E XI.De.13. A *prism* is a solid figure contained by planes two of which, namely those which are opposite, are equal, similar, and parallel, while the rest are parallelograms.

E XI.De.14. When a semicircle with fixed diameter is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a *sphere*.

E XI.De.15. The *axis of the sphere* is the straight line which remains fixed and about which the semicircle is turned.

E XI.De.16. The *center of the sphere* is the same as that of the semicircle.

E XI.De.17. A *diameter of the sphere* is any straight line drawn through the center and terminated in both directions by the surface of the sphere.

E XI.Pr.1. A part of a straight line cannot be in the plane of reference and a part in plane more elevated.

E XI.Pr.3. If two planes cut one another, then their intersection is a straight line.

E XI.Pr.4. If a straight line is set up at right angles to two straight lines which cut one another at their common point of section, then it is also at right angles to the plane passing through them.

E XI.Pr.5. If a straight line is set up at right angles to three straight lines which meet one another at their common point of section, then the three straight lines lie in one plane.

E XI.Pr.6. If two straight lines are at right angles to the same plane, then the straight lines are parallel.

E XI.Pr.8. If two straight lines are parallel, and one of them is at right angles to any plane, then the remaining one is also at right angles to the same plane.

E XI.Pr.9. Straight lines which are parallel to the same straight line but do not lie in the same plane with it are also parallel to each other.

E XI.Pr.10. If two straight lines meeting one another are parallel to two straight lines meeting one another not in the same plane, then they contain equal angles.

E XI.Pr.11. To draw a straight line perpendicular to a given plane from a given elevated point.

E XI.Pr.12. To set up a straight line at right angles to a give plane from a given point in it.

E XI.Pr.13. From the same point two straight lines cannot be set up at right angles to the same plane on the same side.

E XI.Pr.14. Planes to which the same straight line is at right angles are parallel.

E XI.Pr.15. If two straight lines meeting one another are parallel to two straight lines meeting one another not in the same plane, then the planes through them are parallel.

E XI.Pr.16. If two parallel planes are cut by any plane, then their intersections are parallel.

E XI.Pr.17. If two straight lines are cut by parallel planes, then they are cut in the same ratios.

E XI.Pr.18. If a straight line is at right angles to any plane, then all the planes through it are also at right angles to the same plane.

E XI.Pr.19. If two planes which cut one another are at right angles to any plane, then their intersection is also at right angles to the same plane.
"

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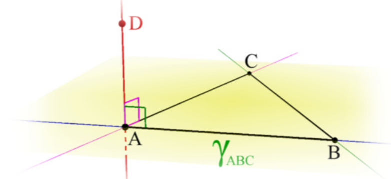
6.1.2. Additional A Priori Judgments to the Euclidean Stereo Space Geometry

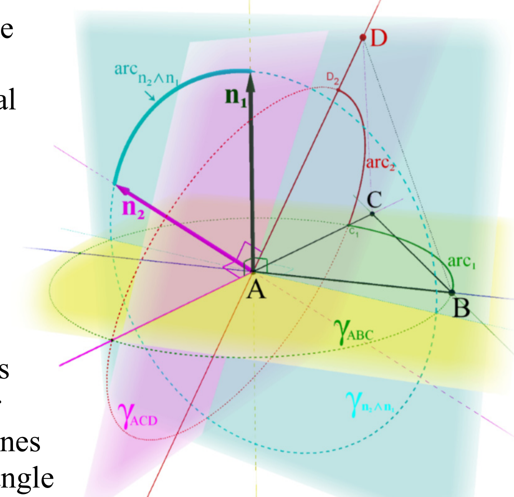
Figure 6.1 Concept of a Space: r-w



r. Four points → one space object.
s. One external point→ one normal.

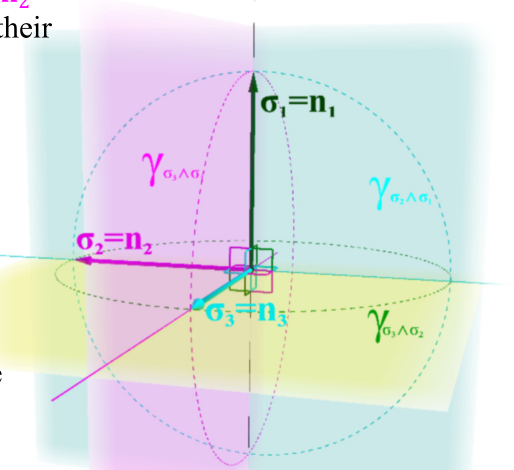
r. Postulate: A space object (solid) is uniquely defined by four points A,B,C,D , which do not lie in one and the same plane. The plane γ_{ABC} through A,B,C, as in 5.1.1.2,k., is seen as a foundation for a fourth point $D \notin \gamma_{ABC}$ on a line ℓ_{AD} through A outside and perpendicular to the plane γ_{ABC} (E XI.De.3, E XI.Pr.12). Such lines from all points $X \in \gamma_{ABC}$ generate the entire stereo space outside the plane.

