angular momentum components from oscillation in these planes can form a locality ***entity*** Ψ_3 that makes physical existence possible. Anyway, the idea that a singular point can have any objective existence is an illusion and can certainly not be a physical ***entity*** with any ***quality*** in 𝔍-space except the zero vector $\mathbf{0} = 0$ as the scalar 0. In practice, we can use single points as an exclusive location center for every single individual fermion ***entity*** $\Psi_{½} \leftarrow \Psi_3$.
Classically this is called a point particle, and its ***quantities*** are linked to this in our naïve minds.

6.5.13. The Fundamental Concept of *Direction* Locality in Space

Why is the plane idea of the quaternion basis of the even geometric algebra $\mathbb{H}\sim\mathcal{G}_{0,2}(\mathbb{R})$ so mandatory to get a fundamental understanding of the invisible structure of locality?
First, it is an a priori issue for topology, as foundation principia for ***primary qualities*** of 𝔍-space.
Gottfried Leibniz addressed the issue by the concept of *geometria situs* (for the object idea) and *analysis situs* (for the subject idea), he approached this issue as pure relations, an issue later Euler operationalised by geometric relations (e.g. polyhedrons structures: $V{-}E{+}F{=}2$).
The first to criticise this as an inadequate concept was Immanuel Kant 1768, [11]p.361-372
Concerning the ultimate ground of the differentiation of directions in space,[387]

(6.506) Von dem Ersten Grunde des Unterschiedes der Gegenden im Raume.

In the newer analyses, we have ***categorised*** the concept of ***direction*** into several ***grades***.
For all these ***primary qualities of grades of directions***, there are just two states of orientation per ***grade*** dimension.
For the concept ***direction*** (der Gegenden im Raume) we resume our idea of this concept:
We first take a given line segment[388] as object AD and endow it with an orientation AD or DA as a ***direction of first grade*** and declare it as a 1-vector ***direction*** $\mathbf{a} = \overrightarrow{AD} = -\overrightarrow{DA}$.
Then we take the product of two independent of these $\mathbf{a}, \mathbf{b}$ and make a bivector $\mathbf{a}\wedge\mathbf{b}$ ***direction of second grade*** with the possibility of two angular orientations in a plane by $\mathbf{a}\wedge\mathbf{b}$ or $\mathbf{b}\wedge\mathbf{a} = -\mathbf{a}\wedge\mathbf{b}$ by reversing the product.
The exterior (outer) product $\mathbf{a}\wedge\mathbf{b}$ was first introduced by Hermann Grassmann in 1844.
Further the product of three 1-vectors to a trivector ***direction of third grade***, and the possibility of two chiral volume orientations $\mathbf{a}\wedge\mathbf{b}\wedge\mathbf{c}$ and its reverse orientation $\mathbf{c}\wedge\mathbf{b}\wedge\mathbf{a} = -\mathbf{a}\wedge\mathbf{b}\wedge\mathbf{c}$.
– Etc. for higher ***grades***.
This fundamental ***direction*** concept gives extra ***qualities*** to the traditional dimensionality of each ***grade***. The lowest ***grade zero*** is the ***quality*** without the ***direction*** of something.
The even closed geometric algebra $\mathcal{G}_{0,2}$ of ***grades pqg***-0 and ***pqg***-2, $\langle A\rangle^{+} = \langle A\rangle_0 + \langle A\rangle_2$, e.g.
$\mathbf{ab} = \mathbf{a}\cdot\mathbf{b} + \mathbf{a}\wedge\mathbf{b}$ instrumentalise the action ***direction quality*** issues as the ideas:
– of spinors, and rotors as unitary spinors,
– of angular momentum,
– and just the pure angular idea, as a ***quality*** *of change* as a circle arc area.
By this, we have the foundation idea of a plane as a subject in a space of physics.
In all, this idea of *angular change* concerns the action of
Kepler's second law, as the conservation of chronometric areas, or
just normalized as the *chronometric angular radian unit*[389] giving ***one quantum of action*** $\hbar{=}1$.
If we count the action of one whole cycle in a plane unit circle, we say that this ***quantum*** is $h{=}1$.

