The odd 3-multivector $\langle A\rangle^-_{1,3}$ represents a physical extended[158, 293] ***entity*** with a ***pqg***-1 length ***quantity*** **a** with magnitude $|\mathbf{a}|$ and a ***direction*** $\hat{\mathbf{a}}=\mathbf{a}/|\mathbf{a}|$, that is joined with a chiral pseudoscalar ***pqg***-3 volume ***quantity*** $\upsilon\boldsymbol{i}$ with a scalar volume $\upsilon=\pm|\upsilon|$ factor (coordinate) as magnitude $|\upsilon\boldsymbol{i}|$ and ***direction*** $\boldsymbol{i}$. A negative $\upsilon<0$ means a sinistral volume contra a dextral volume $\upsilon>0$.
We now have a chiral pseudoscalar unit $\boldsymbol{i}$ representing the ***primary quality of third grade***, a chiral ***pqg***-3***direction*** that gives the necessary extended structure of 𝔍-space of physics.

6.3.3. Operational Structure of the Trivector Chiral Volume Pseudoscalar $\boldsymbol{i}$ of 𝔍 Space

In the text above, we have invented a unit subject $\boldsymbol{i}$, that when it with a multiplication operation on a 1-vector object $\mathbf{b}$ creates a transversal plane subject bivector $\boldsymbol{i}\mathbf{b}$.
To instrumentalise this we choose locally an orthonormal standard frame basis set $\{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\boldsymbol{\sigma}_3\}$ so that $\mathbf{b}=\beta\boldsymbol{\sigma}_3$. Then we can write $\boldsymbol{i}\mathbf{b}=\beta\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1=\beta\boldsymbol{i}$, where $\boldsymbol{i}=\boldsymbol{i}_3=\boldsymbol{i}\boldsymbol{\sigma}_3$ is the transversal unit.
When we further choose the object ***direction*** $\boldsymbol{\sigma}_1=\mathbf{u}\perp\mathbf{b}$ and consider another 1-vector $\mathbf{r}\perp\mathbf{b}$, $\mathbf{r}\nparallel\mathbf{u}$, we can construct a 1-spinor multivector $\langle A\rangle^+_{0,2}=\mathbf{r}\cdot\mathbf{u}+\mathbf{r}\wedge\mathbf{u}=\mathbf{ru}=\rho\mathbf{vu}=\rho U=\rho e^{\boldsymbol{i}\theta}$, with $\mathbf{r}=\rho\mathbf{v}$.
Here we repeat that the scalar ***quantity*** $\mathbf{r}\cdot\mathbf{u}=\rho\cos\theta$ specifies the inner covariant scalar part from the 1-rotor $\mathbf{vu}$ arc angle θ between the two chosen object 1-vectors $\mathbf{u}$ and $\mathbf{v}$. External to this scalar the bivector $\mathbf{r}\wedge\mathbf{u}=\boldsymbol{i}\rho\sin\theta=\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\rho\sin\theta$ exist extended in the plane $\boldsymbol{i}=\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$.
The 1-spinor $\langle A\rangle^+_{0,2}=\mathbf{ru}$ has an amplitude, magnitude $\rho=|\mathbf{ru}|$ as a transversal plane ***quantity***.
We now look at the normalized 1-spinor as an exponential expanded (5.90),(5.191) 1-rotor

(6.68) $$\frac{\langle A\rangle^+_{0,2}}{|\langle A\rangle^+_{0,2}|}=\mathbf{vu}=U=e^{\boldsymbol{i}\theta}=\exp(\boldsymbol{i}\theta)=1+\frac{(\boldsymbol{i}\theta)}{1!}+\frac{(\boldsymbol{i}\theta)^2}{2!}+\frac{(\boldsymbol{i}\theta)^3}{3!}+\frac{(\boldsymbol{i}\theta)^4}{4!}+\frac{(\boldsymbol{i}\theta)^5}{5!}+\cdots$$

