(6.442) $\quad \Psi_{\text{auto},\mathbb{H}} = \pm\frac{1}{2} \pm\frac{1}{2}\boldsymbol{i}_1 \pm\frac{1}{2}\boldsymbol{i}_2 \pm\frac{1}{2}\boldsymbol{i}_3 \in\mathbb{H} \quad\Rightarrow\quad \mathrm{L}^{\Psi_{1/2}}_{\text{auto}\pm} = \hbar\left(\Psi_{\text{auto},\mathbb{H}} \mp \frac{1}{2}\right), \quad (\hbar = 1).$

The first expression can be combined in 2×2×2×2 = 16 ways.
In the last expression $\mathrm{L}^{\Psi_{1/2}}_{\text{auto}\pm}$, we have just removed the scalar part from the versor and have only the pure ***second grade*** bivector ***pqg***-2 ***direction*** part left that can combine in 2×2×2 = 8 ways.
In all, three angular momentum components $\mathrm{L}_k = \frac{1}{2}\boldsymbol{i}_k$ and a scalar $\frac{1}{2}$ make a substance for a four-dimensional $1, 3 \sim 1{+}3$ of real even ***grades*** 0, 2 in $\mathbb{H}\sim \mathcal{G}_{0,2}(\mathbb{R})$ as a dextral quaternion basis

(6.443) $\quad \{1,\ \boldsymbol{i}_1, \boldsymbol{i}_2,\ \boldsymbol{i}_3 := \boldsymbol{i}_1\boldsymbol{i}_2\} = \quad 2\{\tfrac{1}{2},\ \mathrm{L}_1,\ \mathrm{L}_2,\ \mathrm{L}_3 = 2\mathrm{L}_1\mathrm{L}_2\},$

where this four-dimensional S^3-symmetry idea is measured by one quantum count $\hbar{=}\hbar^{-1}{=}1$, just as the constant chronometer in Kepler's second law gives constant angular areas.
Kepler's principle permits plane elliptical movement of a satellite point, but here we concern with a circular symmetry distribution of matter without a central force because there is nothing in the center[366] than the geometric point of the intersection of the three supported planes. We recall the three oscillating perpendicular areas shown in Figure 6.21 that we connect to the orthogonal basis for the algebra $\mathbb{H}\sim \mathcal{G}_{0,2}(\mathbb{R})$ that we have shown is strongly interconnected.
Maybe an elliptical shape is possible because in (6.440) we only demand $\lambda_\mu\lambda_\mu = 1, \mu{=}0{,}1{,}2{,}3, (\sum_\mu)$ to have a versor. The individual components operators λ_μ can fluctuate freely as long as they fulfil $\lambda_\mu\lambda_\mu{=}1$, for the interpretation of (6.440) as a versor $\widetilde{\Psi}_{\mathbb{H}}{=}\Psi_{\mathbb{H}}$ in an even algebra $\mathcal{G}_{0,2}$, and by that also a 2-rotor of 𝔍-space.[367] We ignore the elliptic possibility due to the lag of central force in the quantum mechanics interpretation of the oscillating 1-rotor S^1-symmetry $\{\theta{\to}e^{i\theta} \mid \text{for } \forall\theta \in\mathbb{R}\}$, which we expand (threefold) to the S^3-symmetry, due to $\boldsymbol{i}$ as a free ***direction***.
The interconnectivity first expressed in (6.123) and confirmed by the operator interconnectivity (6.291)-(6.293a) make the three ***direction*** operations $\mathrm{L}_k{=}\frac{1}{2}\boldsymbol{i}_k \leftrightarrow \frac{1}{2}e^{+½\boldsymbol{i}_k\phi_k}$ point out a locality by the intersection of three perpendicular oscillator planes as **one** geometric center point with a resulting entangled state of **one** exclusive atomic-elementary[368] ***entity*** $\Psi_{1/2}$.

The angular momentum of **one** specific *free locality* $\Psi_{1/2}$ in 𝔍-space can easily exchange ***direction*** to one external ***direction*** of a field by aligning its rotation conical precession as a projection to this and achieving a component of spin½ angular momentum of ***quantum*** number $m{=} \pm\frac{1}{2}\hbar$ with *specific orientation up* + or *down* − along an external field gradient ***direction***.
Do we use the traditional picture with 1-vector gradient field $b(x_3)\mathbf{e}_3$ of angular momentum in a laboratory frame ***direction***, we get for $\boldsymbol{\sigma}_3\|\mathbf{e}_3$ the projected spin angular momentum along $\mathbf{e}_3$

- $+\frac{1}{2}\boldsymbol{\sigma}_3 \leftarrow +\frac{1}{2}\hbar\mathbf{e}_3$ for *spin up* (+),
- $-\frac{1}{2}\boldsymbol{\sigma}_3 \leftarrow -\frac{1}{2}\hbar\mathbf{e}_3$ for *spin down* (−).

In the new picture with the transversal bivector gradient field $b(x_3)\boldsymbol{i}\mathbf{e}_3$ of angular momentum in a laboratory frame ***direction*** we get for $\boldsymbol{i}_3\|\boldsymbol{i}\mathbf{e}_3$ the projected spin angular momentum along $\boldsymbol{i}\mathbf{e}_3$ plane.

