The *Parity Inversion* inherits the orientation problem and alters both *chirality* and *spin* simultaneal. This distinguishing of orientation of chirality and spin½ in $\mathfrak{Z}$-space gives us, for the autonomous $\Psi_{½}$ ***entity*** four cases of distinguishable ***qualities***, 2×2=4 for one orientation of the first ***direction*** of $\mathrm{L}_1 = \frac{1}{2}\boldsymbol{i}_1$. The extra four from the *parity inverted* case are like mirrored[370] ***qualities*** for opposite orientations. In the versor case of $\Psi_{\mathbb{H}} \in \mathbb{H} \sim \mathcal{G}_{0,2}(\mathbb{R})$ the *Parity Inversion* is equivalent to *reversion* or *Clifford conjugation* and is used in the constitution demand $\Psi_{\mathbb{H}}\overline{\Psi_{\mathbb{H}}} = \Psi_{\mathbb{H}}\widetilde{\Psi_{\mathbb{H}}} = \Psi_{\mathbb{H}}\Psi_{\mathbb{H}}^{\dagger} = |\Psi_{\mathbb{H}}|^2 = 1$.

Looking at Figure 6.23 the reader should note that for each chirality the four 1-vector *tetraon* ***directions*** $\mathrm{j}^{\Psi_{½}}_{\mathrm{s}...}$ (same display colour) or $\mathrm{j}^{\Psi_{½}}_{\mathrm{d}...}$ form the vertex corners of a *tetrahedron*, and the opposite *parity inverted* 1-vectors have *its* four triangular faces transversal. The circumscribed sphere volume of this tetraon has the radius $\left|\mathrm{j}^{\Psi_{½}}_{\mathrm{s}...}\right| = \left|\mathrm{j}^{\Psi_{½}}_{\mathrm{d}...}\right| = \left|\mathrm{L}^{\Psi_{½}}_{\mathrm{s}...}\right| = \left|\mathrm{L}^{\Psi_{½}}_{\mathrm{d}...}\right| = \sqrt{3/4} = \sqrt{J^2}$

The founding idea for our basis $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3\}$ is defined to be *dextral* by objective construction. By multiplying these basis 1-vectors we lift the $\mathfrak{Z}$-space concept of physics into the closed even geometric algebra of versors $\mathbb{H} \sim \mathcal{G}_{0,2}(\mathbb{R})$. Therefore, we call the quaternion basis $\{1, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3\}$ for *dextral* too, and further that $\left\{\frac{1}{2}, \mathrm{L}_1, \mathrm{L}_2, \mathrm{L}_3\right\}$ (6.443) is the *dextral* foundation of three angular momenta bivectors and a scalar forming the quaternion components.
This we have combined into a versor idea of one four-angular-momentum function (6.439)

(6.445) $$\Psi_{\mathbb{H}} = \lambda_0 + \lambda_1\boldsymbol{i}_1 + \lambda_2\boldsymbol{i}_2 + \lambda_3\boldsymbol{i}_3 \in \mathbb{H} \sim \mathcal{G}_{0,2}(\mathbb{R}),\ \text{where}\ \Psi_{\mathbb{H}}^{\dagger}\Psi_{\mathbb{H}} = 1 \Leftrightarrow \lambda_\mu\lambda_\mu = 1,\ \text{for}\ \mu = 0,1,2,3.$$

In the tradition we always need the *reversed* expression to make an observable, e.g., one $1 = \Psi_{\mathbb{H}}\Psi_{\mathbb{H}}^{\dagger}$, therefor we concern this Clifford conjugation *reversion* as representing the same ***entity*** $\Psi_{½}$ state. The impact of the difference between $\Psi_{\mathbb{H}}$ and $\Psi_{\mathbb{H}}^{\dagger}$ is in the need to multiply from left or right in a product (vice versa). Both $\Psi_{\mathbb{H}}$ and $\Psi_{\mathbb{H}}^{\dagger} = \widetilde{\Psi_{\mathbb{H}}}$ are antagonists for observable existence, $\widetilde{\Psi_{\mathbb{H}}}\Psi_{\mathbb{H}} = 1$.
Compared with the autonomous snapshot (6.442) we see that the scalar component results in two cases $\langle\lambda_0\rangle_{½} = \pm\frac{1}{2}$, which can have an impact on the number of ***entity*** states of $\Psi_{½}$.
We then expect eight different ***categorical qualities*** of ***entities*** $\Psi_{½}$ in physical $\mathfrak{Z}$-space.

