The energy change between these electron orbital levels results in a spectral energy exchange after Rydberg's formula for the *Hydrogen* spectrum

(6.529) $$\hbar\omega \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right).$$

From this, we conclude to know that the binding energy level of the *Hydrogen* electron is

(6.530) $$E_n \approx -\frac{13.6\text{ eV}}{n_2^2}, \qquad \text{in general, for atoms } E_n \propto -\frac{Z^2}{n_2^2}$$

We distinguish the different atom nucleus charge ***quantities*** by their atomic number Z, which corresponds to the number of electrons in each atom.

6.7.3.3. Atomic Shells and Subshells

This (6.530) gives electrons $\Psi_{½}$ in energy levels of a *principal **quantum** number* $n \in \mathbb{N}$.
These so-called energy *shells* do not imply any specific ***direction*** at all.
For the ***directional*** *orbital angular momentum,* we introduce a *subshell **quantum** number*

(6.531) $$\ell \in \mathbb{N}, \ \ell < n,$$

that is counted exclusively from spin½ ***quantum*** *number* $m_{½} = \pm½$ for each electron $\Psi_{½}$.

For the unexcited H*ydrogen* first *shell* $n = 1$, the electron does not possess any orbital angular momentum $\ell = 0$ and its spin½ angular momentum $|L_{½}| = \hbar½$ is compensated by the nucleus or easier by a sibling H*ydrogen* atom in a H_2 molecule.
For the unexcited He*lium*, where there can be two electrons $\Psi_{½}$ with *shell* number $n = 1 \Rightarrow l = 0$.
Both have no orbital angular momentum $\ell = 0$ (no *subshell*). The two-electron spin½ balance each other in a spherical symmetric atom.
In the Bohr-Rutherford atomic model, it is common to separate and ignore the inner spin½ angular momentum of each ***entity*** $\Psi_{½}$. The reason is that spin is neutralised in spin pairs e.g., as (6.527).
The classical electron was viewed as a point particle. The atomic electrons are now just an oscillating ***entity*** in *shells* and *subshells* revolving around the nucleus.
For shell principal **quantum** number $n > 1$ we can have subshell numbers $\ell = 0,1,2, \ldots n-1$.
For the ***direction*** projection of the orbital angular momentum, we have the quantum integer number m_l where $|m_\ell| \le \ell \Rightarrow m_\ell = -\ell \ldots -1,0,1 \ldots \ell$.
We make the resultant $m = m_\ell \pm ½$ that is a non-integer, half-valued, for each single electron $\Psi_{½}$.
From the orbital angular momentum number ℓ of each electron $\Psi_{½}$ we find the total spin as

(6.532) $$j = |\ell \pm ½| \qquad \Rightarrow \qquad \ell = j + ½ \quad \text{or} \quad \ell = j - ½.$$

6.7.3.4. Categories of Atomic Quantum Numbers

For the overall wavefunction Ψ for the atomic electrons, we use the Casimir scalar operator J^2 in eigenvalue equation $J^2\Psi = j(j+1)\Psi$, (6.335), (6.347). Ignoring the electron spin½ we get by $j \to \ell$ the overall eigenvalue $\ell(\ell+1)$ for electron orbital ***directional*** states in atoms.

- The Bohr atom model gives ***direction***-free scalar energy *principal shell numbers* $n \in \mathbb{N}$.
- The ***directional*** angular revolving *subshell quantum numbers* $\ell \in \mathbb{N}, \ \ell < n$.
- The ***direction*** projection of angular momentum with magnetic impact $m_\ell \in \mathbb{N}, \ |m_\ell| < n$.
- The ***directional*** angular spin½ of each electron with magnetic emergence $m_{½} = \pm½ \to \pm½\hbar e/m_e$.

These four *number **quantities*** are in combination exclusive unique for each electron in an atom.
In Table 6.3 we have tried to show the connection between these ***quantum*** *numbers* for each electron participating in one atom.
Internal in one atom the extension exclusion principle demand only one state of each occupied.

