Research on the a priori of Physics
Geometric Critique of Pure Mathematical Reasoning
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6.3. The 3-space Structure *Quality* Described by Multivectors

A general arbitrary 3-multivector is constructed from four ***primary qualities of grades*** ***pqg***-0 + ***pqg***-1 + ***pqg***-2 + ***pqg***-3. The approach tries to add these different qualities together.
A classical space solid object we have the tradition to define in a Euclidean vector space $(V_3,\mathbb{R})\sim\mathbb{R}_1^3$ fixed to a standard *orthonormal dextral basis* $\{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\boldsymbol{\sigma}_3\}$ of three 1-vector objects.
All 1-vectors in this Euclidean vector space possess the ***primary qualities of first grade*** (***pqg***-1).
This basis implies the transversal bivector basis $\{\boldsymbol{i}_1,\boldsymbol{i}_2,\boldsymbol{i}_3\}=\{\boldsymbol{\sigma}_3\boldsymbol{\sigma}_2,\ \boldsymbol{\sigma}_1\boldsymbol{\sigma}_3,\ \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\}$, (6.31), (6.34) as a generator of all planes possessing ***pqg***-2 ***quality***. On top of that we have trivector chiral pseudoscalar unit $\boldsymbol{\imath}:=\boldsymbol{\sigma}_3\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ generating the chiral volume possessing ***pqg***-3 ***quality***.
In all, we expand a multivector form from the geometric algebra $\mathcal{G}_3=\mathcal{G}_3(\mathbb{R})=\mathcal{G}(V_3,\mathbb{R})$ as

(6.59) $$A=\ \alpha\ +\underbrace{x_1\boldsymbol{\sigma}_1+x_2\boldsymbol{\sigma}_2+x_3\boldsymbol{\sigma}_3}\ +\ \underbrace{\beta_1\boldsymbol{i}_1+\beta_2\boldsymbol{i}_2+\beta_3\boldsymbol{i}_3}\ +\ \upsilon\boldsymbol{\imath}$$
$$A=\langle A\rangle_0+\quad\langle A\rangle_1\quad+\quad\langle A\rangle_2\quad+\ \langle A\rangle_3$$

Refer to:	(5.59)	(6.29)	(6.32), (6.33), (6.36)	(6.15) in 6.2.2

We have separated the multivector concept into four different ***primary quality grades***

(6.60) $\langle A\rangle_0=\alpha,$ ***pqg***-0, scalar, $\dim(\mathbb{R})=1,$

(6.61) $\langle A\rangle_1=\mathbf{a}\ =x_1\boldsymbol{\sigma}_1+x_2\boldsymbol{\sigma}_2+x_3\boldsymbol{\sigma}_3,$ ***pqg***-1, 1-vector, $\dim(V_3)=3,$

(6.62) $\langle A\rangle_2=\mathbf{b}\boldsymbol{\imath}=(\beta_1\boldsymbol{\sigma}_1+\beta_2\boldsymbol{\sigma}_2+\beta_3\boldsymbol{\sigma}_3)\boldsymbol{\imath}=\ \beta_1\boldsymbol{i}_1+\beta_2\boldsymbol{i}_2+\beta_3\boldsymbol{i}_3,$ ***pqg***-2, bivector, $\dim(V_3)=3,$

(6.63) $\langle A\rangle_3=\upsilon\boldsymbol{\imath},$ *chiral volume pseudoscalar*, ***pqg***-3, trivector, $\dim(\mathbb{R})=1.$

The multivector idea over a geometric product algebra has a linear addition structure of multiple ***primary qualities of grades*** we call it $\mathcal{G}_3(\mathbb{R})=\mathcal{G}(V_3,\mathbb{R})$ over a Euclidean vector space V_3, with which we try to describe the *local topological form structure* of the physical 3-space.
This linear addition structure for multivectors over a real field $\mathbb{R}$ we express

(6.64) $$A\quad=\quad\alpha+\mathbf{a}+\mathbf{b}\boldsymbol{\imath}+\upsilon\boldsymbol{\imath}.$$

The linear algebra $\mathcal{G}_3=\mathcal{G}_3(\mathbb{R})$ has more dimension from the different ***grade qualities***, in all