- $+\frac{1}{2}\boldsymbol{i}_3 \leftarrow +\frac{1}{2}\hbar\boldsymbol{i}\mathbf{e}_3$ for *progressive spin* (+) (*up*),
- $-\frac{1}{2}\boldsymbol{i}_3 \leftarrow -\frac{1}{2}\hbar\boldsymbol{i}\mathbf{e}_3$ for *retrograde spin* (−) (*down*).

In this new picture, the spin½ angular momentum along the Kepler plane performs a *progressive* or *retrograde* cyclic oscillating rotation as an intrinsic ***quality*** of a fundamental local ***entity*** $\Psi_{1/2}$.
Following the tradition, we will in this book when it is concerned spin½ often use the terms *up* or *down,* even when we mean the cyclic oscillating in the *lane path* ⊙ of a unit circle.[369] For a local ***entity*** $\Psi_{1/2}$ in 𝔍-space, which we describe by the even versor quaternion algebra $\mathbb{H}{\sim}\mathcal{G}_{0,2}(\mathbb{R})$, there

[366] We remember the Rayleigh distribution (3.143) and (6.398) of a circular oscillator.
[367] We remember that the plane ***directions*** of such a rotor is fluctuating in lab 𝔍 space, with precession by interacting with a field.
[368] By atomic-elementary, we mean that a fundamental indivisible ***entity*** of certain ***qualities*** is ***one quantum*** of ***entity*** existence.
[369] *Lane path*, because there is *no* satellite to *orbit* only a circle symmetric ⊙ with a radial Rayleigh probability distribution.

are more possible ***qualities*** than spin½. We have 16 possibilities in the 4-dimensional (6.442).
When we first ignore the scalar freedom and only consider the spatial perpendicular ***directions*** into consideration, we only have 8 possibilities of different ***qualities***.

6.5.8.7. Eight Qualitative States of a Spin½ *Entity* Ψ½ in 𝔍 Space

The autonomous total angular momentum can be built in eight different ways in combination of the orthogonal angular momentum ***direction*** components (6.443) in an ***entity*** $\Psi_{1/2}$ volume with different *chirality* and *spin* as follows:

The total bivector angular momentum } ~ { and the dual 1-vector forms

Total, Components, Chirality, Spin₃,

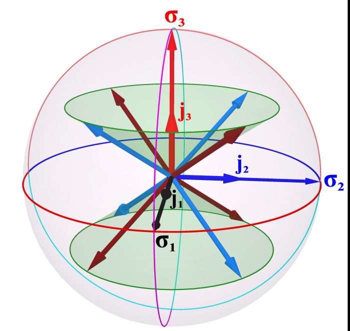
(6.444)
$$\left.\begin{array}{rlll}
 & \mathrm{L}^{\Psi_{1/2}}_{s-++} = -\mathrm{L}_1 + \mathrm{L}_2 + \mathrm{L}_3, & \text{sinistral}, & +\\
 & \mathrm{L}^{\Psi_{1/2}}_{s---} = -\mathrm{L}_1 - \mathrm{L}_2 - \mathrm{L}_3, & \text{sinistral}, & -\\
 & \mathrm{L}^{\Psi_{1/2}}_{d--+} = -\mathrm{L}_1 - \mathrm{L}_2 + \mathrm{L}_3, & \text{dextral}, & +\\
 & \mathrm{L}^{\Psi_{1/2}}_{d-+-} = -\mathrm{L}_1 + \mathrm{L}_2 - \mathrm{L}_3, & \text{dextral}, & -\\
\mathrm{L}^{\Psi_{1/2}\dagger}_{d-+-} = & \mathrm{L}^{\Psi_{1/2}}_{s+-+} = +\mathrm{L}_1 - \mathrm{L}_2 + \mathrm{L}_3, & \text{sinistral}, & +\\
\mathrm{L}^{\Psi_{1/2}\dagger}_{d--+} = & \mathrm{L}^{\Psi_{1/2}}_{s++-} = +\mathrm{L}_1 + \mathrm{L}_2 - \mathrm{L}_3, & \text{sinistral}, & -\\
\mathrm{L}^{\Psi_{1/2}\dagger}_{s---} = & \mathrm{L}^{\Psi_{1/2}}_{d+++} = +\mathrm{L}_1 + \mathrm{L}_2 + \mathrm{L}_3, & \text{dextral}, & +\\
\mathrm{L}^{\Psi_{1/2}\dagger}_{s-++} = & \mathrm{L}^{\Psi_{1/2}}_{d+--} = +\mathrm{L}_1 - \mathrm{L}_2 - \mathrm{L}_3, & \text{dextral}, & -
\end{array}\right\} = \boldsymbol{i}\left\{\begin{array}{ll}
\mathrm{j}^{\Psi_{1/2}}_{s-++} = -\mathrm{j}_1 + \mathrm{j}_2 + \mathrm{j}_3 & \\
\mathrm{j}^{\Psi_{1/2}}_{s---} = -\mathrm{j}_1 - \mathrm{j}_2 - \mathrm{j}_3 & \\
\mathrm{j}^{\Psi_{1/2}}_{d--+} = -\mathrm{j}_1 - \mathrm{j}_2 + \mathrm{j}_3 & \\
\mathrm{j}^{\Psi_{1/2}}_{d-+-} = -\mathrm{j}_1 + \mathrm{j}_2 - \mathrm{j}_3 & \\
\mathrm{j}^{\Psi_{1/2}}_{s+-+} = +\mathrm{j}_1 - \mathrm{j}_2 + \mathrm{j}_3 & = \overline{\mathrm{j}^{\Psi_{1/2}}_{d-+-}}\\
\mathrm{j}^{\Psi_{1/2}}_{s++-} = +\mathrm{j}_1 + \mathrm{j}_2 - \mathrm{j}_3 & = \overline{\mathrm{j}^{\Psi_{1/2}}_{d--+}}\\
\mathrm{j}^{\Psi_{1/2}}_{d+++} = +\mathrm{j}_1 + \mathrm{j}_2 + \mathrm{j}_3 & = \overline{\mathrm{j}^{\Psi_{1/2}}_{s---}}\\
\mathrm{j}^{\Psi_{1/2}}_{d+--} = +\mathrm{j}_1 - \mathrm{j}_2 - \mathrm{j}_3 & = \overline{\mathrm{j}^{\Psi_{1/2}}_{s-++}}
\end{array}\right.$$

