6.6.1.3. Three Linear Independent Internal Components of Angular Momenta of Fermion Structure

Since Immanuel Kant 1747, *"Von der Kraft der Körper überhaupt"* [390] argued for three dimensions from Kepler's chronometric area law and Newtons' squared distance law, we have fundamentally assumed 𝔍-space of physics as canonical three-dimensional.

The principle of angular momentum as chronometric areas we concept make as bivectors and constitute in duality with 1-vectors. For a unit, we write the symbols $i = \imath u$ and express the three-dimensional linear independent ***directions*** by the dual orthogonal basis $\{i_1, i_2, i_3\} = \imath\{\sigma_1, \sigma_2, \sigma_3\}$. To keep it reasonable we demand such a basis to be autonomous for just one indivisible fermion. Above in Chapter 6.5, we have argued that spin½ stats of fermion ***entity*** $\Psi_{½}$ can consist of three orthogonal circular oscillating 1-spinors possessing half radian angular momenta $\pm\frac{1}{2}i_k$ components, that in their indivisible composition of each orientation perform distinguishable properties as described in § 6.5.8.7 by combinations (6.444): **1. Chirality**, **2. Spin orientation**:

1. The serious impact differentiation is chirality volume with orientations *dextral* and *sinistral*.
2. The spin *up* or *down* orientation is an easy differentiation. It shifts with no internal effort. External spin orientation can just be shifted by turning the ***entity*** $\Psi_{½}$ opposite by rotating π around one external axis perpendicular to the external spin axis, which can be detected (measured). This turn may of course have an impact on the relation to the surroundings and be observable.
3. We find that *parity inversion* of all 1-vectors is just in duality with *versor reversion*, this gives $\Psi_{\mathbb{H}}\widetilde{\Psi}_{\mathbb{H}} = \widetilde{\Psi}_{\mathbb{H}}\Psi_{\mathbb{H}} = \Psi_{\mathbb{H}}\Psi_{\mathbb{H}}^{\dagger} = \Psi_{\mathbb{H}}^{\dagger}\Psi_{\mathbb{H}} = |\Psi_{\mathbb{H}}|^2 = 1.$
   This *reversion* operation gives no new observable different state of any fermion ***entity*** $\Psi_{½}$.

In the tradition, we say that a spin½ fermion ***entity*** state $\Psi_{½} \to |\lambda, m\rangle \to |\frac{3}{4}, \pm\frac{1}{2}\rangle$ has the ***quantum*** state ***direction*** number $j = \frac{1}{2}$ with orientation number $m = \pm\frac{1}{2}$ and the total quantum number $\lambda = \frac{3}{4} = j(j+1)$ for the total squared angular momentum $J^2 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = \frac{3}{4}$. (Casimir).

The symmetric autonomy model of three orthogonal oscillating 1-spinors possessing half radian angular momenta $\pm\frac{1}{2}|i_k|$ has square sum $\left(\frac{1}{2}\right)_1^2 + \left(\frac{1}{2}\right)_2^2 + \left(\frac{1}{2}\right)_3^2 = \frac{3}{4}$ with a lack $\frac{1}{4} = \left(\frac{1}{2}\right)_0^2 \sim \lambda_0^2$ in one whole for a versor form of four-angular-momenta (6.456)

(6.511) $$\Psi_{\mathbb{H}} = \lambda_0 + \lambda_1 i_1 + \lambda_2 i_2 + \lambda_3 i_3 \in \mathbb{H} \sim \mathcal{G}_{0,2}(\mathbb{R}), \quad \text{where } \Psi_{\mathbb{H}}^{\dagger}\Psi_{\mathbb{H}} = \lambda_0^2 + \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 1$$

The dialectic complement development oscillation plays their trio music in harmony, as (6.447)

(6.512) $$\psi_{k\pm}^{½} = \varrho_k e^{\pm i_k ½\phi_k} = \varrho_k(\cos ½\phi_k \pm i_k \sin ½\phi_k), \qquad \text{(6.408)-(6.414) and (6.431)-(6.432).}$$

This '*playing music trio*'[391] of three orthogonal oscillators each with amplitudes λ_k combined in a scalar oscillation with amplitude λ_0 and what we in all will call a four impact $\sum \lambda_\mu^2 = \lambda_\mu \lambda_\mu = 1$ from the internal frequency energies, autonomous possibly chronometric $|\omega_{¼}| = 1$ each.

There are four dimensions, three angular momentum bivectors ***directions*** and one common scalar for the four oscillators. Even though the ***direction qualities*** are ***quantitative*** linear independent, their oscillation spinors are interconnected and perform one unified indivisible versor 2-rotor wavefunction of the type (6.451) $\psi_{+½} = U = +u_0 + u_1 i_1 + u_2 i_2 + u_3 i_3$, $u_\mu u_\mu = 1$, and $UU^{\dagger} = 1$.

6.6.2. Fermions have by Tetraon Symmetry from the Platonic Tetrahedron Idea

Although the possible circle oscillations by 1-spinors internal in any fermion ***entity*** $\Psi_{½}$ are inter-connected. It is possible to accept multiple of them as long as they sum synchronise to a versor 2-rotor structure of the even geometric algebra $\mathcal{G}_{0,2}(\mathbb{R})$. Or alternatively the isomorphic resulting in unit spinor representation of the Special Unitary group $SU(2)$ by complex 2×2 matrices.

We have seen, that four autonomy ***directions*** 1-vectors $\mathbf{u}_\mu$ orientated ***outwards*** in a local *regular tetraon basis structure* symmetric in 𝔍-space balance itself to null. But the four ***direction*** different

[390] See the German text in footnote 178, or https://korpora.zim.uni-duisburg-essen.de/kant/aa01/023.html .