(6.65) $\dim(\mathcal{G}_3)=1+3+3+1=8.$ In generell, $\dim(\mathcal{G}_n)=\sum_{r=0}^{n}\binom{n}{r}\ =\ 2^n$

The mixed basis for the whole linear algebra $\mathcal{G}_3$ is $\{1,\ \boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\boldsymbol{\sigma}_3,\ \boldsymbol{\sigma}_3\boldsymbol{\sigma}_2,\ \boldsymbol{\sigma}_1\boldsymbol{\sigma}_3,\ \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1,\ \boldsymbol{\sigma}_3\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\}$.

6.3.2. The Even and the Odd Geometric Algebra

When we split this algebra in even $\mathcal{G}_3^+$ and odd $\mathcal{G}_3^-$, so $\mathcal{G}_3=\mathcal{G}_3^-+\mathcal{G}_3^+$, we write the multivector

(6.66) $$A=\langle A\rangle^++\langle A\rangle^-=\ \langle A\rangle_{0,2}^++\langle A\rangle_{1,3}^-=\ \underbrace{\alpha+\mathbf{b}\boldsymbol{\imath}}_{+}\ +\ \underbrace{\mathbf{a}+\upsilon\boldsymbol{\imath}}_{-},$$

where the even algebra $\mathcal{G}_3^+$ is a *closed* multiplication spinor algebra, while $\mathcal{G}_3^-$ is open, not closed.
The spinor multivector subject can be expressed as a 1-rotor multiplied with a real dilation factor

(6.67) $$\langle A\rangle^+=\ \langle A\rangle_{0,2}^+=\ \langle A\rangle_0+\langle A\rangle_2=\ \alpha+\mathbf{b}\boldsymbol{\imath}\ =\ \mathbf{r}\cdot\mathbf{u}+\mathbf{r}\wedge\mathbf{u}=\ \mathbf{ru}=\rho U=\ \rho e^{\boldsymbol{i}\theta}\quad\in\mathcal{G}_3^+(\mathbb{R}),$$

as in (5.163) origin in (5.97), where the rotation is in the plane of the bivector $\mathbf{b}\boldsymbol{\imath}=\mathbf{r}\wedge\mathbf{u}$.
You may define two object 1-vectors whose product spinor $\mathbf{ru}$ determines the rotation, and dilation $\rho=|\mathbf{ru}|$. (We prefer to choose $|\mathbf{u}|=1$ so that $\rho=|\mathbf{r}|$ if it is possible.)[297]
If you determine the 1-vector $\mathbf{b}=-\boldsymbol{\imath}(\mathbf{r}\wedge\mathbf{u})$ as an object it will represent the rotation axis for $\langle A\rangle^+$.
This rotation axis has a transversal plane ***direction*** in which $\mathbf{r}$ and $\mathbf{u}$ exist.

The odd algebra $\mathcal{G}_3^-(\mathbb{R})$ part substance of 3-space gives us subjects $\langle A\rangle_{1,3}^-=\mathbf{a}+\upsilon\boldsymbol{\imath}$.
The geometric 1-vector can give us a straight rectilinear translation as in §4.4.2.13 and § 5.3.7.2 by the subject $\mathbf{t}=\mathbf{a}$, or by the 1-vector object $\mathbf{a}$ representing a ***pqg***-1 ***direction*** and a straight line-segment magnitude $|\mathbf{a}|$ for a physical ***entity***. The new ***pqg***-3 subject $\upsilon\boldsymbol{\imath}$ gives us a ***chiral-directed*** volume of a solid spatial ***entity*** object, as a ***chiral quality pqg***-3.

[297] I think that there could be a little problem here; if we find $\mathbf{r}$ and $\mathbf{u}$ colinear $\mathbf{r}\wedge\mathbf{u}=0\Leftrightarrow\mathbf{b}=0$ then the spinor is a pure scalar $\mathbf{r}\cdot\mathbf{u}$, and by that, does a pure scalar spinor gives any sense, even if it has a final magnitude $|\mathbf{ru}|>0$? Oscillations will resolve this.