6.4.9. Oscillations in $\mathfrak{Z}$-space

The general claim in this book (concerning chapter I.) for an ***entity*** Ψ to exist is, that we demand it to contain at least one ***quantum*** of some frequency energy $\hbar\omega$.

6.4.9.1. Review of the Quantum Mechanical Circle Oscillator

In section 3.3 we introduced the plane excited circle oscillator I. (3.148) and (3.163)

(6.225) $\psi_\pm^{\odot} = a_{\odot\pm}^{\dagger}|0{,}0\rangle = |1,\pm1\rangle = \ 2\tilde{r}(\rho)\odot\ e^{\pm i\omega t},$

where $\odot$ is the transversal plane symmetry factor with $|e^{i\theta}|=|\odot|=1$, and the radial distribution is auto-normalized $\langle 1,\pm1|1,\pm1\rangle = \int_0^\infty (\,2\,\tilde{r}(\rho)\,)^2 e^{\pm i\phi}e^{\mp i\phi}d\rho = \int_0^\infty \frac{4}{\sqrt{\pi}}\rho^2 e^{-\rho^2}d\rho = 1.$

This is an eigenvalue solution to the angular momentum quantum operator equation (3.167)

(6.226) $\hat{L}_3|1,\pm1\rangle \doteq \pm1\hbar|1,\pm1\rangle.$

Where the angular momentum operator $\hat{L}_3$ (3.103) govern a rotating state (6.225) in a plane that is transversal to a 1-vector $\vec{L}_3^+$ ***direction*** for one quantum $|\vec{L}_3^+|=\hbar=1$ of the angular momentum, in an analogy with the classical angular momentum 1-vector as (3.171)

(6.227) $\vec{L}_3^+ = \widehat{\vec{\omega}} = \vec{n} = \mathbf{n} \sim \hat{L}_3 \quad \sim \hat{\vec{1}},$ ***One Quantum***.[328]

For simplification, we use the autonomous angular frequency energy $\omega = 1$ so that the phase angle (e.g. $\varphi, \phi, \psi \sim\omega t$) is the same as the development parameter internal in the ***entity***.

Removing the radial and the circular distributing factors $2\tilde{r}(\rho)\odot$ we have a pure unitary oscillator

(6.228) $\hat{\psi}_\pm = |\pm1\rangle = \ e^{\pm i\psi}$ or $\hat{\phi}_\pm = |\pm1\rangle = \ e^{\pm i\phi},$

where there is no specified physical ***direction*** in a $\mathfrak{Z}$-space. (Descartes: No Extension.)

In opposition to this traditional view, we led the transversal angular momentum ***quantum*** represent the ***quality of direction*** by the unit 1-vector $\mathbf{n}$ normal to the angular rotation plane ***direction*** unit $\boldsymbol{i}_{\perp\mathbf{n}} = \boldsymbol{i}_{\mathbf{n}} = \boldsymbol{\imath}\mathbf{n}$, whence we define a frame ***direction***, e.g. $\boldsymbol{\sigma}_3 \coloneqq \mathbf{n}$, whereby we have the transversal bivector plane ***direction*** $\boldsymbol{i}_3 = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 = \boldsymbol{\imath}\boldsymbol{\sigma}_3 = \boldsymbol{\imath}\mathbf{n}$ for the angular rotation plane.

By this ***primary quality of direction***, we rewrite (6.228)

(6.229) $\hat{\psi}_{\pm\boldsymbol{i}_3} = |\pm1\rangle_3 = e^{\pm\boldsymbol{i}_3\psi} \ = \ \hat{\psi}_{\pm\boldsymbol{\sigma}_3} = e^{\pm\boldsymbol{\imath}\boldsymbol{\sigma}_3\psi}.$

We know from the definitions, that this 1-rotor exists in the plane ***direction*** spanned by $\{\boldsymbol{i}_3\}$.

This quantum oscillating rotor possesses an angular momentum ***direction*** from which we endow the angular momentum quantum operator idea for $\hat{L}_3$ with a bivector ***direction***

(6.230) $\mathrm{L}_3 = \hbar\lambda_3\boldsymbol{i}_3 = \hbar\lambda_3\boldsymbol{\imath}\boldsymbol{\sigma}_3,$ as a *transversal bivector angular momentum operator*,

where $\lambda_3 \in\mathbb{R}$ is a *real scalar operator* span from the supporting ***direction*** $\boldsymbol{i}_3 = \boldsymbol{\imath}\boldsymbol{\sigma}_3$.

From this, we now rewrite the eigenvalue equation (6.226) for the angular momentum

(6.231) $(\hbar\lambda_3\boldsymbol{i}_3)|\pm1\rangle_3 \doteq \pm1\hbar\boldsymbol{i}_3|\pm1\rangle_3$

of ***one*** free subton state that defines an eigenvalue ***direction*** $\boldsymbol{i}_3 = \boldsymbol{\imath}\boldsymbol{\sigma}_3$ for the operator $\mathrm{L}_3 = \hbar\lambda_3\boldsymbol{i}_3$.

