has by comparing with (6.369) and (6.421) the ***pqg***-1 ***direction*** eigenvalue equation

(6.492) $$(J_+J_- - J_-J_+)\left|\tfrac{3}{4},\pm\tfrac{1}{2}\right\rangle_3 = 2\hbar \mathbf{j}_3\left|\tfrac{3}{4},\pm\tfrac{1}{2}\right\rangle_3 \doteq \pm\hbar^2\boldsymbol{\sigma}_3\left|\tfrac{3}{4},\pm\tfrac{1}{2}\right\rangle_3,$$

for change of orientation in the 1-vector ***direction*** as in (6.370), that in the dual bivector ***direction*** can be expressed by the *free subton* eigenvalue equation (6.401) $\mathbf{L}_3|1,\pm1\rangle_3 = \pm1\hbar\boldsymbol{i}_3|1,\pm1\rangle_3$ as an expression of ***direction*** orientation exchange $\pm1\hbar$ with the surroundings.
Neither does the operators $J_+ = J_-^\dagger$ and $J_- = J_+^\dagger$ *anticommute* by the operator construction (6.325)

(6.493) $$\tfrac{1}{2}\left(J_+J_+^\dagger + J_+^\dagger J_+\right) = J_\perp^2 = \mathbf{j}^2 - \mathbf{j}_3^2 = \mathbf{j}_1^2 + \mathbf{j}_2^2 = \mathbf{j}_\perp^2,$$

that as a ***pqg***-0 real scalar operator by (6.338)-(6.339) gives a positive expectation value

(6.494) $$\langle\lambda,m|J^2 - J_3^2|\lambda,m\rangle = \left\langle\tfrac{3}{4},\pm\tfrac{1}{2}\right|J_\perp^2\left|\tfrac{3}{4},\pm\tfrac{1}{2}\right\rangle \sim \hbar^2\lambda - \hbar^2m^2 = \hbar^2\tfrac{3}{4} - \hbar^2\left(\tfrac{1}{2}\right)^2 = \hbar^2\tfrac{1}{2}$$

It is worth noting that in the dual picture (6.324) $\mathbf{L}^2 - \mathbf{L}_3^2 = \mathbf{L}_\perp^2 = \mathbf{L}_1^2 + \mathbf{L}_2^2$, this is negative $-\hbar^2\frac{1}{2}$,
The internal part $\mathbf{L}_\perp = \boldsymbol{\imath}J_\perp = \boldsymbol{\imath}\mathbf{j}_\perp$ of one ***entity*** $\Psi_{½}$ represent the unobservable part of the fluctuating ***direction*** of the oscillation, which is transcendental to all external knowledge.
The interesting thing is when we multiply this by its reversed oscillating part $-\boldsymbol{\imath}\mathbf{j}_\perp$, we get

(6.495) $$(-\mathbf{L}_\perp)\mathbf{L}_\perp = (-\boldsymbol{\imath}\mathbf{j}_\perp)\boldsymbol{\imath}\mathbf{j}_\perp = \mathbf{j}_\perp^2 = J_\perp^2 \rightarrow \hbar^2\tfrac{1}{2}.$$

This is half the contribution to the one indivisible ***entity*** $\Psi_{½}$ in $\mathfrak{Z}$-space.
This half part we represent by the oscillating bivector 1-spinor (6.154) using $\rho = \sqrt{½}$,

(6.496) $$Q_1\boldsymbol{i}_2 = \sqrt{½}\,\boldsymbol{i}_\perp(½\psi) = u_1\boldsymbol{i}_1 + u_2\boldsymbol{i}_2 = \sqrt{½}\,e^{-\boldsymbol{i}_3½\psi}\boldsymbol{i}_2.$$

The other orthogonal half part we represent by the 1-spinor (6.146) using $\varrho = \sqrt{½}$,

(6.497) $$Q_3 = u_0 + u_3\boldsymbol{i}_3 = \sqrt{½}\,e^{+\boldsymbol{i}_3½\phi}.$$

The last we presume interacts by angular momentum with the external surroundings after the principle expressed by (6.492) governed by the ***direction*** expressed in (6.369)-(6.370) and (6.379)

(6.498) $$\mathbf{L}_3 \longrightarrow \langle\mathbf{L}_3\rangle_{½} = \pm\tfrac{1}{2}\hbar\boldsymbol{i}_3 = \pm\tfrac{1}{2}\hbar\boldsymbol{\imath}\boldsymbol{\sigma}_3, \quad \text{and dual} \quad \mathbf{j}_3 \longrightarrow \langle\mathbf{j}_3\rangle_{½} = \pm\tfrac{1}{2}\hbar\boldsymbol{\sigma}_3.$$