The magnitude of this 1-rotor (a unitary spinor) $\mathbf{vu}=U$ is always one $UU^\dagger=|\mathbf{vu}|^2=1$.
The even exponents (5.92) of the bivector $(\boldsymbol{i}\theta)^n$ contribute to the pure scalar $\mathbf{v}\cdot\mathbf{u}=\cos\theta$, and the odd exponents (5.93) contribute to the bivector $\mathbf{v}\wedge\mathbf{u}=\boldsymbol{i}\sin\theta$. Here take $\hat{\boldsymbol{\omega}}=\boldsymbol{\sigma}_3$ and forget implicit the basis $\{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\boldsymbol{\sigma}_3\}$ and instead use $\hat{\boldsymbol{\omega}}\perp\boldsymbol{i}$ as the frame for ***directions***. From this we define the 1-vector $\mathbf{b}=\beta\hat{\boldsymbol{\omega}}$, from this we scale the transversal bivector $\boldsymbol{\theta}=\boldsymbol{i}\mathbf{b}=\boldsymbol{i}\beta$ for a rotation angular area around $\mathbf{b}$. For the 1-rotor we have $\theta=½\beta$ and $½\boldsymbol{\theta}=\boldsymbol{i}\theta$, with a magnitude $\theta=|\boldsymbol{i}\theta|$ as the radian arc measure θ of the 1-rotor angle, see Figure 5.47 where we have that angular sector areal is the half $|½\boldsymbol{\theta}|$ of this with the ***pqg***-2 ***direction*** bivector $\boldsymbol{\theta}=2\boldsymbol{i}\theta$, with magnitude $2\theta=\beta$.
Hereby we express the regular rotation with this angular bivector as (5.193) from a 1-vector object $\mathbf{x}\in\mathcal{G}_3(\mathbb{R})$

(6.69) $$\underline{\mathcal{R}}\mathbf{x}=U\mathbf{x}U^\dagger=e^{½\boldsymbol{\theta}}\mathbf{x}e^{-½\boldsymbol{\theta}}=e^{\boldsymbol{\theta}}\mathbf{x}=e^{\boldsymbol{i}\theta}\mathbf{x}e^{-\boldsymbol{i}\theta}=e^{\boldsymbol{i}2\theta}\mathbf{x}$$

In comparing this with the magnitude $|\boldsymbol{i}\mathbf{b}|=|\mathbf{b}|=|\beta|$ the reader is encouraged to consider the idea:
the first choose $\beta=2\theta$, so that the rotation axis ***direction*** 1-vector $\mathbf{b}$ has magnitude $|\mathbf{b}|=|2\theta|$ so that the 1-rotor is $U=e^{½\boldsymbol{i}\mathbf{b}}$ and the regular rotation is

(6.70) $$\underline{\mathcal{R}}\mathbf{x}=U\mathbf{x}U^\dagger=e^{½\boldsymbol{i}\mathbf{b}}\mathbf{x}e^{-½\boldsymbol{i}\mathbf{b}}=e^{\boldsymbol{i}\mathbf{b}}\mathbf{x}\quad\in\mathcal{G}_3(\mathbb{R})$$

Now we have defined an operator $\underline{\mathcal{R}}=e^{\boldsymbol{i}\mathbf{b}}$ that rotate a 1-vector $\mathbf{x}$ ***direction*** in 𝔍-space around another 1-vector $\mathbf{b}$ ***direction***, where the magnitude $|\mathbf{b}|=|2\theta|$, equals the rotation angle. Then the transversal bivector $\boldsymbol{\theta}=\boldsymbol{i}\mathbf{b}$ exposes the rotation $\underline{\mathcal{R}}\mathbf{x}=U\mathbf{x}U^\dagger=e^{\boldsymbol{i}\mathbf{b}}\mathbf{x}$ as shown in Figure 6.12, which is a perspective that has its projection on plane $\boldsymbol{i}$ in Figure 5.47.

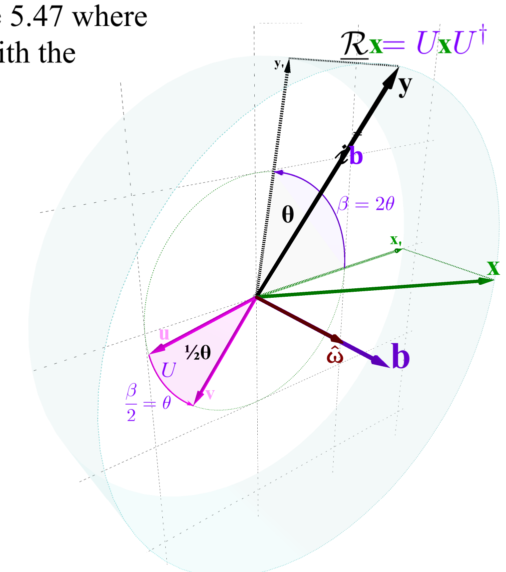

Figure 6.12 Regular rotation around $\mathbf{b}$ by an angle $\beta=2\theta=|\mathbf{b}|$, driven by the unitary spinor rotor $U=e^{½\boldsymbol{i}\mathbf{b}}$, through the operator $\underline{\mathcal{R}}=e^{\boldsymbol{i}\mathbf{b}}$. 1-vector rotation: $\mathbf{x}\to\mathbf{y}=\underline{\mathcal{R}}\mathbf{x}=U\mathbf{x}U^\dagger=e^{\boldsymbol{i}\mathbf{b}}\mathbf{x}$. The rotation 1-vector $\mathbf{b}$ and its bivector $\boldsymbol{\theta}=\boldsymbol{i}\mathbf{b}$ ***direction*** $\boldsymbol{i}=\boldsymbol{i}\hat{\boldsymbol{\omega}}$. Then $U=e^{½\boldsymbol{i}\hat{\boldsymbol{\omega}}\beta}=e^{½\boldsymbol{i}\mathbf{b}}$.