After this internal autonomous idea of three orthogonal angular momentum components founded in circular oscillations, we shift back to the dialectic complement idea of angular development of these and find as an alternative to the versor function (6.445), the development fluctuating versor wavefunction (6.425) $\psi_{½} = \hat{Q} = u_0 + u_3\boldsymbol{i}_3 + u_1\boldsymbol{i}_1 + u_2\boldsymbol{i}_2 \in \mathbb{H}$.
This is a merge by (6.414) $\psi_{\pm½} = \psi^{½}_{3\pm} + \psi^{½}_{2\pm} + \psi^{½}_{1\pm}$ given by ideas in § 6.5.8.2 and (6.415)-(6.416), where the u_μ's are trigonometric functions of a common development parameter.

6.5.9. The Internal Auto Synchronisation of an Indivisible-Atomic-Elementary *Entity*

The whole idea of one fundamental physical ***entity*** $\Psi_{½}$ presumes as an a priori demand, that all angular development internal in $\Psi_{½}$ is synchronised. That would say, that there has to be one common development parameter t given from one angular frequency energy reference ω. For simplicity we choose auto reference $|\omega| = 1$, then we reduce to the phase development $\varphi \leftarrow \omega t$.
We choose to define the internal ***direction*** $\boldsymbol{i}_3 = \boldsymbol{\imath}\mathbf{e}_3$ defined by the measurable spin½ ***direction*** $\mathbf{e}_3$ in a stationary lab frame $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$. The $\Psi_{½}$ internal autonomous quaternion frame $\{1, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3\}$ is a circle oscillator rotated as (6.252) by the phase angle development $\phi_3 = \pm\phi_{3,\omega} = \omega t$.

(6.446) $$\boldsymbol{i}_k(\phi_3) = \boldsymbol{\imath}\sigma_k(\phi_3) = e^{\boldsymbol{i}_3\phi_3}\boldsymbol{\imath}\mathbf{e}_k.$$

The rotation is along $\boldsymbol{i}_3$ around $\boldsymbol{\sigma}_3$ which is the undisturbed spin½ ***direction*** possessing the spin½ angular momentum $\mathrm{L}_3 = \pm\frac{1}{2}\boldsymbol{i}_3$. Now we include the two other internal autonomous orthogonal oscillating ***directions*** $e^{\boldsymbol{i}_1\phi_1}$ and $e^{\boldsymbol{i}_3\phi_3}$ that possess angular momentum $\mathrm{L}_1 = \pm\frac{1}{2}\boldsymbol{i}_1$ and $\mathrm{L}_2 = \pm\frac{1}{2}\boldsymbol{i}_2$.

[370] An analogy intuition perception of yourself in a plane mirror does not make you believe that your picture is another person.

The synchronisation demand, that the development of the other angles is $\phi_1 = \omega t + \theta_1$ and $\phi_2 = \omega t + \theta_2$. The overall symmetry phase factor $\odot_{\boldsymbol{i}_3} = \{\theta \to e^{\boldsymbol{i}_3\theta} | \forall\theta \in [0, 2\pi[\}$ gives $\phi_3 = \omega t + \theta$.
Now the synchronisation demand can be written

(6.447) $$\left.\begin{aligned} \phi_3 &= \pm\omega t + \theta \quad + 2\pi n \\ \phi_2 &= \pm\phi_3 + \theta_2 + 2\pi n \\ \phi_1 &= \pm\phi_2 - \pi + \theta_1 + 2\pi n \end{aligned}\right\}\ \theta, \theta_2, \theta_1 \in [0, 2\pi[\ \text{and}\ n \in \mathbb{Z},$$

were we for intuition use $\theta_1 = 0$ for choosing $\phi_1 = \pm\phi_2 \pm \pi$ as 0 when ϕ_3 is the driver.
We will not go further into what happens detailed inside ***entity*** $\Psi_{½}$, because the phase relation will be uncertain. What we have learned is that the normalized wavefunction for an ***entity*** $\Psi_{½}$ can be expressed as a versor quaternion (6.422) and (6.435)

(6.448) $$U = \hat{Q} = \boxed{\psi_{+½} = \left|\tfrac{3}{4}, +\tfrac{1}{2}\right\rangle = u_0 + u_1\boldsymbol{i}_1 + u_2\boldsymbol{i}_2 + u_3\boldsymbol{i}_3} \in \mathbb{H}$$

This is also what we call a 2-rotor from the even $\mathcal{G}_{0,2}(\mathbb{R})$ algebra isomorph with the $SU(2)$ group.