The ***direction*** bivector eigenvalue just replaces the traditional complex number eigenvalue.

Consider the fact that $\boldsymbol{i}_3 = e^{\boldsymbol{i}_3\pi/2}$ operating by $\boldsymbol{i}_3$ rotate by a phase shift $\pi/2$ in its own plane ***direction***, then $\boldsymbol{i}_3|\pm1\rangle_3 = e^{\boldsymbol{i}_3\pi/2}e^{\pm\boldsymbol{i}_3\psi} = e^{\boldsymbol{i}_3\varphi}$, where $\pm\varphi = (\pi/2 \pm \psi).$

Whence we have the *real scalar operator* angular momentum eigenvalue equation

(6.232) $(\hbar\lambda_3\boldsymbol{i}_3)e^{\pm\boldsymbol{i}_3\psi} \doteq \pm1\hbar\boldsymbol{i}_3e^{\pm\boldsymbol{i}_3\psi} \ \Rightarrow\ \lambda_3e^{\pm\boldsymbol{i}_3\varphi} = \pm1e^{\pm\boldsymbol{i}_3\varphi} \ \Rightarrow\ e^{\mp\boldsymbol{i}_3\varphi}\lambda_3e^{\pm\boldsymbol{i}_3\varphi} = \pm1.$

The ***direction*** is now given in the wavefunction $\boldsymbol{i}_3(e^{\pm\boldsymbol{i}_3\psi}) = (e^{\pm\boldsymbol{i}_3\psi})\boldsymbol{i}_3$ by $\boldsymbol{i}_3 = \boldsymbol{\imath}\boldsymbol{\sigma}_3$ and too implicit in the plane exponential function $e^{\pm\boldsymbol{i}_3\psi}$ ***direction***. (an analogy to [18] (8.25) p.271.)

From (6.232) we see that $|\langle\lambda_3\rangle| = 1$. Do we count $\hbar = 1$[329] we simply get $\lambda_3 = \pm1$ and by that just $\pm\boldsymbol{i}_3e^{\pm\boldsymbol{i}_3\psi} = \pm1\ \boldsymbol{i}_3e^{\pm\boldsymbol{i}_3\psi}$ as a tautology for a free subton excitation.

This subton excitation performs a free (external) angular momentum ***direction***.

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[328] Counting the cyclical ***direction*** with the causal counting operator $\hat{\vec{1}}$ we get the causal ***direction*** like angular momentum.

[329] $\hbar=1$ just means as we know, that energy and frequency is the same ***quality*** measured by the same ***quantity*** unit.

This is the foundation to create and by that define a frame ***direction*** for physical ***entities***.

When we use the 1-vector ***direction*** $\vec{L}_3^+ = \hbar\boldsymbol{\sigma}_3$, we have the transversal bivector $\mathrm{L}_3^+/\hbar = \boldsymbol{i}_3 = \boldsymbol{\imath}\boldsymbol{\sigma}_3$, representing the oscillating angular momentum with the ***direction quality of one quantum***.

6.4.9.2. Multi Excitations of Angular Momentum Internal in One *Entity*

When two angular momentum excitations as components of an ***entity*** exist in different planes , we know from § 6.3.5.2 that their product does not commute. Therefore, we use the canonical sandwich method by half-angle rotor operations as indicated (6.164)-(6.166)

(6.233) $\psi_{k\pm}^{1/2} = U_{\phi_k} = \ e^{+\frac{1}{2}\boldsymbol{i}_k\phi_k}.$

The question now is, will this affect the angular momentum operator components ~(6.230)

(6.234) $\mathrm{L}_k = \lambda_k\boldsymbol{i}_k = \lambda_k\boldsymbol{\imath}\boldsymbol{\sigma}_k.$

A guess could be that $\lambda_k \rightarrow \sim$½. We will qualify this for an ***entity*** $\Psi_{1/2}$ below in section 6.5. ↓

6.4.9.3. Intuition of Two Perpendicular Exited Circle Oscillators Inside one Entity

Presuming an ***entity*** Ψ_3 in $\mathfrak{Z}$-space we demand at least one ***quantum*** excitation of angular momentum. We imagine a local frame by $\boldsymbol{\sigma}_3 \sim \vec{L}_3^+$, i.e., we choose a local ***direction*** for Ψ_3 in our intuition, which is represented by the plane circular oscillator of the unitary circle group $\odot_3 = \{U_\theta\colon \theta \rightarrow e^{\boldsymbol{i}_3\theta} \in U(1) |\ \forall\theta \in\mathbb{R}\}$ that exists in the even geometric algebra $\mathcal{G}_{0,2}(\mathbb{R})$ for the $\mathfrak{Z}$-space,

As (6.31) and (6.119) $\boldsymbol{i}_3 = \boldsymbol{\imath}\boldsymbol{\sigma}_3$ we have that the chiral volume pseudoscalar $\boldsymbol{\imath}$ turn the angular momentum 1-vector ***direction*** $\boldsymbol{\sigma}_3 = \vec{L}_3^+/\hbar$ into its dual transversal *bivector*, that is the true internal representative for the free ***direction for one quantum of angular momentum*** $(\hbar=1)$,

(6.235) $\boldsymbol{i}_3 = \boldsymbol{\imath}\boldsymbol{\sigma}_3 = \boldsymbol{\imath}\vec{L}_3^+/\hbar = \ \mathrm{L}_3^+/\hbar.$ (a unit bivector).