Together these two form the versor (6.145f)

(6.499) $$U = \hat{Q} = Q_3 1 + Q_1\boldsymbol{i}_2 = u_0 + u_3\boldsymbol{i}_3 + u_1\boldsymbol{i}_1 + u_2\boldsymbol{i}_2 \quad \in\mathbb{H} \sim \mathcal{G}_{0,2}(\mathbb{R}).$$

It is well known from (6.436) that a product with the Clifford conjugated of this is one

(6.500) $$|U|^2 = U\widetilde{U} = UU^\dagger = u_\mu u_\mu = 1.$$

For each of the two orthogonal spinors, we write their squared magnitudes

(6.501) $$|Q_1\boldsymbol{i}_2|^2 = Q_1\boldsymbol{i}_2\widetilde{Q_1\boldsymbol{i}_2} = (u_1\boldsymbol{i}_1 + u_2\boldsymbol{i}_2)(-u_1\boldsymbol{i}_1 - u_2\boldsymbol{i}_2) = \sqrt{½}\,e^{-\boldsymbol{i}_3½\psi}\boldsymbol{i}_2\sqrt{½}\,e^{+\boldsymbol{i}_3½\psi}(-\boldsymbol{i}_2) = \tfrac{1}{2},$$

(6.502) $$|Q_3|^2 = Q_3\widetilde{Q_3} = (u_0 + u_3\boldsymbol{i}_3)(u_0 - u_3\boldsymbol{i}_3) = \sqrt{½}\,e^{+\boldsymbol{i}_3½\phi}\sqrt{½}\,e^{-\boldsymbol{i}_3½\phi} = \tfrac{1}{2}.$$

Combined these squared magnitudes form a Pythagorean sum of orthogonal parts

(6.503) $$|U|^2 = |Q_3|^2 + |Q_1\boldsymbol{i}_2|^2 = \tfrac{1}{2} + \tfrac{1}{2} = 1$$

This we once again demand as a ***category*** of the fundamental ***quantity*** of one indivisible observable ***entity*** $\Psi_{½}$ ***quality*** of $\mathfrak{Z}$-space. This split in two parts of orthogonal magnitudes has been an essential characteristic of a spin½ fermion. This agrees with the treatment in section 6.4.5 above, of traditional commonly used complex 2×2 matrix form, that by multiplication stays inside the special unitary group $SU(2)$. By this we conclude isomorphism between $SU(2)$, the lifted Pauli group and the geometric algebra idea of versor group $\mathbb{H}\sim\mathcal{G}_{0,2}(\mathbb{R})$.
Here we judge this interpretation (6.501)-(6.503) to have two orthogonal angular development parameters (ϕ,ψ) for two cyclic oscillations that autonomously possess two angular momenta ***directions*** $\mathbf{L}_3^{\Psi_{½}} = \pm\frac{1}{2}\boldsymbol{\imath}\boldsymbol{\sigma}_3$ and orthogonal $\mathbf{L}_\perp^{\Psi_{½}}$. (Simplified this is two orthogonal $U(1)$ circular oscillators.)
In laboratory frame $\{\mathbf{e}_k\}$ we express this as $\mathbf{L}_3^{\Psi_{½}} = \pm\frac{1}{2}\boldsymbol{\imath}\hbar\mathbf{e}_3$ and $\mathbf{L}_\perp^{\Psi_{½}} = \sqrt{½}\,e^{+\boldsymbol{i}_3½(\phi-½\pi)}\boldsymbol{\imath}\hbar\mathbf{e}_2$. In this model, two angular momenta are sufficient, of which only the former is externally observable.

6.5.12. The strange Intuition of Locality in $\mathfrak{Z}$-space

When we humans try to image things, that are not direct objects to our perception we make use of the idea of a plane. We find a surface of an object, e.g., a stone, a wall, a board (Tabula) or even a piece of paper and draw something on it. The simplest thing from geometry is to draw lines in the plane surface (the use of a ruler is allowed). We get two intersecting lines that meet at a point, which makes our fundamental intuition of locality.