We have chosen the name $\hat{\boldsymbol{\omega}}=\boldsymbol{\sigma}_3$ for the unit ***pqg***-1 ***direction***. When we use $\hat{\boldsymbol{\omega}}$ as our angular reference frequency count $|\hat{\boldsymbol{\omega}}|\equiv1$ and choose the phase development parameter $\beta=(t-x_3/c)$ we have the bivector exponential oscillation operator $e^{\boldsymbol{i}\hat{\boldsymbol{\omega}}(t-x_3/c)}=e^{\boldsymbol{i}\boldsymbol{\sigma}_3\beta}$ in the transversal plane to $\hat{\boldsymbol{\omega}}$. This expresses the classical plane wave form, which is transversal to the propagation.

We desire is to make $\hat{\boldsymbol{\omega}}$ the normal rotation oscillation plane. We, therefore, led the unit bivector $\boldsymbol{i}$ represent this transversal plane, then we have $\boldsymbol{i}=\boldsymbol{i}\hat{\boldsymbol{\omega}}\in\mathcal{G}^+_3(\mathbb{R})$, and by that $\hat{\boldsymbol{\omega}}=-\boldsymbol{i}\boldsymbol{i}$, where we simply have chosen $[|\hat{\boldsymbol{\omega}}|^{-1}=1]$ as the unit for the phase angle parameter β.
For $\forall\beta\in\mathbb{R}$ we have a bivector argument in the rotation exponential $e^{\boldsymbol{i}\beta}=e^{\boldsymbol{i}\hat{\boldsymbol{\omega}}\beta}\in\mathcal{G}^+_3(\mathbb{R})$.
The tradition uses the designation $\mathbf{n}$ for a plane normal therefore $\hat{\mathbf{b}}\coloneqq\mathbf{n}=\hat{\boldsymbol{\omega}}$, as the 1-vector ***direction*** for a rotation axis object $\mathbf{b}=\beta\hat{\mathbf{b}}\in\mathcal{G}^-_3(\mathbb{R})$, where the magnitude $\beta=|\mathbf{b}|$ represents an angle in the transversal plane ***pqg***-2***direction*** for the bivector $\boldsymbol{i}\mathbf{b}=\beta\boldsymbol{i}\hat{\mathbf{b}}=\boldsymbol{i}\beta$ subject in $\mathcal{G}^+_3(\mathbb{R})$.
This is in the everyday world illustrated by pointing your index finger or arm towards the paper, a screen, a wall or even the celestial sky and you will realise that there is something transversal and by that, a space around your pointing, and we describe this connecting structure by the ***pqg***-3 ***directional*** chiral volume pseudoscalar unit $\boldsymbol{i}$ as ***a primary quality of third grade*** (***pqg***-3).
The idea $\boldsymbol{i}$ has obtained the name pseudoscalar because 1-dimensionality as (6.28), and in $\mathcal{G}_3(\mathbb{R})$ *commutes* with both 1-vectors and bivectors: $\boldsymbol{i}\mathbf{x}=\mathrm{X}=\mathbf{x}\boldsymbol{i}$ and $\boldsymbol{i}\mathrm{B}=-\mathbf{b}=\mathrm{B}\boldsymbol{i}$, due to (6.33), (6.36) and the definition (6.21) just like a scalar multiplication. The reverse order orientation definition (6.23) $\tilde{\boldsymbol{i}}=-\boldsymbol{i}$ gives the pseudo ***quality***, together with the fact that $\boldsymbol{i}^2=-1$.
In this context, we remember that ordinary scalars commute with every kind of multivector.

