6.5.8.4. Versor Eigenwave-Function as for the Stationary State Existence of an *Entity* $\Psi_{½}$

The ***directional*** bivector eigenvalue-equations (6.369)⟵(6.334) joined by the Hermitian scalar ***pqg***-0 operator eigenvalue-equations (6.371)⟵(6.335)

(6.421) $$\mathrm{L}_3\left|\tfrac{3}{4},\pm\tfrac{1}{2}\right\rangle_3 \doteq \pm\tfrac{1}{2}\hbar\boldsymbol{i}_3\left|\tfrac{3}{4},\pm\tfrac{1}{2}\right\rangle_3 \qquad \text{and} \qquad \mathrm{L}^2\left|\tfrac{3}{4},\pm\tfrac{1}{2}\right\rangle \doteq -\hbar^2\lambda\left|\tfrac{3}{4},\pm\tfrac{1}{2}\right\rangle,$$

have the ***directional*** versor eigenstate (6.414)

(6.422) $$\boxed{\psi_{+½} = \left|\tfrac{3}{4},+\tfrac{1}{2}\right\rangle = u_0+u_3\boldsymbol{i}_3+u_1\boldsymbol{i}_1+u_2\boldsymbol{i}_2}. \qquad \text{Similar for } \psi_{-½} = \left|\tfrac{3}{4},-\tfrac{1}{2}\right\rangle.$$

The reader should verify by recalling § 6.5.5.2 that the operators we have defined by $\mathrm{L}_3 = \hbar\lambda_3\boldsymbol{i}_3$, and $\mathrm{L}^2 = \hbar^2\lambda_r^2 = \hbar^2\lambda$, for $\hbar=1$ give these real eigenvalue coefficients pattern

$$j=\tfrac{1}{2} \;\Rightarrow\; \lambda = j(j+1) = \tfrac{3}{4} \;\Rightarrow\; m = \pm\tfrac{1}{2} \;\cong \lambda_3 \;\Leftarrow\; \lambda_3^2 = \tfrac{1}{4},$$

and for the first of (6.421) the ***directional*** transversal bivector eigenvalues $\pm\tfrac{1}{2}\hbar\boldsymbol{i}_3$.[363]

We see that our versor wavefunction (6.422) is already normalized by using the reversed

(6.423) $$\psi_{+½}^{\dagger} = \left\langle\tfrac{3}{4},+\tfrac{1}{2}\right| = u_0-u_3\boldsymbol{i}_3-u_1\boldsymbol{i}_1-u_2\boldsymbol{i}_2$$

We have the unitary product for a versor inside the S^3-spherical symmetry

(6.424) $$\left\langle\tfrac{3}{4},\pm\tfrac{1}{2}\middle|\tfrac{3}{4},\pm\tfrac{1}{2}\right\rangle = \psi_{\pm½}^{\dagger}\psi_{\pm½} = \hat{Q}^{\dagger}\hat{Q} = \left|\hat{Q}\right|^2 = u_0^2+u_1^2+u_2^2+u_3^2 = 1.$$

The designation $\pm½$ in the versor wavefunction $\psi_{\pm½}$ can be omitted because it is just a versor

(6.425) $$\psi_{½} = \hat{Q} = u_0+u_3\boldsymbol{i}_3+u_1\boldsymbol{i}_1+u_2\boldsymbol{i}_2 \quad \in\mathbb{H}.$$

We use $\psi_{½}$ here for the versor to remember the mutual internal oscillations of the ***entity*** $\Psi_{½}$, that has states depending on the mutual angular development of the u_μ's.

6.5.8.5. The One Eigen-Versor Separated in 1-Spinor Angular-Momentum-Wavefunctions

For the versor state $\psi_{½}$ of an ***entity*** $\Psi_{½}$, we now separate it in the three orthogonal ***directions*** planes spanned from $\{\boldsymbol{i}_k\}$, $\psi_{½} = \psi_{3\pm}^{½}+\psi_{2\pm}^{½}+\psi_{1\pm}^{½} \sim (\psi_{1\pm},\psi_{2\pm},\psi_{3\pm})$.

We have for each $\psi_{k\pm}$ circular wavefunction complementary ***directional*** angular momentum operators (6.279) we know are interconnected as (6.291)-(6.293a) for the circle oscillators

(6.426) $$(\mathrm{L}_1,\ \mathrm{L}_2,\ \mathrm{L}_3) = \hbar(\lambda_1\boldsymbol{i}_1,\ \lambda_2\boldsymbol{i}_2,\ \lambda_3\boldsymbol{i}_3),$$

with a total of $\mathrm{L} = \mathrm{L}_1+\mathrm{L}_2+\mathrm{L}_3 = \lambda_1\boldsymbol{i}_1+\lambda_2\boldsymbol{i}_2+\lambda_3\boldsymbol{i}_3$ (6.263), that we presume is not observable.