(6.507) $$h = 2\pi\hbar$$

We recall, there is no *chronometer* without a choice of an reference cyclic oscillator! (Chapter I.).

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[387] I would rather translate the title as "From the first causal foundation for distinguishability of directions in space".
[388] We presume the line segment is algebraically instrumentalised as Descartes did with a positive real number for its magnitude (Hestenes [10]p.5-7). And that the negative reals represent the opposite orientation of the positive. (refer section 4.4,(vii)).
[389] We note the conic section arc curve development measure is $[radius^{-1}]$. Angular area development is then $radius\cdot[radius^{-1}]$.

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6.6. Identical *Entities* in 𝔍-Space

The ***category*** of *identical* ***entities*** is dependent on all possible distinguishable ***primary qualities*** for these. For the geometric algebra $\mathcal{G}_3(\mathbb{R}) = \mathcal{G}(V_3,\mathbb{R})$ of physical 𝔍-space, we have learned that there are four different ***primary qualities grades*** (***pqg***-0-1-2-3) with four dimensions of ***odd grade*** and dual four dimensions of ***even grade***, as described in § 6.3, etc. Each of these dimensions can have two orientations that appear as $\pm$ in their eight ***quantities***. – : $\mathcal{G}_3(\mathbb{R}){=}\mathcal{G}_3^-(\mathbb{R}){+}\mathcal{G}_3^+(\mathbb{R})$:

- The ***odd grade*** algebra $\mathcal{G}_3^-(\mathbb{R})$:
  $\langle A\rangle^- = \langle A\rangle_1 + \langle A\rangle_3$ has three 1-vectors and one chiral volume pseudoscalar dimensions
- The ***even grade*** algebra $\mathcal{G}_3^+(\mathbb{R}) = \mathcal{G}_{0,2}(\mathbb{R}) \sim\mathbb{H}$:
  $\langle A\rangle^+ = \langle A\rangle_0 + \langle A\rangle_2$ has one scalar and three bivector dimensions

The two subalgebras are dual dependent $\langle A\rangle^+ = \boldsymbol{i}\langle A\rangle^- \Leftrightarrow \langle A\rangle^- = -\boldsymbol{i}\langle A\rangle^+$, where $\boldsymbol{i} \sim\langle A\rangle_3$.

Products of even elements $\langle A\rangle^+$ in $\mathcal{G}_3^+(\mathbb{R})$ stay closed inside the even algebra $\mathcal{G}_{0,2}(\mathbb{R}) = \mathcal{G}_3^+(\mathbb{R})$ as their ***quantities*** are autonomous internal for the indivisible fermion ***entity*** $\Psi_{½}$ in 𝔍-space.
The odd ***primary direction qualities*** are often easier to interpret intuitively; but the product of odd elements $\langle A\rangle^-$ in $\mathcal{G}_3^-(\mathbb{R})$ are lifted into the even algebra and stay closed trapped there; they are returned by the dextral chiral pseudoscalar $\boldsymbol{i} \sim\langle A\rangle_3$ that is given by the volume extension of the right-hand rule. This is inherited from of three linear independent line dimensions of odd ***grade*** 1-vector ***directions*** of one ***entity*** $\Psi_{½}$.The straight ***direction*** $\mathbf{a} = \overrightarrow{AB} \sim\langle A\rangle_1$ is given by the line extension e.g., as your arm or local mind is pointing at a star. These as odd element have by multiplication by the odd $\boldsymbol{i}$ dual transversal angular plane bivector ***directions***, that as even elements also consist of three linear independent dimensions inside their even algebra $\mathbb{H}\sim\mathcal{G}_{0,2}(\mathbb{R})$.
Due to this duality, we can restrict ourselves to the closed even algebra idea $\mathbb{H}\sim\mathcal{G}_{0,2}(\mathbb{R})$ to classify the ***category*** of *identical* fermion ***entities*** $\Psi_{½}$ by combination of internal interconnected ***quantities*** of the four dimensions of ***even grade primary qualities***, which is characteristic for 𝔍-space.

