3}{2}$	
	a ure		2	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	
	.e. 1		2	1	+1	$+\frac{1}{2}$	$\frac{3}{2}$	
			2	1	+1	$-\frac{1}{2}$	$\frac{1}{2}$	
	ri atl		2	1	-1	$+\frac{1}{2}$	$\frac{3}{2}$	
	iori		2	1	-1	$-\frac{1}{2}$	$\frac{1}{2}$	
	na.		3	0	0	$+\frac{1}{2}$	$\frac{1}{2}$	
	0 Itid		3	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	
•	\mathbf{f}		3	1	0	$+\frac{1}{2}$	$\frac{3}{2}$	
	P I R		3	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	
r	h		3	1	+1	$+\frac{1}{2}$	$\frac{3}{2}$	
	hysics easoning		3	1	+1	- 1/2	$\frac{1}{2}$	
	S1		3	1	-1	$+\frac{1}{2}$	$\frac{3}{2}$	
	CS ing		3	1	1	$-\frac{1}{2}$	$\frac{1}{2}$	
			3	2	0	$+\frac{1}{2}$	$\frac{3}{2}$	
17.			3	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{3}{2}$	
1/2·			3	2	+1	$+\frac{1}{2}$	2	
			3	2	+1	$-\frac{1}{2}$	$\frac{3}{2}$	
	Je		3		-1	$+\frac{1}{2}$	$\frac{1}{2}$	
	n lit		3	$\frac{2}{2}$	+2	$-\frac{1}{2}$	2	
	S. S		3		+2	$+\frac{1}{2}$ $-\frac{1}{2}$	$ \frac{5}{2} \frac{3}{2} \frac{5}{2} \frac{3}{2} \frac{5}{2} \frac{3}{2} \frac{5}{2} \frac{5}{2} \frac{5}{2} \frac{5}{2} $	
	ηĹ		3		-2	$+\frac{1}{2}$	<u>2</u> <u>5</u>	
	$\frac{1}{2}$		3	2	-2	$-\frac{1}{2}$	$\frac{2}{\frac{3}{2}}$	
)				2	2	2	
ι_e .) 2		Atoms in 1 For the gr	ractise	e of H <i>vdr</i>	ogen and	He <i>lium</i> . v	ve
	\sim		-	shell n =	•	0		
	20		-	ical symn		-		
	1d			number Z			-	
	r (Im Z = 3				
	S			1 filled in				
	resen		for Deryl	lium Z =	т, sneti N	$-2, t_3$	$-i_4 - 0$, n
	n							

December 2022

.3 Atomic quantum numbers, exclusive scalar shell, directional subshell, magnetic projection, and spin¹/₂.