We have seen, that the three component scalar operators λ_k for $\psi_{½}\sim|\lambda,\pm½\rangle$ must fulfil

(6.427) $$\lambda_k\lambda_k = \lambda_1^2+\lambda_2^2+\lambda_3^2 = \lambda_r^2 = \lambda = \tfrac{3}{4}$$

for the Hermitian scalar operator L^2 in agreement with (6.335) and (6.421).

For these wavefunctions components $\psi_{k\pm}^{½}$, that possesses angular momentum $\langle\mathrm{L}_k\rangle_{½}$, we note individually magnitudes $\left\langle\psi_{k\pm}^{½}\middle|\psi_{k\pm}^{½}\right\rangle = \tfrac{1}{4}$ due to (6.407), (6.399) in association with their definition in section I. 3.3.1, (We avoid the parity inversion balance factor individual for these components).

Then we write the simplified wavefunctions as 1-spinors dilated ½ from the 1-rotor idea (6.241)

(6.428) $$\psi_{k+}^{½} = \tfrac{1}{2}e^{+½\boldsymbol{i}_k\phi_k} = \tfrac{1}{2}U_{\phi_k}, \qquad \text{and} \qquad \psi_{k-}^{½} = \tfrac{1}{2}e^{-½\boldsymbol{i}_k\phi_k} = \tfrac{1}{2}U_{\phi_k}^{\dagger}$$

These three oscillating 1-spinors auto-commute individually with their own plane ***direction*** $\boldsymbol{i}_k$.

The regular oscillating rotation components in each plane, originally generated by rotor sandwiches from definition (5.193), are now used with 1-spinors in these planes,

(6.429) $$\psi_{k\pm}^{½}\,\boldsymbol{i}_k\psi_{k\mp}^{½} = \left(\psi_{k\pm}^{½}\right)^2\boldsymbol{i}_k = \psi_{k\pm}\boldsymbol{i}_k = \tfrac{1}{4}U_{\phi_k}^2\boldsymbol{i}_k = \tfrac{1}{4}e^{\pm\boldsymbol{i}_k\phi_k}\boldsymbol{i}_k \sim \tfrac{1}{4}e^{\pm\boldsymbol{i}_k(\phi_k+½\pi)}.$$

Here we have used that the unit area ***direction*** $\boldsymbol{i}_k = e^{\boldsymbol{i}_k½\pi}$ is founded in just[364] an angular phase $½\pi$, as a factor for that ***direction*** $\boldsymbol{i}_k = \boldsymbol{i}\sigma_k = e^{\pm½\boldsymbol{i}_k\pi} \longleftarrow e^{\pm½\boldsymbol{i}_k\phi_k}$ given by that wave component

[363] This corresponds to the traditional complex number eigenvalue $\pm\frac{i}{2}\hbar$ where the ***direction*** is *transcendental* to our ideas.

[364] This angular phase area as a foundation unit principle was first formulated in Kepler's second law as a constant ***primary quality***.

idea from the unitary group $U(1)$ that possesses the S^1 circle symmetry

(6.430) $$S^1 \leftrightarrow \{\forall(x,y)\in\mathbb{R}^2 \,|\, x^2+y^2=1\} = \{(\cos\varphi,\sin\varphi)\in\mathbb{R}^2 \,|\, \cos^2\varphi+\sin^2\varphi=1,\ \varphi\in\mathbb{R}\} = \{U_\varphi = e^{i\varphi}\in\mathbb{C} \,|\, \varphi\in\mathbb{R}\}.$$

From this plane symmetry idea of 1-rotors, we take the ½, and use these three perpendicular circular 1-spinors (6.428), which are oscillating multivectors of the form

(6.431) $$\psi_{k+}^{½} = \tfrac{1}{2}e^{+½\boldsymbol{i}_k\phi_k} = \tfrac{1}{2}(\cos½\phi_k+\boldsymbol{i}_k\sin½\phi_k)$$

These components of transversal plane states consist each of an oscillating scalar plus an oscillating transversal bivector in the ***direction*** $\boldsymbol{i}_k = \boldsymbol{i}\sigma_k = e^{\boldsymbol{i}_k½\pi}$.

6.5.8.6. The Versor Quaternion Spin½ *entity* $\Psi_{½}$

Adding these (6.431) orthogonal 1-spinors as (6.409), we achieve a quaternion (6.131), (6.136)

(6.432) $$Q = \tfrac{1}{2}\textstyle\sum_{k=1}^{3}\cos½\phi_k + \tfrac{1}{2}\sum_{k=1}^{3}\boldsymbol{i}_k\sin½\phi_k = u_0+\mathrm{B}.$$

The first sum is a real scalar

(6.433) $$u_0 = \tfrac{1}{2}\textstyle\sum_{k=1}^{3}\cos½\phi_k$$

The last sum is a bivector where we use $u_k = \tfrac{1}{2}\sin\phi_k$, (with $\pm$ hidden inside ϕ_k)

