

There are all even elements of $\mathbb{H} \sim \mathcal{G}_{0,2}(\mathbb{R}) \sim \mathcal{G}_3^+(\mathbb{R}) \leftrightarrow SU(2)$.

The scalar components in (6.178) both scale the elements and stand inside the quaternion group as elements of their basic form (6.131)

$$Q = q_0 + q_k \mathbf{i}_k \in \mathbb{H}, \quad k = 1, 2, 3,$$

where we know

- $\forall q_0 \in \mathbb{R}$ represent the real scalar field: s , and
- $q_k \mathbf{i}_k = \sum_{k=1}^3 (q_k \mathbf{i}_k)$ for $\forall q_k \in \mathbb{R}$ represent the bivector field: \mathbf{B} .

These two **qualities** represent a spinor of the form: A scalar plus a bivector, $s + \mathbf{B}$

What do oscillating quaternion spinors do?

- One 1-spinor oscillator can represent or endow an angular momentum **direction**.
- Another can represent or endow the precession of the angular momentum **direction**.

Both cases are dependent on the idea of angular **qualities** that are parameterised by phase angular arguments as real **quantities** that change the **directions**.

The first change 1-vector **directions** in a bivector plane,

the second change in the **direction** of that plane.

In all these form a quaternion 2-spinor.

Therefore, we for these arks:

What impact does the angular concept have on the intuition of a 3-space idea?

We start with a look at the Euler angles for rotation in a standard frame system.

6.4.6. Euler Angles for a Rotor in 3-space

We return to the mixed basis (6.119) for the geometric algebra of 3-space unfolded as

$$(6.179) \quad \{1, \sigma_1, \sigma_2, \sigma_3, \mathbf{i}_1 := \sigma_3 \sigma_2 = \mathbf{i} \sigma_1 = \mathbf{i}_2 \mathbf{i}_3, \mathbf{i}_2 := \sigma_1 \sigma_3 = \mathbf{i} \sigma_2 = \mathbf{i}_3 \mathbf{i}_1, \mathbf{i}_3 := \sigma_2 \sigma_1 = \mathbf{i} \sigma_3 = \mathbf{i}_1 \mathbf{i}_2, \mathbf{i} = \sigma_3 \sigma_2 \sigma_1\}.$$

We have in (6.145) from (6.144) for 3-space introduced the rotor for the spherical locality

$$(6.180) \quad U_{\mathbf{i}_n} = e^{(n_1 \mathbf{i}_1 + n_2 \mathbf{i}_2 + n_3 \mathbf{i}_3) \frac{1}{2} \varphi} = e^{\mathbf{i} n \frac{1}{2} \varphi}, \quad 319$$

with the 1-vector **direction** $\mathbf{n} = n_1 \sigma_1 + n_2 \sigma_2 + n_3 \sigma_3$ with the transversal bivector **direction**

$$(6.181) \quad \mathbf{i}_n = \mathbf{i} \mathbf{n} = n_1 \mathbf{i}_1 + n_2 \mathbf{i}_2 + n_3 \mathbf{i}_3.$$

We now intuit a physical **entity** Ψ in 3-space where all the **qualities**

pqg-0, pqg-1, pqg-2, and pqg-3 are participating.

We will see what happens when we rotate this.

We have in (5.193) formulated the general **canonical form** for any **orthogonal transformation**

$$(6.182) \quad \underline{\mathcal{R}} \mathbf{x} = \mathbf{U} \mathbf{x} \mathbf{U}^\dagger,$$

that applies to all (multi)vectors in space \mathbb{G} , also outside the rotor \mathbf{U} plane **direction**. See Figure 6.12.

I.e., for all 1-vectors objects $\mathbf{x} = x_1 \sigma_1 + x_2 \sigma_2 + x_3 \sigma_3$ and thereby all bivectors subjects³²⁰

$\mathbf{X} = \beta_1 \mathbf{i}_1 + \beta_2 \mathbf{i}_2 + \beta_3 \mathbf{i}_3$ in entire 3-space of physics, for a regular rotation we write,

$$(6.183) \quad \mathbf{y} = \underline{\mathcal{R}} \mathbf{x} = \mathbf{U} \mathbf{x} \mathbf{U}^\dagger, \quad \mathbf{Y} = \underline{\mathcal{R}} \mathbf{X} = \mathbf{U} \mathbf{X} \mathbf{U}^\dagger \quad \text{or} \quad \Psi' = \underline{\mathcal{R}} \Psi = \mathbf{U} \Psi \mathbf{U}^\dagger.$$

The scalars are rotation invariant and pseudoscalars are invariant for regular rotations.

The rotation performed by the rotor (6.180) is

$$(6.184) \quad \underline{\mathcal{R}}_{\mathbf{n}} \Psi = U_{\mathbf{i}_n} \Psi U_{\mathbf{i}_n}^\dagger = e^{\mathbf{i} n \frac{1}{2} \varphi} \Psi e^{-\mathbf{i} n \frac{1}{2} \varphi} = e^{\mathbf{i} n \varphi} \Psi = e^{(n_1 \mathbf{i}_1 + n_2 \mathbf{i}_2 + n_3 \mathbf{i}_3) \varphi} \Psi,$$

rotated around one 1-vector \mathbf{n} in plane \mathbf{i}_n . – Here we use that one angular rotor anticommute through any **entity** Ψ by its own reverse in the same plane **direction**, refer (5.193), (6.70).

What happens when we rotate through two or more independent planes parametrised by their separate angular (development) parameters as demanded above?

One circle plane rotation has the normal 1-vector \mathbf{n} as its turning axis **direction**.

Rotation by another independent plane circular rotation will change the **direction** of this \mathbf{n} .

This problem we illustrate between two standard frame orthonormal bases

$$(6.185) \quad \{\sigma_1, \sigma_2, \sigma_3\} \leftrightarrow \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$$

For the two reverse transformations, we have due to (6.92) and (6.95) the **canonical forms**

$$(6.186) \quad \sigma_j = \mathbf{U} \mathbf{e}_j \mathbf{U}^\dagger \quad \text{and} \quad \mathbf{e}_k = \mathbf{U}^\dagger \sigma_k \mathbf{U}$$

These 1-vectors we use as an object for intuition in Figure 6.17 of an **entity** Ψ subject:

³¹⁹ Don't trust the rewriting of (6.180) $\approx e^{i_1 n_1 \frac{1}{2} \varphi} e^{i_2 n_2 \frac{1}{2} \varphi} e^{i_3 n_3 \frac{1}{2} \varphi}$ due to, the three operators existing in independent planes. ³²⁵

³²⁰ The 1-vector **object** we intuit as an arrow from one point to another, while the bivector stays as a plane amoeba **subject**.