(6.240) $\boldsymbol{i}_2(\phi) = \boldsymbol{\imath}\sigma_2(\phi) = \boldsymbol{\imath}e^{\boldsymbol{i}_3\phi}\mathbf{e}_2 = e^{\boldsymbol{i}_3\phi}\boldsymbol{\imath}\mathbf{e}_2$

Here we recall once again that the angular development is $\phi = \omega t$, where we just choose ω=1 as auto-norm. We consider a double-created orthogonal ***quantum*** excitation of two rotations: The one angular wavefunction is the rotation oscillating function (we involve $\odot_3$ circle group)

(6.241) $U_\phi^2 = e^{\boldsymbol{i}_3\phi} \quad\leftrightarrow\quad \hat{\phi}_+ = |{+1}\rangle_3 = e^{i\phi} \quad\Rightarrow\quad \boxed{U_\phi := e^{½\boldsymbol{i}_3\phi}} \quad\leftrightarrow\quad \hat{\phi}_+^{½} = e^{½i\phi}.$

The other rotation oscillator that is in a plane orthogonal to this (we involve $\odot_2$ circle group)

(6.242) $U_\psi^2 = e^{\boldsymbol{i}_2\psi} \quad\leftrightarrow\quad \hat{\psi}_+ = |{+1}\rangle_2 = e^{i\psi} \quad\Rightarrow\quad \boxed{U_\psi := e^{½\boldsymbol{i}_2\psi}} \quad\leftrightarrow\quad \hat{\psi}_+^{½} = e^{½i\psi}.$

These two rotation oscillators cannot combine directly due to, that there do not commute $U_\psi U_\phi \not\approx U_\phi U_\psi$ as described[325] in § 6.3.5.2, shown in Figure 6.14. Instead, we use the canonical the sandwich method with (6.202) for a resulting 2-rotor product of two 1-rotors

(6.243) $U{=}U_{\phi,\psi} := U_\phi U_\psi = e^{½\boldsymbol{i}_3\phi}e^{½\boldsymbol{i}_2\psi},$

and from this the full rotation of any 1-vector $\mathbf{x}$

(6.244)
$$\mathcal{R}_{\phi,\psi}\mathbf{x} = U\mathbf{x}U^\dagger = U_\phi U_\psi \mathbf{x} U_\psi^\dagger U_\phi^\dagger$$
$$= e^{½\boldsymbol{i}_3\phi}e^{½\boldsymbol{i}_2\psi}\mathbf{x}\,e^{-½\boldsymbol{i}_2\psi}e^{-½\boldsymbol{i}_3\phi}$$
$$= e^{½\mathrm{B}(\phi,\psi)}\mathbf{x}\,e^{-½\mathrm{B}(\phi,\psi)} = e^{\mathrm{B}(\phi,\psi)}\mathbf{x} = e^{\boldsymbol{\imath}\mathbf{r}(\phi,\psi)}\mathbf{x}.$$

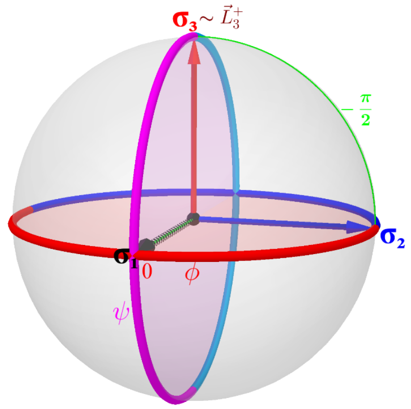

Figure 6.18 The two orthogonal circle oscillators in the plane ***directions*** $\boldsymbol{i}_3 \equiv \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 = \boldsymbol{\imath}\boldsymbol{\sigma}_3$ and $\boldsymbol{i}_2 \equiv \boldsymbol{\sigma}_1\boldsymbol{\sigma}_3 = \boldsymbol{\imath}\boldsymbol{\sigma}_2$ with the azimuth angular development parameters ϕ and from the polar angle $\psi = \theta - \pi/2$. (ϕ,ψ) both starting from $\mathbf{e}_1 = \boldsymbol{\sigma}_1(0)$ The two oscillators are shown as unitary circular rings in an uneven parity inversion contradiction (red-blue) and (magenta-blue) just as in Figure 3.8.

We display the two orthogonal rotations in Figure 6.18 as snapshots of the circle oscillators $U_\phi := e^{½\boldsymbol{i}_3 0}$, $U_\psi := e^{½\boldsymbol{i}_2 0}$. The rule here for the basis vector $\boldsymbol{\sigma}_1(0) = \mathbf{e}_1$ is to be the start node $\mathbf{n}_{0,0} = \mathbf{e}_1$ for the oscillations $\phi = 0$, $\psi = 0$ (compare Figure 6.17, 2-3, for the Euler angles, where we choose the orthogonality by setting $\theta = -\pi/2$).
The node $\mathbf{n}_{\phi,0} = \boldsymbol{\sigma}_1 = e^{\boldsymbol{i}_3\phi}\mathbf{e}_1$ is the intersection between the two orthogonal oscillator planes in the ***entity*** $\Psi_{\phi,\psi}$, and those follow its autonomous frame $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3\}$.
The intersection between these circle oscillator planes is $\boldsymbol{\sigma}_1$.
We use the even $\mathcal{G}_{0,2}$ algebra for these planes in $\mathfrak{Z}$-space.
The resulting variable bivector argument in (6.244) is