How do we intuit locality in free space where two imagine lines properly never meet? In the physical world as a premise for geometry two non-parallel imaginary planes intersects in a line, just as two walls meet in a corner of a room. Where this corner line meets the third-floor plane in a vertex. Such three non-parallel planes mutually intersect in just one point as the *locus situs* center. Of course, such an idea of imaginary planes has no objective existence in physics.[383]
The same as for non-objective existence for imaginary straight rectitude line segments.
Anyway, since Kepler's second law, for the substance of space we have accepted the line segment from the sun to a planet, joined by a line segment for the chronometric movement. These two forms are by multiplication a constant angle area. This we call for the angular momentum, the constant area that performs the plane subject, as angular chronometric movement of the planet.
We note Kepler's second law is a ***primary quality***, that is independent of the emergent concept of mass later introduced by Newton, which we now judge as a ***secondary quality***.[384]
This idea of a plane ***primary quality of second grade*** (***pqg***-2) substance for the idea of a fundamental angular momentum we use to create something in space. Is this plane subject imaginary? Yes, just as imaginary as the unit i in the complex number plane. In Geometric Algebra, we have just invented the unit bivector idea $\boldsymbol{i}$,[385] which as a fundamental subject holds the plane substance given by a possible excitation of ***one quantum*** of angular momentum. We need the idea of the intersection of three plane subjects to establish a center of *locus situs* from the void and by that the idea of at least three (possibly orthogonal) components of angular momentum.
The idea, three intersecting planes (⫲) establish a *locus situs* of the intersection point in $\mathfrak{Z}$-space.[386]
The third plane ***direction*** is implicitly given intersection of the two planes by the interconnectivity (6.123). We express this third plane with association to (6.126) as

(6.504) $$\boldsymbol{i}_1 \coloneqq \boldsymbol{\sigma}_3\boldsymbol{\sigma}_2 \quad\text{and}\quad \boldsymbol{i}_2 \coloneqq \boldsymbol{\sigma}_1\boldsymbol{\sigma}_3 \quad\Longrightarrow\quad \boldsymbol{i}_3 = \boldsymbol{\imath}\boldsymbol{\sigma}_3.$$

These are the fundamental three orthogonal unit plane segments that together with the real scalar idea form the quaternion basis of the concept of mutual interconnectivity (6.134)

(6.505) $$\{1,\boldsymbol{i}_1,\boldsymbol{i}_2,\boldsymbol{i}_3\} = \{+1 = \boldsymbol{i}_3\boldsymbol{i}_2\boldsymbol{i}_1,\ \ \boldsymbol{i}_1 = \boldsymbol{i}_2\boldsymbol{i}_3,\ \ \boldsymbol{i}_2 = \boldsymbol{i}_3\boldsymbol{i}_1,\ \ \boldsymbol{i}_3 = \boldsymbol{i}_1\boldsymbol{i}_2\},$$

for the even closed geometric algebra $\mathcal{G}_{0,2}$ describing the fundamental structure of the oscillating rotations with angular momentum in $\mathfrak{Z}$-space. (These oscillating spinors have ***pqg***-0 and ***pqg***-2 ***qualities***.)
When we humans think of these planes as objects, their intersection form an origo point.
These objects are indirect given by definition (6.31) $\boldsymbol{i}_1 \coloneqq \boldsymbol{\sigma}_3\boldsymbol{\sigma}_2$ for the first plane-segment, $\boldsymbol{i}_2 \coloneqq \boldsymbol{\sigma}_1\boldsymbol{\sigma}_3$ for the second, that intersects the first in the line segment object $\boldsymbol{\sigma}_3$, which has the dual plane-segment $\boldsymbol{\imath}\boldsymbol{\sigma}_3 = \boldsymbol{i}_3 \coloneqq \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$, that is transversal to $\boldsymbol{\sigma}_3$. All three are formed from the basis objects $\{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\boldsymbol{\sigma}_3\}$, that need an implicit origo to be an object for the plane construction.
This human need for objectification results in an infinite regress in a cyclic ring argue for a point origo. If we humans can accept the three planes as intersecting subjects as the idea of the substance of an intersection center of *locus situs*, then the interconnected entanglement of

[383] Recall the traditional idea of the imaginary unit $i \in\mathbb{C}$ for the idea of the complex plane, which is a subject from mathematics.
[384] The mass m is a proportionality factor for a force field concept (that acts synchronously all over an infinite universe eternally).
[385] You can intuit some displayed objects as object examples in Figure 5.14
[386] In reality, we only need two orthogonal planes to describe the hole $\mathfrak{Z}$ space if we control the orientation $(\pm\boldsymbol{\imath})$ and do not need a specific origin for a location. No *locus situs*.