These different combinations of bivectors result in the following impact:

1. The shift of sign in one ***direction*** changes the chirality with *dextral* or *sinistral* orientation.
2. The shift of sign in two of three ***directions*** changes the *spin up* (+) or *down* (−) orientation.
3. The shift of sign in three ***directions*** is a *parity inversion* of 1-vectors or just a *versor reversion* that consists of the *reversion* of the total bivector, that shifts both above cases 1. & 2.

This combination of the 𝔍-space ***directions quality*** results in two main ***Categories***: *Chirality,* and *Spin* for the volume of one ***entity*** $\Psi_{1/2}$

Chirality is the fundamental ***quality*** aspect of 𝔍-space we first defined as the unit pseudoscalar orientation (6.22) and (6.23) of the sequential order of operations $\boldsymbol{i} = \boldsymbol{\sigma}_3\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 = -\,\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\boldsymbol{\sigma}_3$. Or $1 = \boldsymbol{i}_3\boldsymbol{i}_2\boldsymbol{i}_1 = -\,\boldsymbol{i}_1\boldsymbol{i}_2\boldsymbol{i}_3$.
It is sufficient to change the sign on one (k) of the three basis elements $\boldsymbol{\sigma}_k$ and $\boldsymbol{i}_k$ to change the *chiral* volume orientation.



The external impact of *spin*½ shift $\pm\mathrm{L}_3$ of the projection on an external ***direction*** happens without internal effort when one other component change too, e.g., $-\frac{1}{2}\mathrm{L}_1 = \mathrm{L}_3\mathrm{L}_2 = -\mathrm{L}_3(-\mathrm{L}_2)$ in (6.444) due to interconnectivity. (Two of three orthogonal circle oscillators shift orientation.)

What happens when the orientation of the total bivector expressed angular momentum is *reversed* in the same way as the versor $\Psi_{\mathbb{H}}$ (6.439) to the four-angular-momentum (6.441) $\Psi^{\dagger}_{\mathbb{H}} = \widetilde{\Psi}_{\mathbb{H}}$?
Then all three orthogonal components of the circle oscillators are *reversed*, expressed as bivector angular momenta $\widetilde{\mathrm{L}_k} = \mathrm{L}^{\dagger}_k = -\mathrm{L}_k$.
The dual picture of this is the total 1-vector is *parity inverted* by Clifford conjugating $\overline{\mathrm{j}_k} = \overline{\mathrm{j}_k} = -\mathrm{j}_k$.
See a 1-vector intuition of autonomy in Figure 6.23.

Figure 6.23 An autonomous ***entity*** $\Psi_{1/2}$ has in all 8 internal ***directional*** cases founded in in the dextral basis $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3\} = 2\{\mathbf{j}_1, \mathbf{j}_2, \mathbf{j}_3\}$. In (6.444) combined to a total $\mathbf{j}^{\Psi_{1/2}}_{....} = \pm\,\mathbf{j}_1 \pm \mathbf{j}_2 \pm \mathbf{j}_3$ of angular momentum 1-vectors, shown as intuition 1-vector objects.
Chirality: sinistral $\mathbf{j}^{\Psi_{1/2}}_{s...}$, or dextral $\mathbf{j}^{\Psi_{1/2}}_{d...}$; with spin₃ *up* (+) or *down* (−) in a lab; and the *parity inversion* of these.
The total angular momentum ***direction*** perform a conic precession oscillation seen from the lab frame; cyclic rotated in all by the 1-rotor $\odot_3 e^{\pm\boldsymbol{i}_3\phi_3}$.