(6.245) $\mathrm{B}(\phi,\psi) = \boldsymbol{i}_3\phi + \boldsymbol{i}_2(\phi)\psi = \phi\boldsymbol{i}_3 + \psi e^{\boldsymbol{i}_3\phi}\boldsymbol{\imath}\mathbf{e}_2,$

also represented by its joined dual 1-vector argument

(6.246) $\mathbf{r}(\phi,\psi) = \phi\boldsymbol{\sigma}_3 + \psi\boldsymbol{\sigma}_2(\phi) = \phi\boldsymbol{\sigma}_3 + \psi e^{\boldsymbol{\imath}\phi\boldsymbol{\sigma}_3}\mathbf{e}_2,$

that has an oscillating ***direction*** in the sphere, which normalized unit 1-vector ***direction*** can be expressed

(6.247) $\mathbf{u}(\phi,\psi) = \mathbf{r}(\phi,\psi)/\sqrt{\phi^2+\psi^2}\,,$ see this in Figure 6.19.

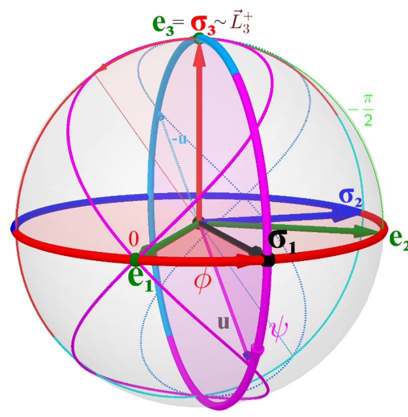

Figure 6.19 An arbitrary intuition of the two perpendicular circle oscillators. The outermost oscillator in (6.244) drives the inner oscillator plane synchronously around $\psi = \phi \sim \pm\,\omega t$. The angular ***direction*** axis $\mathbf{u}(\phi,\psi)$ form a 8 curve on the unit sphere for every 2π turn in the two driving oscillators. – This example starts at $\mathbf{e}_1$ and shows $\boldsymbol{\sigma}_1(35°) = \mathbf{n}_{35°,0°} \Rightarrow \mathbf{u}(35°,35°)$.

As we have chosen to know, the angular phase $\phi,\psi \sim \pm\omega t$ is developing monotonously. This picture with an oscillating 1-vector seems weird though the result is periodic. Anyway, we will try to display it for the intuition.
For the ***entity*** $\Psi_{\phi,\psi}$ to give meaning we must presume that the angular frequency energy ω=1 autonomously is the same for both ϕ and ψ. In that, we write $\phi = \pm\omega t + \theta_3$ and $\psi = \pm\omega t + \theta_2$, (arbitrary start relative to $\mathbf{e}_1$) we can set $\theta_0 = \theta_2 - \theta_3$, and write $\psi = \pm\phi + \theta_0 \sim \pm\omega t + \theta_0$ for any angular development internal in $\Psi_{\phi,\psi}$.

In a fictive snap $\theta_0 = 0$ where we set $\psi = \phi \sim \pm\omega t$, we can intuit the angular development as displayed in Figure 6.19, where the angular point on the unit sphere is pointed out by the unit 1-vector (6.247) $\mathbf{u}(\phi,\psi)$ describing a 8 curve on the sphere surface.
This angular surface curve has an opposite placed odd parity 1-vector $-\mathbf{u}(\phi,\psi)$ describing a 8 curve too for the oscillation. The reader should note that $\mathbf{u}(n\pi,n\pi) = \mathbf{u}(0,0) = \boldsymbol{\sigma}_1$, $n \in \mathbb{Z}$, so, for every cycle in the resulting oscillation, the n numbered phase; the pointing axis $\mathbf{u}$ passes the x in the 8 bows two times, one in each branch for even versus odd n number, the same for 8.
This oversimplified snap synchronised angular parameter $\psi = \phi$ example used for (6.201) gives

(6.248) $u_0 = +\sqrt{½}\cos\phi,\quad u_1 = -u_0,\quad u_2 = 0,\quad u_3 = +\sqrt{½}\sin\phi.$

This makes $u_2\boldsymbol{i}_2 = 0$ disappear and we get the versor quaternion 2-rotor

(6.249) $\hat{Q} = U = u_0 + u_1\boldsymbol{i}_1(\phi) + u_2\boldsymbol{i}_2(\phi) + u_3\boldsymbol{i}_3 = \sqrt{½}\big((\cos\phi + \boldsymbol{i}_3\sin\phi) - \boldsymbol{i}_1(\phi)\cos\phi\big)$