(6.434) $$\mathrm{B} = \tfrac{1}{2}\sin½\phi_1\,\boldsymbol{i}_1 + \tfrac{1}{2}\sin\phi_2\,\boldsymbol{i}_2 + \tfrac{1}{2}\sin\phi_3\,\boldsymbol{i}_3 = u_1\boldsymbol{i}_1+u_2\boldsymbol{i}_2+u_3\boldsymbol{i}_3 = \boldsymbol{i}\mathrm{n}\sqrt{1-u_0^2}.$$

Now we generalise this on the quaternion form for an indivisible physical ***entity*** $\Psi_{½}$ of 3-space

(6.435) $$Q = u_0+u_1\boldsymbol{i}_1+u_2\boldsymbol{i}_2+u_3\boldsymbol{i}_3.$$

To make this quaternion a 2-rotor versor of the closed quaternion group $\mathcal{G}_{0,2}(\mathbb{R})$ we demand that

(6.436) $$|Q|^2 = Q\tilde{Q} = QQ^{\dagger} = u_0^2+u_1^2+u_2^2+u_3^2 = u_\mu u_\mu = 1 \quad\Rightarrow\quad Q=\hat{Q}=U.$$

We compare this with the complementary angular momentum operator form (6.263)⟶(6.316)

(6.437) $$\mathrm{L} = \hbar(\lambda_1\boldsymbol{i}_1+\lambda_2\boldsymbol{i}_2+\lambda_3\boldsymbol{i}_3) \quad \text{autonomous} \quad \mathrm{L}_{\text{auto}\pm}^{\Psi_{½}} = \hbar\left(\pm\tfrac{1}{2}\boldsymbol{i}_1\pm\tfrac{1}{2}\boldsymbol{i}_2\pm\tfrac{1}{2}\boldsymbol{i}_3\right), \quad (\hbar=1).$$

Where we for the indivisible physical ***entity*** $\Psi_{½}$ are demanded by (6.427) to set

(6.438) $$\lambda_k\lambda_k = \lambda_1^2+\lambda_2^2+\lambda_3^2 = \tfrac{3}{4}, \quad \text{autonomous} \quad \lambda = \tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4} = \tfrac{3}{4}.$$

as the eigenvalue for the Hermitian scalar operator L^2 from the equation $\mathrm{L}^2\left|\tfrac{3}{4},\pm\tfrac{1}{2}\right\rangle = -\hbar^2\tfrac{3}{4}\left|\tfrac{3}{4},\pm\tfrac{1}{2}\right\rangle$.

At first sight this seems weird, why is the magnitude of the indivisible total active angular momentum not one whole 1, but ¾. When we look at the versor quaternion form (6.435) we see beside the three bivector basis ***directions*** a scalar u_0 (6.433). I suggest this is essential for the total. From this idea, we try to construct a versor for what I now call a four-angular-momentum

(6.439) $$\Psi_{\mathbb{H}} = \lambda_0+\lambda_1\boldsymbol{i}_1+\lambda_2\boldsymbol{i}_2+\lambda_3\boldsymbol{i}_3 \;\in\mathbb{H} \sim \mathcal{G}_{0,2}(\mathbb{R}),$$

where we add a synthetic essential scalar λ_0 to the expression (6.437) so that we achieve one whole for an active angular 3-space indivisible physical ***entity*** $\Psi_{½}$.

(6.440) $$|\Psi_{\mathbb{H}}|^2 = \Psi_{\mathbb{H}}\widetilde{\Psi}_{\mathbb{H}} = \Psi_{\mathbb{H}}\Psi_{\mathbb{H}}^{\dagger} = \lambda_0^2+\lambda_1^2+\lambda_2^2+\lambda_3^2 = \lambda_\mu\lambda_\mu = 1, \quad (\hbar=1).$$

We have here used the reversed four-angular-momentum versor,

(6.441) $$\Psi_{\mathbb{H}}^{\dagger} = \lambda_0-\lambda_1\boldsymbol{i}_1-\lambda_2\boldsymbol{i}_2-\lambda_3\boldsymbol{i}_3 \;\in\mathbb{H}$$

that also is the Clifford conjugated $\widetilde{\Psi}_{\mathbb{H}}$ in $\mathbb{H}\sim\mathcal{G}_{0,2}(\mathbb{R})$ of the versor (6.439) $\Psi_{\mathbb{H}}$.

We have above guessed, that each of the four quaternion state component dimensions – of the real even closed geometric algebra $\mathcal{G}_{0,2}(\mathbb{R})$ for a *free* autonomous[365] indivisible physical ***entity*** $\Psi_{½}$ in a S^3-symmetric space structure – have the values $\langle\lambda_\mu\rangle_{½} = \lambda_\mu = \pm\tfrac{1}{2}$ and squared $\langle\lambda_\mu^2\rangle_{½} = \lambda_\mu^2 = \tfrac{1}{4}$.

Then in the simplified orthogonal perpendicular autonomous picture, we write

[365] By *free* autonomous, we mean the highest internal symmetry for one indivisible existence without any interaction with external surroundings and no external ***quantitative*** norms. An initial frame is *free*. External classically this is called a *force-free* frame.