We remember that $\boldsymbol{i}_3 = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ where we have both $\boldsymbol{\sigma}_2$ and $\boldsymbol{\sigma}_1$ is perpendicular to $\boldsymbol{\sigma}_3$, and further that their transversal plane bivectors $\boldsymbol{i}_3, \boldsymbol{i}_2$ and $\boldsymbol{i}_1$ are mutual perpendicular.

We choose the orthonormal 1-vector dextral basis $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3\}$ as the autonomous frame for our fundamental ***entity*** Ψ_3. Implied from this we have its autonomous quaternion basis $\{1, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3\}$.

With this knowledge we compare this to the perpendicular unitary 1-rotors

$\boxed{U_\phi \coloneqq e^{\frac{1}{2}\boldsymbol{i}_3\phi} \text{ and } U_\psi \coloneqq e^{\frac{1}{2}\boldsymbol{i}_2\psi}}$ in the ***directions*** $\boldsymbol{i}_3$ and $\boldsymbol{i}_2$, which was essential for the spherical coordinates in section 6.4.8.

This unitary 1-rotor $\boxed{U_\phi \coloneqq e^{\frac{1}{2}\boldsymbol{i}_3\phi}}$ oscillation manages the plane ***direction*** $\boldsymbol{i}_3$, whereby all 1-vectors rotates along this $\boldsymbol{i}_3$ plane. We now choose to rotate the internal autonomous frame $\{\boldsymbol{\sigma}_j\}$ relative to an external dextral frame $\{\mathbf{e}_j\}$ that's fixed to the surroundings. This plane rotation oscillation is performed by the 1-rotor U_ϕ using the canonical form (6.70)

(6.236) $\boldsymbol{\sigma}_j = U_\phi\mathbf{e}_jU_\phi^\dagger \qquad = \underline{\mathcal{R}}_\phi\mathbf{e}_j.$

Because the 1-rotor is in the $\boldsymbol{i}_3$ plane we have $\mathbf{e}_3 = \boldsymbol{\sigma}_3$ as the steady ***direction*** for our ***entity*** Ψ_3.

When we e.g., take the start reference from a fixed external ***direction*** 1-vector $\mathbf{e}_1$, we rotate

(6.237) $\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_1(\phi) = \underline{\mathcal{R}}_\phi\mathbf{e}_1 = U_\phi\mathbf{e}_1U_\phi^\dagger = U_\phi U_\phi\mathbf{e}_1 = U_\phi^2\mathbf{e}_1 = \ e^{\boldsymbol{i}_3\phi}\mathbf{e}_1.$

Then seen autonomous from the ***entity*** Ψ_3 frame $\{\boldsymbol{\sigma}_j\}$ the surrounding frame is rotating reversed

(6.238) $\mathbf{e}_1(\phi) = U_\phi^\dagger\boldsymbol{\sigma}_1U_\phi = \ e^{-\boldsymbol{i}_3\phi}\boldsymbol{\sigma}_1,$ and $\mathbf{e}_2(\phi) = U_\phi^\dagger\boldsymbol{\sigma}_2U_\phi = \ e^{-\boldsymbol{i}_3\phi}\boldsymbol{\sigma}_2$

Seen from the external lab world this frame $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ is fixed.

For the orthogonal 1-rotor circle oscillation $\boxed{U_\psi \coloneqq e^{\frac{1}{2}\boldsymbol{i}_2\psi}}$ around the perpendicular 1-vector axis $\boldsymbol{\sigma}_2$ with the angular momentum quantum $\boldsymbol{\sigma}_2 = \vec{L}_2^+/\hbar$ in the 1-vector ***direction***

(6.239) $\boldsymbol{\sigma}_2 = \boldsymbol{\sigma}_2(\phi) = \underline{\mathcal{R}}_\phi\mathbf{e}_2 = e^{\boldsymbol{i}_3\phi}\mathbf{e}_2 \qquad = \boldsymbol{i}_3e^{\boldsymbol{i}_3\phi}\mathbf{e}_1.$ (note $\boldsymbol{\sigma}_2 = \boldsymbol{i}_3\boldsymbol{\sigma}_1 \Leftrightarrow \mathbf{e}_2 = \boldsymbol{i}_3\mathbf{e}_1$).

And the dual transversal angular momentum ***direction*** bivector for this $\mathrm{L}_2^+/\hbar = \boldsymbol{i}_2 = \boldsymbol{\imath}\boldsymbol{\sigma}_2$.

Then we designed the external picture so that this plane for angular momentum has its ***direction*** rotating in the following oscillating way

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