Table 6.3 Atomic quantum numbers, exclusive scalar *shell*, ***directional*** *subshell*, magnetic projection, and spin½.

shell n	*subshell* ℓ	project m_ℓ	spin½ $m_{½}$	$j = \lvert\ell + m_{½}\rvert$	$m = m_\ell + m_{½}$	$\lambda = j(j+1)$	orbital $\ell(\ell+1)$
1	0	0	$+\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{3}{4}$	0
1	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{3}{4}$	0
2	0	0	$+\frac{1}{2}$	$\frac{1}{2}$	$+\frac{3}{2}$	$\frac{3}{4}$	0
2	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{3}{4}$	0
2	1	0	$+\frac{1}{2}$	$\frac{3}{2}$	$+\frac{1}{2}$	$\frac{15}{4}$	2
2	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{4}$	2
2	1	+1	$+\frac{1}{2}$	$\frac{3}{2}$	$+\frac{3}{2}$	$\frac{15}{4}$	2
2	1	+1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{4}$	2
2	1	−1	$+\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{15}{4}$	2
2	1	−1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	$\frac{3}{4}$	2
3	0	0	$+\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{3}{4}$	0
3	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{4}$	0
3	1	0	$+\frac{1}{2}$	$\frac{3}{2}$	$+\frac{1}{2}$	$\frac{15}{4}$	2
3	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{4}$	2
3	1	+1	$+\frac{1}{2}$	$\frac{3}{2}$	$+\frac{3}{2}$	$\frac{15}{4}$	2
3	1	+1	$-\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{3}{4}$	2
3	1	−1	$+\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{15}{4}$	2
3	1	−1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	$\frac{3}{4}$	2
3	2	0	$+\frac{1}{2}$	$\frac{5}{2}$	$+\frac{1}{2}$	$\frac{35}{4}$	6
3	2	0	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{15}{4}$	6
3	2	+1	$+\frac{1}{2}$	$\frac{5}{2}$	$+\frac{3}{2}$	$\frac{35}{4}$	6
3	2	+1	$-\frac{1}{2}$	$\frac{3}{2}$	$+\frac{1}{2}$	$\frac{15}{4}$	6
3	2	−1	$+\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{35}{4}$	6
3	2	−1	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	$\frac{15}{4}$	6
3	2	+2	$+\frac{1}{2}$	$\frac{5}{2}$	$+\frac{5}{2}$	$\frac{35}{4}$	6
3	2	+2	$-\frac{1}{2}$	$\frac{3}{2}$	$+\frac{3}{2}$	$\frac{15}{4}$	6
3	2	−2	$+\frac{1}{2}$	$\frac{5}{2}$	$-\frac{3}{2}$	$\frac{35}{4}$	6
3	2	−2	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{5}{2}$	$\frac{15}{4}$	6

6.7.3.5. Atoms in Practise

For the ground state of H*ydrogen* and He*lium*, we have $\ell = 0$. There can only be two electrons in this scalar *shell* $n = 1$. The He*lium* nucleus is where two electrons exclude each other by extension in a spherical symmetric completion fulfilling (6.527) and achieving $j = 1$ for He*lium* with $Z = 2$.
For atom number $Z \in \mathbb{N}$ we index the electron by $i = 1,2,3, \ldots Z$. Each individual electron has $j_i = \frac{1}{2}$.
For Li*thium* $Z = 3$ with $j_{\text{Li}} = j_1 + j_2 + j_3 = |\ell_1 \pm ½| + |\ell_2 \mp ½| + |\ell_3 \pm ½| = \frac{3}{2}$, $\ell_1 = \ell_2 = \ell_3 = 0$,
shell $n = 1$ filled in balance, and *shell* $n = 2$, $\ell_3 = 0$ has $j_3 = \frac{1}{2}$ and one electron spin $m_{½,3} = \pm½$.
For Be*ryllium* $Z = 4$, *shell* $n = 2$, $\ell_3 = \ell_4 = 0$, $m_{½,3} + m_{½,4} = 0$.

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