6.6.1.1. Classification of Fermions in the Structure of 𝔍-space

We have above through chapter 6 described the structure of 𝔍-space by geometric algebra $\mathcal{G}_3(\mathbb{R})$.
The even closed subalgebra $\mathbb{H}\sim\mathcal{G}_{0,2}(\mathbb{R}) \subset \mathcal{G}_3(\mathbb{R})$ carries, in essence, the structure, created on the dextral chirality unit (6.21) $\boldsymbol{i} \coloneqq \boldsymbol{\sigma}_3\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ for the Cartesian orthonormal basis $\{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\boldsymbol{\sigma}_3\}$ forming $\mathcal{G}_3^-(\mathbb{R})$ spanned by linear combination from basis $\{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\boldsymbol{\sigma}_3,\boldsymbol{i}\}$ as the substructure. Multiplication gives the lifted algebra $\mathcal{G}_{0,2}(\mathbb{R})$ spanned by linear combination from basis $\{1,\boldsymbol{i}_1,\boldsymbol{i}_2,\boldsymbol{i}_3\}$, this is just the quaternion basis for the four-dimensional unitary versor concept $U{=}\,u_0{+}u_1\boldsymbol{i}_1{+}u_2\boldsymbol{i}_2{+}u_3\boldsymbol{i}_3 \in\mathbb{H}$.
These two algebras the odd and the closed even gives in duality together $\mathcal{G}_3(\mathbb{R}){=}\,\mathcal{G}_3^-(\mathbb{R}){+}\mathcal{G}_{0,2}(\mathbb{R})$ the full structural basis (6.119) $\{1,\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\boldsymbol{\sigma}_3,\boldsymbol{i}_1,\boldsymbol{i}_2,\boldsymbol{i}_3,\boldsymbol{i}\}$ linearly spanned to all elements (6.120), from the $2^3{=}\,8$ mixed dimensions. The duality reduces this dextral structure to the fundamental interconnectivity expressed in (6.123) for the versor (6.145), (6.168) etc. of four dimensions (6.505).

6.6.1.2. The Simple Versor Form and the $SU(2)$ Algebra of the Complex 2×2 Matrix Group

The versor wavefunction form (6.454) $U = \psi_{+½} = \hat{Q} = u_0{+}u_3\boldsymbol{i}_3{+}u_2\boldsymbol{i}_2{+}u_1\boldsymbol{i}_1 = Q_3 1{+}Q_1\boldsymbol{i}_2$ ←(6.14
describe a possible angular development that the two orthogonal circular oscillating 1-spinors

(6.508) $$Q_3 = (u_0{+}u_3\boldsymbol{i}_3) = \varrho e^{+\boldsymbol{i}_3 ½\phi} \in\mathbb{H}, \qquad \leftarrow (6.170) \leftarrow (6.146),$$

(6.509) $$Q_1 = (u_2{-}u_1\boldsymbol{i}_3) = \rho e^{-\boldsymbol{i}_3 ½\psi} \in\mathbb{H}, \qquad \leftarrow (6.171) \leftarrow (6.147).$$

This model is a '*playing music duo*' of two spinning plane circular oscillators.
These have ***directions*** $\boldsymbol{i}_3{\rightarrow}1$ and $\boldsymbol{i}_2$ as a basis $\{1,\boldsymbol{i}_2\}$ for the equivalent isomorphic complex 2×2 matrix group representation $SU(2)$ (6.175) for the autonomy state of one fermion ***entity*** $\Psi_{½}$

(6.510) $$\begin{bmatrix} u_0{+}u_3\boldsymbol{i}_3 & -u_2{-}u_1\boldsymbol{i}_3 \\ u_2{-}u_1\boldsymbol{i}_3 & u_0{-}u_3\boldsymbol{i}_3 \end{bmatrix} \leftrightarrow U = Q_3 1 + Q_1\boldsymbol{i}_2 = u_0{+}u_1\boldsymbol{i}_1{+}u_2\boldsymbol{i}_2{+}u_3\boldsymbol{i}_3, \ \left\{\Updownarrow \begin{matrix} UU^{\dagger}{=}1 \\ u_\nu u_\nu{=}1 \end{matrix}\right..$$

These representations do not give much insight into the ***category*** of differentiating the *identities* of the ***primary category*** of indivisible fermion ***entities***. But the angular momentum idea does work.

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