6.3.4. Rotation in 𝔍-space

We got the insight that the essential concept of an 𝔍-space ***entity*** is rotation oscillation in a plane given by at least two unit 1-vector ***directions*** $\mathbf{u}$ and $\mathbf{v}$ forming a rotor concept by a product $\mathbf{vu}$.
Here we normalize the even spinor in $\mathcal{G}^+_3$ by a rotor $\langle A\rangle^+_{0,2}=\mathbf{vu}$. (with spinor radius $|\mathbf{ru}|=|\mathbf{vu}|=1$.)
It is obvious, that a permutation reverses the orientation of the rotor plane ***pqg***-2 ***direction***.
This we from (5.191) and (5.192) express as unitary operators

(6.71) $$U=\mathbf{vu}=\mathbf{v}\cdot\mathbf{u}+\mathbf{v}\wedge\mathbf{u}=e^{+½\boldsymbol{\theta}}=e^{+½\boldsymbol{i}\mathbf{b}}=e^{+½\boldsymbol{i}\hat{\mathbf{b}}\beta}=e^{+½\boldsymbol{i}\beta}=e^{+\boldsymbol{i}\theta}$$

(6.72) $$U^\dagger=\mathbf{uv}=\mathbf{v}\cdot\mathbf{u}-\mathbf{v}\wedge\mathbf{u}=e^{-½\boldsymbol{\theta}}=e^{-½\boldsymbol{i}\mathbf{b}}=e^{-½\boldsymbol{i}\hat{\mathbf{b}}\beta}=e^{-½\boldsymbol{i}\beta}=e^{-\boldsymbol{i}\theta}$$

Here we introduce the bivector $½\boldsymbol{i}\mathbf{b}=½\boldsymbol{\theta}$ which represents the 1-rotor area with ***direction***, as an argument for the 2-multi-vector exponential function $e^{\pm½\boldsymbol{i}\mathbf{b}}$, around the axis of rotation $\mathbf{b}=\beta\hat{\boldsymbol{\omega}}$.
Hence, the regular rotation is a linear transformation along the transversal plane $\boldsymbol{i}\hat{\boldsymbol{\omega}}$ is

(6.73) $$\underline{\mathcal{R}}\mathbf{x}=U\mathbf{x}U^\dagger=e^{½\boldsymbol{i}\mathbf{b}}\mathbf{x}e^{-½\boldsymbol{i}\mathbf{b}}=e^{\boldsymbol{i}\mathbf{b}}\mathbf{x}=e^{\boldsymbol{i}\hat{\boldsymbol{\omega}}\beta}\mathbf{x}=UU\mathbf{x}=U^2_\beta\mathbf{x}\qquad\in\mathcal{G}_3(\mathbb{R}).$$

Refer to section 5.4.5. The unitary U demand that $|U|^2=1$, by (6.71), (6.72) we have

(6.74) $$|U|^2=UU^\dagger=\mathbf{vuuv}=e^{+½\boldsymbol{i}\mathbf{b}}e^{-½\boldsymbol{i}\mathbf{b}}=1.$$

One essential thing here is that rotor U does not commutate with 1-vectors $\mathbf{x}$.
In (6.73) we have the two-sided multiplication operation (sandwiching) $\underline{\mathcal{R}}\mathbf{x}=U\mathbf{x}U^\dagger\in\mathcal{G}_3(\mathbb{R})$
We can also have situations where we use Left or Right multiplication operations:

(6.75) $$\underline{\mathcal{R}}_L\mathbf{x}=U\mathbf{x}\quad\text{or}\quad\underline{\mathcal{R}}_R\mathbf{x}=U^\dagger\mathbf{x}=\mathbf{x}U\qquad\in\mathcal{G}_3(\mathbb{R}).$$

Anyway U, $U^\dagger$, U^2, exist in the same plane ***direction***, a transversal plane to one and the same 1-vector ***direction***, therefore we may always implicit presume a unit 1-vector ***direction*** $\hat{\boldsymbol{\omega}}$ for the transversal plane $\boldsymbol{i}\hat{\boldsymbol{\omega}}$. To be explicit we can write $U_{\theta\hat{\boldsymbol{\omega}}}$, $U^\dagger_{\theta\hat{\boldsymbol{\omega}}}$, $U^2_{\beta\hat{\boldsymbol{\omega}}}$, and therefore
$\underline{\mathcal{R}}_{\beta\hat{\boldsymbol{\omega}}}\mathbf{x}=U_{\theta\hat{\boldsymbol{\omega}}}\mathbf{x}U^\dagger_{\theta\hat{\boldsymbol{\omega}}}=U^2_{\beta\hat{\boldsymbol{\omega}}}\mathbf{x}$, with $\beta=2\theta$. As an operator we have an explicit example

(6.76) $$\underline{\mathcal{R}}_{\beta\hat{\boldsymbol{\omega}}}=U^2_{\beta\hat{\boldsymbol{\omega}}},\quad\text{where we have}\quad U_{\theta\hat{\boldsymbol{\omega}}}=\underline{\mathcal{R}}_{L,\theta\hat{\boldsymbol{\omega}}}\ \text{from the left, and}\ U^\dagger_{\theta\hat{\boldsymbol{\omega}}}=\underline{\mathcal{R}}_{R,\theta\hat{\boldsymbol{\omega}}}\ \text{from the right.}$$