6.3 A	tomic quar	ntum numbe	ers, exclusiv	e scalar she	ell, direction	nal subshel	l, magnetic	projection,
	shell	subshell	project	spin ¹ / ₂	<i>j</i> =	<i>m</i> =	$\lambda =$	orbital
	n	ℓ	m_ℓ	$m_{1/2}$	$\ell + m_{\frac{1}{2}}$	$m_{\ell}+m_{\frac{1}{2}}$	j(j+1)	$\ell(\ell+1)$
Ī	1	0	0	$+\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{3}{4}$	0
	1	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{3}{4}$	0
Ī	2	0	0	$\frac{-\frac{1}{2}}{+\frac{1}{2}}$	$\frac{1}{2}$	$+\frac{1}{2}$ $+\frac{1}{2}$ $+\frac{3}{2}$	$\frac{3}{4}$	0
	2	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{3}{4}$	0
Ī	2	1	0	$+\frac{1}{2}$	$\frac{3}{2}$	$+\frac{1}{2}$	$\frac{15}{4}$	2
	2	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{4}$	2
Ī	2	1	+1	$+\frac{1}{2}$	$\frac{3}{2}$	$+\frac{3}{2}$	$\frac{15}{4}$	2
	2	1	+1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{4}$	2
Ī	2	1	-1	$ \begin{array}{r} -\frac{1}{2} \\ +\frac{1}{2} \\ +\frac{1}{2} \\ -\frac{1}{2} \\ \end{array} $	$ \begin{array}{r} 1\\ 2\\ 2\\ 1\\ 2\\ 2\\ 1\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\$	$-\frac{1}{2}$	$ \begin{array}{r} 3 \\ 3 \\ 4 \\ $	2
	2	1	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	$\frac{3}{4}$	2
Ī	3	0	0	$+\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{3}{4}$	0
	3	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{4}$	0
Ī	3	1	0	$+\frac{1}{2}$	$\frac{3}{2}$	$+\frac{1}{2}$	$\frac{15}{4}$	2
	3	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{4}$	2
Ī	3	1	+1	$+\frac{1}{2}$	$\frac{3}{2}$	$+\frac{3}{2}$	$\frac{15}{4}$	2
	3	1	+1	$-\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{3}{4}$	2 2
Ī	3	1	-1	$+\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{15}{4}$	
	3	1	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	$\frac{3}{4}$	$\frac{2}{6}$
Ī	3	2	0	$+\frac{1}{2}$	$\frac{5}{2}$	$+\frac{1}{2}$	$\frac{35}{4}$	6
	3	2	0	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{15}{4}$	6
Ī	3	2	+1	$+\frac{1}{2}$	$\frac{5}{2}$	$+\frac{3}{2}$	$\frac{35}{4}$	6
	3	2	+1	$-\frac{1}{2}$	$\frac{3}{2}$	$+\frac{1}{2}$	$\frac{15}{4}$	6
Ī	3	2	-1	$+\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{35}{4}$	6
	3	2	-1	$ \begin{array}{r} -\frac{1}{2} \\ +\frac{1}{2} \\ +\frac{1}{2} \\ -\frac{1}{2} \\ +\frac{1}{2} \\ +$	$\frac{3}{2}$	$-\frac{3}{2}$	$\frac{\frac{15}{4}}{\frac{35}{4}}$	6
Ī	3	2	+2	$+\frac{1}{2}$	$\frac{5}{2}$	$+\frac{5}{2}$	$\frac{35}{4}$	6
	3	2	+2	$-\frac{1}{2}$	$\frac{3}{2}$	$+\frac{3}{2}$	$\frac{15}{4}$	6
Ī	3	2	-2	$+\frac{1}{2}$	$ \frac{\frac{1}{2}}{\frac{3}{2}} \frac{\frac{1}{2}}{\frac{1}{2}} \frac{\frac{3}{2}}{\frac{1}{2}} \frac{\frac{3}{2}}{\frac{1}{2}} \frac{\frac{5}{2}}{\frac{3}{2}} \frac{\frac{5}{2}}{\frac{3}{2}} $	$ \begin{array}{c} +\frac{1}{2} \\ +\frac{1}{2} \\ -\frac{1}{2} \\ +\frac{3}{2} \\ -\frac{1}{2} \\ +\frac{3}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ +\frac{1}{2} \\ -\frac{1}{2} \\ +\frac{1}{2} \\ -\frac{1}{2} \\ +\frac{1}{2} \\ -\frac{1}{2} \\ +\frac{3}{2} \\ +\frac{3}{2} \\ +\frac{3}{2} \\ -\frac{3}{2} \\ -\frac{3}{2} \\ -\frac{3}{2} \\ -\frac{5}{2} \end{array} $	$\frac{\frac{15}{4}}{\frac{35}{4}}$	6
	3	2	-2	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{5}{2}$	$\frac{15}{4}$	6
۲.								

.5. Atoms in Practise

For the ground state of Hydrogen and Helium, we have $\ell = 0$. There can only be two electrons in this scalar shell n = 1. The Helium nucleus is where two electrons exclude each other by extension in a spherical symmetric completion fulfilling (6.527) and achieving j = 1 for Helium with Z = 2. For atom number $Z \in \mathbb{N}$ we index the electron by i = 1, 2, 3, ..., Z. Each individual electron has $j_i = \frac{1}{2}$. For Lithium Z = 3 with $j_{\text{Li}} = j_1 + j_2 + j_3 = |\ell_1 \pm \frac{1}{2}| + |\ell_2 \mp \frac{1}{2}| + |\ell_3 \pm \frac{1}{2}| = \frac{3}{2}, \ \ell_1 = \ell_2 = \ell_3 = 0,$ shell n = 1 filled in balance, and shell n = 2, $\ell_3 = 0$ has $j_3 = \frac{1}{2}$ and one electron spin $m_{\frac{1}{2}3} = \pm \frac{1}{2}$ For Beryllium Z = 4, shell n = 2, $\ell_3 = \ell_4 = 0$, $m_{\frac{1}{2},3} + m_{\frac{1}{2},4} = 0$.

© Jens Erfurt Andresen, M.Sc. NBI-UCPH,

-317

For quotation reference use: ISBN-13: 978-8797246931

For quotation reference use: ISBN-13: 978-8797246931