s. A straight line perpendicular to the plane is called a normal to the plane. From any point outside the plane can be raised just one normal to the plane. Contradiction, when a line through D and A not normal line ℓ_{AD} is inclined to plane γ_{ABC}, (E XI.De.5.).



t. The *inclination* of a plane object γ_{ACD} to a plane object γ_{ABC}, with definition of a mutual angle.

t. Two planes have a mutual inclination (E XI.De.6.) defined as the angle between their normal lines or between their two normal 1-vectors $|\mathbf{n}_2| = |\mathbf{n}_1| = 1$. For the object planes γ_{ABC} and γ_{ACD} we have the angular measure $\mathrm{arc}_1 = ∡BAC$ and $\mathrm{arc}_2 = ∡CAD$ for the two angles internal in the two planes with a radius reference $|AB| = 1$. We have the two normal 1-vector objects $\mathbf{n}_1$ and $\mathbf{n}_2$ support spanning the two normal lines through A to the planes γ_{ABC} and γ_{ACD}. The mutual angle $\mathrm{arc}_{\mathbf{n}_2\wedge\mathbf{n}_1} = ∡(\mathbf{n}_2, \mathbf{n}_1)$ define the angle between the planes. That angle exists in the plane of $\mathbf{n}_2\wedge\mathbf{n}_1$ perpendicular to the other planes. These unit normal 1-vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ represents the two axes of arcus circular rotations in their respective planes $\mathrm{arc}_1 \subset \gamma_{ABC}$ and $\mathrm{arc}_2 \subset \gamma_{ACD}$.

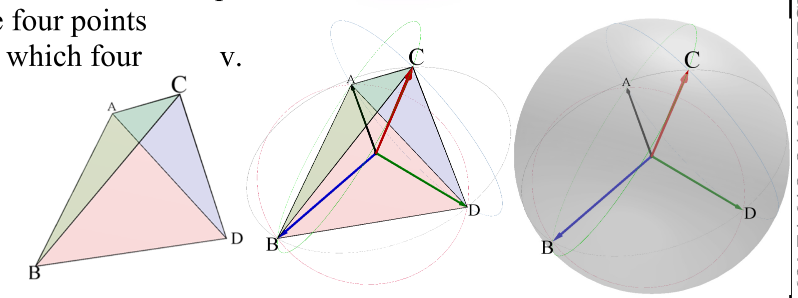


u. The orthonormal basis set and their transversal planes.

u. Two plane that intersects in a line has both their normal 1-vectors perpendicular to that line. This line is spanned from a third unit 1-vector $\mathbf{n}_3$ that is perpendicular $\mathbf{n}_3 \perp \mathbf{n}_1$ and $\mathbf{n}_3 \perp \mathbf{n}_2$. There is now a symmetry to choose $\mathbf{n}_2 \perp \mathbf{n}_1$. In geometric algebra we rename such an orthonormal basis $\sigma_1 = \mathbf{n}_1, \sigma_2 = \mathbf{n}_2$ and $\sigma_3 = \mathbf{n}_3$, when $\sigma_j \cdot \sigma_k = \delta_{jk}$, $\delta_{jk} = 1$ when $j = k$ else $\delta_{jk} = 0$ for $j, k = 1,2,3$. Their three supported lines ℓ_{σ_j} form $12 = 3 \cdot 4$ mutual equal right angles in a symmetry. Their three transversal normal planes also have mutual perpendicular angles.

v. Four points forming a *tetrahedron* fix a solid in space. The circumscribed sphere to the four points A,B,C,D forms a center O from which four 1-vectors called a *tetraon* point out four points on the sphere that define the stereo $\mathfrak{Z}$-space symmetry $S^2 \to S^3$ ***outwards***. The regular tetrahedron is a Platonic ideal.

v.



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