[391] The word '*music*' is used instead of *a picture* we cannot see. Its external estimated '*pitch*' frequency may be $\omega_{¼} \sim ½mc^2/\hbar$.

possible oscillating circular 1-spinors (6.486) possess each two different orientations of their angular momentum bivectors $\mathbf{S}_\mu = \pm\frac{1}{2}\imath\mathbf{u}_\mu$ parallel to the faces of the belonging *regular tetrahedron* with unit circumscribed sphere describing the S^3 symmetry $u_\nu u_\nu = 1$ of the versor idea $\mathbb{H} \sim \mathcal{G}_{0,2}(\mathbb{R})$ in § 6.6.1.2. We recall the four dual ***outwards directions*** from a local ***entity*** $\Psi_{½}$ center with the two orientations $\pm\frac{1}{2}\mathbf{u}_\mu$ that we combine as (6.473) and achieved the scalar ***quantities*** (6.474)

(6.513) $$q = \mathbf{u}_\kappa \sum_{\mu=0}^{3} P_{\mathbf{u}_\kappa}\left(\pm\tfrac{1}{2}\mathbf{u}_\mu\right) = 1, \ \tfrac{2}{3}, \ \tfrac{1}{3}, \ 0, \ -0, \ -\tfrac{1}{3}, \ -\tfrac{2}{3}, \ -1.$$ [392]

The condition for this is $\mathbf{u}_\mu^2 = 1$ and the regular tetraon demand $\mathbf{u}_\mu \cdot \mathbf{u}_\nu = \frac{1}{3}$, for $\mu \neq \nu$; $\kappa, \mu, \nu = 0,1,2,3$. We use the idea of full spherical symmetry to remove any preferred projection ***direction*** $\mathbf{u}_\kappa$ to the surroundings by multiplication with this ***direction*** unit $\mathbf{u}_\kappa$ of the regular tetraon basis, which is a free autonomy for any fermion ***entity*** $\Psi_{½}$.

These $2^3 = 8$ real scalar ***quantities*** characterise differences in the ***category*** of identical ***entities*** $\Psi_{½}$. This is possible a *charge* ***quantity*** cargo endowment ***quality*** for each fermion ***entity*** $\Psi_{½}$.

We see in (6.473) that the fractional q's have three generic cases each may give a ***qualitative*** difference between ***entities*** when they interacts with their siblings. This is possibly what is called *chromatic* ***qualities*** in the Standard Model. In all, we have sixteen 2^4 different ***quality*** types of fermions in the ***category*** of identical fermions ***entities*** $\Psi_{½}$ in 𝔍-space of physics, which possess the full geometric algebra $\mathcal{G}_3(\mathbb{R}) = \mathcal{G}_3^-(\mathbb{R}) + \mathcal{G}_{0,2}(\mathbb{R})$. This Geometric Algebras is the foundation of the ideology of ***entities*** $\Psi_{½}$ that are *fermions* with the ***primary quality of spin one-half*** (*spin½*).

The chirality ***quality*** differentiation in the identity ***category*** is included in the sixteen $2^4 = 16$ different types of fermions $\Psi_{½}$ in (6.473), in that three independent ***directions*** are sufficient to determine chirality. It is up to the reader to do this in the specific lab measurement situation.

The external spin property is not an internal differentiation identifier for any fermion $\Psi_{½}$.

The one-half spin½ ***quality*** appears when the fermion interacts with the surroundings in a state, what it de facto always do (else we or its interacting siblings do not know it exists). We have argued that an internal shift of two of three orthogonal oscillating orientations does the spin shift. For the regular tetraon music quartet we need a shift for two out of four oscillating orientations, to make the internal spin½ change that has an external impact (is observable):

- For both charges $q = \pm 1$ the spin½ shift is straight internal effortless.
- For fractional charged the spin½ shift is joined by a 'colour' change.

But for charge $q = \pm 0$ this internal spin½ shift is double, a total shift of all four oscillating orientations. Maybe we can associate this with neutrino oscillations?

6.6.3. The *Categorical Quality* of Spin½ Fermions in 𝔍-space

From the Stern-Gerlach 1922 experiment and many other, we have measured different location impacts of the fermion ***quality*** for the individual existence of ***entities*** $\Psi_{½}$.

- The *reality* is that we actually have detected such ***entity*** $\Psi_{½}$ as spin½ fermions.

For each ***entity***, we by autonomy have to define a locality in an idea of a 𝔍-space. Presuming this established by the intersection point of at least three planes[393] of circle oscillator components it might be orthogonalized with the angular momenta $\pm\frac{1}{2}i_1, \pm\frac{1}{2}i_2, \pm\frac{1}{2}i_3$, which by the dextral chiral pseudoscalar $\imath$ have six ***outwards*** orientated ***directions*** $+\sigma_1, +\sigma_2, +\sigma_3, -\sigma_1, -\sigma_2, -\sigma_3$. Each of these should in an external view indicate six interconnected oscillators, but the parity inversion of each will either cancel out or be one and the same orientated angular momentum. The out cancelling of one of the orthogonal components would disagree with (6.289) and make the idea of such a fermion disappear from the ***quality category*** of one ***entity*** $\Psi_{½}$ in 𝔍-space.

[392] This simple formulation by any symmetric ***direction*** projection was first formulated by the author of this book august 2nd 2019.

[393] We will sometimes call these Kepler planes in association with a classical oscillation of a satellite around its planet.