What we have learned from (6.444) is that a *spin*½ shift orientation from $m = +½$ to $m = -½$ change the *orientation* in *two* of the three orthogonal Kepler plane ***directions***, e.g., if we have

(6.449) $$\psi_{+½} = \left|\tfrac{3}{4}, +\tfrac{1}{2}\right\rangle = u_0 + u_1\boldsymbol{i}_1 + u_2\boldsymbol{i}_2 + u_3\boldsymbol{i}_3 \in \mathbb{H},\quad \text{for}\ m = +½,$$

then for a *spin*½ orientation shift, we can use

(6.450) $$\psi_{-½} = \left|\tfrac{3}{4}, -\tfrac{1}{2}\right\rangle = u_0 + u_1\boldsymbol{i}_1 - u_2\boldsymbol{i}_2 - u_3\boldsymbol{i}_3 \in \mathbb{H},\quad \text{for}\ m = -½,$$

in the $\boldsymbol{i}_3$ plane ***direction*** of the measurement.
The change of sign in one ***direction*** turns the *chiral* orientation from dextral to sinistral, vice versa.
The *reversal* of a versor 2-rotor (this is equivalent to parity inversion in the dual 1-vector space)

(6.451) $$U = \psi_{+½} = +u_0 + u_1\boldsymbol{i}_1 + u_2\boldsymbol{i}_2 + u_3\boldsymbol{i}_3, \quad \text{and} \quad U^{\dagger} = \psi^{\dagger}_{+½} = +u_0 - u_1\boldsymbol{i}_1 - u_2\boldsymbol{i}_2 - u_3\boldsymbol{i}_3.$$

The idea for the wavefunction for a physical ***entity*** $\Psi_{½}$ is that all four versor coordinates u_μ are oscillating trigonometric reals,[371] in a kind of synchronisation.
The strange thing with a versor $U \in \mathbb{H}$ is, that it is a 2-rotor in the geometric algebra $\mathcal{G}_{0,2}(\mathbb{R})$ containing a ***pqg***-0 scalar plus a ***pqg***-2 bivector, that looks like (6.143)

(6.452) $$U = u_0 + u\boldsymbol{i}_{\mathrm{n}}, \quad \text{where} \quad \boldsymbol{i}_{\mathrm{n}} = \boldsymbol{\imath}\mathbf{n} = \left(u_1\boldsymbol{i}_1 + u_2\boldsymbol{i}_2 + u_3\boldsymbol{i}_3\right)/\sqrt{1 - u_0^2}, \ \text{and} \ u^2 = 1 - u_0^2, \ (6.139).$$

This we can interpret as a circle 1-rotor in the $\mathcal{G}_2(\mathbb{R})$ plane of the bivector pseudoscalar $\boldsymbol{i}_{\mathrm{n}}$ ***direction*** that apparently can be written as

(6.453) $$U_{\varphi_{\mathrm{n}}} = u_0 + u\boldsymbol{i}_{\mathrm{n}} = 1\cos ½\varphi_{\mathrm{n}} + \boldsymbol{i}_{\mathrm{n}}\sin ½\varphi_{\mathrm{n}} = e^{+\boldsymbol{i}_{\mathrm{n}}½\varphi_{\mathrm{n}}} = e^{\boldsymbol{\imath}\mathbf{n}½\varphi_{\mathrm{n}}}.$$

This simplified circle oscillation is *insufficient* in that the bivector ***direction*** $\boldsymbol{i}_{\mathrm{n}} = \boldsymbol{\imath}\mathbf{n}$ is dependent on the oscillating versor coordinate functions (u_1, u_2, u_3). Therefore, we need the full versor algebra $\mathcal{G}_{0,2}(\mathbb{R}) \subset \mathcal{G}_3(\mathbb{R})$ to describe a normalized wavefunction for an ***entity*** $\Psi_{½}$ in $\mathfrak{Z}$-space of physics, also containing the even ***grades*** as a ***pqg***-0 scalar plus ***pqg***-2 bivectors. We recall the two orthogonal 1-spinors from (6.415)-(6.416) Q_3 and Q_1 in a basis $\{1, \boldsymbol{i}_2\}$ and get their combination (6.145f)

(6.454) $$U = \psi_{+½} = \hat{Q} = u_0 + u_3\boldsymbol{i}_3 + u_2\boldsymbol{i}_2 + u_1\boldsymbol{i}_1 = (u_0 + u_3\boldsymbol{i}_3)1 + (u_2 - u_1\boldsymbol{i}_3)\boldsymbol{i}_2 = Q_3 1 + Q_1\boldsymbol{i}_2$$

Here we cannot go further into the versor wavefunction than it has a unitary structure (6.451) for just *one* indivisible ***entity*** that we express as

(6.455) $$UU^{\dagger} = U^{\dagger}U = \psi_{+½}\psi^{\dagger}_{+½} = \psi^{\dagger}_{+½}\psi_{+½} = 1 = u_0^2 + u_1^2 + u_2^2 + u_3^2 = |U|^2 = 1.$$

To get an intuition for the internal spatial structure of one ***entity*** $\Psi_{½}$ we return to the concept of angular momentum.

[371] That as the tradition can be expressed in a complex 2×2 matrix mentioned §6.4.5. Or here as two 1-spinors (6.145)-(6.147), (6.415)-(6.416). These properly are governed by two synchronised angular development parameters, up to a phase factor.

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