Compare this to Figure 6.15 and Figure 6.16, and let the autonomy stay on the plane $\boldsymbol{i}_2$ in relation to external frame $\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$.
The fictive snap of the two-parameter synchronised oscillation phase $\theta = \theta_1 = \theta_2 = \theta_3$ is spoiled by the symmetry factors $e^{\boldsymbol{i}\theta}$ for $\forall\theta \in [0,2\pi[$ in all ***directions*** $\forall\boldsymbol{i} \in \mathfrak{Z}$ – from the unitary circle group $U(1)$, e.g. $\odot_2 = \{U_\theta\colon \theta \to e^{\boldsymbol{i}_2\theta} \in U(1) \,|\, \forall\theta \in \mathbb{R}\}$.
This illustrates the traditional uncertainty of quantum mechanics.
Here in $\mathfrak{Z}$-space, we have three orthogonal ***directions*** $\odot_1, \odot_2, \odot_3$ of S^1circular symmetry. We only need to multiply two of these to make a full spherical symmetry S^2 as illustrated in Figure 6.20.
This auto unit sphere for ***entity*** Ψ_3 is a priori the principal transcendental range of a locality in $\mathfrak{Z}$-space.
We remember a radial probability distribution of the ***one quantum circle oscillator***

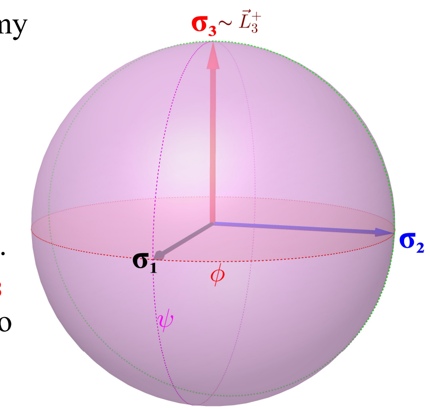

Figure 6.20, The unit sphere for the S^2 *hide* what is going on inside. The even algebra $\mathcal{G}_{0,2}$ has a structure for the versor oscillations that isomorph to the special unitary group $SU(2)$ in $\mathfrak{Z}$ space, that we cannot illustrate as S^3 symmetry.

(6.250) $2\tilde{r}(\rho) = 2\frac{1}{\sqrt{\pi}}\rho e^{-\frac{1}{2}\rho^2} \quad\text{for}\quad \forall\rho \geq 0,\ \text{with}\ \langle 2\tilde{r}(\rho)\rangle = 1$

relative to the ***entity*** autonomy unit radius $\rho = 1$ for its maximum, and the polar radial mean $\int_0^\infty \rho e^{-\frac{1}{2}\rho^2} d\rho = \sqrt{\pi/2}$ for these 1-rotor oscillators.[330]
We remember here that due to the odd density magnitude function (3.143) dependent on $\forall\rho \in \mathbb{R}$ for the central contradiction from the principle of Newton's third law expressed by: $\tilde{r}(\rho) + \tilde{r}(-\rho) = 0$, we get a factor 2 in (3.144) from the balanced radial dependent $\forall\rho \geq 0$, which is illustrated in Figure 3.5. In all planes, the distribution factor is $e^{\boldsymbol{i}\theta}2\tilde{r}(\rho)$, where $\boldsymbol{i}$ represent all arbitrary plane directions in $\mathfrak{Z}$-space. This is the foundation of what in the tradition is called the Heisenberg uncertainty for an information signal from an ***entity*** Ψ_3 in ***quantum mechanics***.

6.4.9.4. Breaking the Spherical Symmetry into One *Direction*

To be precise here in our example we are preoccupied with the ***entity direction*** $\boldsymbol{i}_3 = \boldsymbol{\imath}\boldsymbol{\sigma}_3$, which possesses the uncertain symmetry factor $e^{\boldsymbol{i}_3\theta_3}$ from the $U(1)$ group $\odot_{\boldsymbol{i}_3} = \{\theta \to e^{\boldsymbol{i}_3\theta} | \forall\theta \in [0,2\pi[\}$.
This makes a phase angle shift $\theta = \theta_3 - ½\pi$ indifferent to the start ***direction*** of the rotation axis

(6.251) $\boldsymbol{\sigma}_1 = e^{-\boldsymbol{i}_3½\pi}\boldsymbol{\sigma}_2 \iff \boldsymbol{i}_1 = e^{-\boldsymbol{i}_3½\pi}\boldsymbol{i}_2.$ We use $-\boldsymbol{i}_3 = e^{-\boldsymbol{i}_3½\pi}$ and have as (6.123) $\boldsymbol{i}_1 = -\boldsymbol{i}_3\boldsymbol{i}_2.$

Due to this the oscillator plane that is parallel to the polar axis $\boldsymbol{\sigma}_3$ can as well as (6.240) be represented by the 1-rotor $U_{\psi_1}^\dagger = e^{-½\boldsymbol{i}_1\psi_1}$, driven around $\boldsymbol{\sigma}_3$ by

(6.252) $\boldsymbol{i}_1(\phi) = \boldsymbol{\imath}\sigma_1(\phi) = e^{\boldsymbol{i}_3\phi}\boldsymbol{\imath}\mathbf{e}_1.$

[330] Consult section 3.3.5 for an external estimate of the size of this radius, formula (3.181) and (3.188) says $\frac{1C}{\omega} \to \rho$=1.