6. The Natural Space of Physics

6.1. The Classic Geometric Extension Space $\mathfrak{Z}$

We recall from section 4.2 the name $\mathfrak{G}$ for the idea of total natural space with geometry.
First, for the plane concept $\mathfrak{P}$, we have three points $A, B, C \in \mathfrak{G}$, where the third $C \notin \ell_{AB}$ is not on the line $A, B \in \ell_{AB}$. We form in our minds a triangle $\triangle ABC$ with the circumscribed circle $\odot ABC$ that defines the plane γ_{ABC}. We will from this plane define the natural space $\mathfrak{Z}_{ABCD}$: We presume four points $A, B, C, D \in \mathfrak{G}$, where we can pass judgments $A, B \in \ell_{AB}$ and $C \notin \ell_{AB}$ making a plane $A, B, C \in \gamma_{ABC}$ and $D \notin \gamma_{ABC}$. Point D external to this plane defining an extension space $\mathfrak{Z}_{ABCD}$, as a subject to the concept of the extension space substance $\mathfrak{Z}$.

(6.1) $\mathfrak{Z}_{ABCD} \subset \mathfrak{G}, \quad \mathfrak{Z}_{ABCD} \in \mathfrak{Z}$, where we apply the a priori synthetic judgments:

(6.1a) $A, B, C, D \in \mathfrak{G} \Rightarrow \ell_{AB}, \ell_{BC}, \ell_{CA}, \ell_{AD}, \ell_{BD}, \ell_{CD} \subset \mathfrak{Z}_{ABCD}; \quad AB, BC, CA, AD, BD, CD \subset \mathfrak{Z}_{ABCD};$

(6.1b) $\gamma_{ABC}, \gamma_{ABD}, \gamma_{BCD}, \gamma_{CAD} \subset \mathfrak{Z}_{ABCD}; \quad \triangle ABC, \triangle ABD, \triangle BCD, \triangle CAD \subset \mathfrak{Z}_{ABCD}$

(6.1c) $\text{Tetrahedron}(ABCD) \subset \mathfrak{Z}_{ABCD}; \quad \text{Circumscribe Sphere}(ABCD) \subset \mathfrak{Z}_{ABCD}$

The geometric extension of space per se is a platonic idea and therefore transcendental for the recognition, but for intuition, it is possible to recognize or construct extended structures in natural space $\mathfrak{G}$ of physics.
The simplest symmetry of four different points ABCD is represented by the Platonic solid regular tetrahedron from which the circumscribed sphere is inherited.

3 dimensions and the Concept of Geometric Extension (pqg-3)

Quote [12] : "Euclid's Elements:

E I.De.5. A *surface* is that which has length and breadth only.
E XI.De.1. A *solid* is that which has length, breadth, and depth.
E XI.De.2. A face of a solid is a surface.
E XI.De.3. A straight line is *at right angles* to a plane when it makes right angles with all the straight lines which meet it and are in the plane.
E XI.De.4. A plane is *at right angles* to a plane when the straight lines drawn in one of the planes at right angles to the intersection of the planes are at right angles to the remaining plane.
E XI.De.5. The *inclination* of a straight line to a plane is, assuming a perpendicular drawn from the end of the straight line which is elevated above the plane to the plane, and a straight line joined from the point thus arising to the end of the straight line which is in the plane, the angle contained by the straight line so drawn and the straight line standing up.
E XI.De.6. The *inclination* of a plane to a plane is the acute angle contained by the straight lines drawn at right angles to the intersection at the same point, one in each of the planes.
E XI.De.7. A plane is said to be *similarly inclined* to a plane as another is to another when the said angles of the inclinations equal one another.
E XI.De.8. *Parallel planes* are those which do not meet.
E XI.De.9. *Similar solid figures* are those contained by similar planes equal in multitude.
E XI.De.10. *Equal and similar solid figures* are those contained by similar planes equal in multitude and magnitude.
E XI.De.11. A *solid angle* is the inclination constituted by more than two lines which meet one another and are not in the same surface, towards all the lines, that is, a *solid angle* is that which is contained by more than two plane angles which are not in the same plane and are constructed to one point.
E XI.De.12. A *pyramid* is a solid figure contained by planes which is constructed from one plane to one point.

For quotation reference use: ISBN-13: 978-8797246931