5.8.2.4. The Lorentz 1-Spinor in the Minkowski $\mathcal{B}$-plane

To find a correspondence to the Space-Time-Algebra (STA) we have to introduce an information development unit 1-vector ***direction*** γ_0 and to map the paravector (5.405) to a Minkowski $\mathcal{B}$-plane we simply operator right multiply with γ_0, $\gamma_0^2 \coloneqq 1$ (we use (5.328) with $\mathbf{u} = \boldsymbol{\sigma}_k$)

(5.412) $$\wp = \wp\gamma_0\gamma_0 = (\alpha 1 + \zeta\mathbf{u})\gamma_0\gamma_0 = \alpha + \zeta\mathbf{u}\gamma_0\gamma_0 = \alpha + \zeta\gamma_{\mathrm{u}}\gamma_0 = \alpha + \zeta\mathcal{B}_{\mathrm{u}}.$$

To achieve the STA 1-vector we map this by right multiply operation once again with γ_0

(5.413) $$p_{\mathbf{u}} = \wp\gamma_0 = \alpha\gamma_0 + \zeta\mathcal{B}_{\mathrm{u}}\gamma_0 = \alpha\gamma_0 + \zeta\gamma_{\mathrm{u}}. \qquad \text{(using (5.309) or)}$$

Using this STA 1-vector $p_{\mathbf{u}} = \alpha\gamma_0 + \zeta\gamma_{\mathrm{u}}$ as an argument in an exponential function we write

(5.414) $$\exp(p_{\mathbf{u}}) = e^{\alpha\gamma_0 + \zeta\gamma_{\mathrm{u}}}.$$

The result of this will of course stay in the STA algebra due to the exponential power series (5.388). The two different ***directions*** γ_0 and γ_{u} anticommute $\gamma_0\gamma_{\mathrm{u}} = -\gamma_{\mathrm{u}}\gamma_0$ and are orthogonal $\gamma_0 \cdot \gamma_{\mathrm{u}} = 0$ and therefore independent.

General for $AB \neq BA$ we will be able to write $e^A e^B = e^C \nRightarrow C = A + B$,
but for $C \neq A + B$ we do not have a general solution for C from A and B [10]p.74.

The two exp forms $e^{\alpha\gamma_0}$ and $e^{\zeta\gamma_{\mathrm{u}}}$ have independent 1-vector ***direction*** arguments and their product factors are mostly not commuting

(5.415) $$e^{\alpha\gamma_0} e^{\zeta\gamma_{\mathrm{u}}} \overset{?}{\neq} e^{\zeta\gamma_{\mathrm{u}}} e^{\alpha\gamma_0}.$$

Anyway, by using the paravector concept $\wp = \alpha + \zeta\mathcal{B}_{\mathrm{u}}$ where the development measure parameter is a pure real scalar $\alpha \in \mathbb{R}$.

(5.416) $$\wp_\wedge = e^{\alpha + \zeta\mathcal{B}_{\mathrm{u}}} = e^{\alpha} e^{\zeta\mathcal{B}_{\mathrm{u}}} = e^{\zeta\mathcal{B}_{\mathrm{u}}} e^{\alpha} = e^{\alpha}(\cosh\zeta + \mathcal{B}_{\mathrm{u}}\sinh\zeta).$$

Mapped to the STA 1-vector space $\{\gamma_0, \gamma_{\mathrm{u}}\} \sim \{\gamma_0, \gamma_1, \gamma_2, \gamma_3\}$[286],

(5.417) $$p_{\mathbf{u},\wedge} = \wp_\wedge\gamma_0 = (e^{\alpha}\cosh\zeta + \mathcal{B}_{\mathrm{u}} e^{\alpha}\sinh\zeta)\gamma_0 = (e^{\alpha}\cosh\zeta)\gamma_0 + (e^{\alpha}\sinh\zeta)\gamma_{\mathrm{u}}$$

In (5.416) we associate the Cartesian length ***direction*** with the measure bivector $\mathcal{B}_{\mathrm{u}} \coloneqq \gamma_{\mathrm{u}}\gamma_0$, $\mathcal{B}_{\mathrm{u}}^2 \coloneqq 1$ that in the tradition is a pseudonym with the Euclidean 1-vector $\mathbf{u} \coloneqq \gamma_{\mathrm{u}}\gamma_0$, $\mathbf{u}^2 \coloneqq 1$, in (5.411). When we think of a physical length ***direction quantity*** unit $\mathbf{u}$, we cannot deny a measure unit γ_0.

5.8.3. A Philosophical Conclusion on All This Exercise

When you look at the mathematical constructions you may wonder, what will work when it is used in physics, and give consistency with lab measurements? One a priori rule, that had been mandatory for science since Baruch Spinoza is that Nature is the master in the name of Physic. Any Low of mathematics must obey the a priori structures of Physics in a universal context. When you (we) as humans construct a mathematic system, the Laws of this are dependent on Physics, in the last instance, our thoughts are physical processes. A simple example:
The capability to count numbers as one and one more… of equal identical but distinguishable ***entities*** is possible, not only by humans but all ***entities*** that possess information.
When you let the human science of mathematics be the master, you have lost to religion, and any knowledge about Nature is segregated beyond the transcendental barrier.
The human mathematical idea of continuity must be a map of moving two ***entities*** relative to each other extensively without abruption by annihilation and creation.
To accept the existence of scalars without ***directional*** extension to be a ***quantity*** of an ***entity***, do not need a religious super-instance to be the background for the whole universe.
Anyway, this philosophy is performed by humans and not by animals or any other physical ***entities*** or supercomputers with *artificial* intelligence.
Consciousness is not an *art*, but an a priori fundamental concept idea that emerged from Nature.

[286] Where the communication extension **direction** is $\gamma_{\mathrm{u}} = \upsilon_1\gamma_1 + \upsilon_1\gamma_2 + \upsilon_1\gamma_3$ for $\upsilon_k \in \mathbb{R}$, and $\upsilon_1^2 + \upsilon_2^2 + \upsilon_3^2 = 1 = |\gamma_{\mathrm{u}}|$.

5.9. Concluding Summary on the Algebra for the Geometric Plane Concept

5.9.1. The Euclidean Plane Concept

From the first start for a plane concept, we have Euclid's Elements chapter 5.1 where we emphasised the angular concept feature between 1-vectors. In chapter 5.2 from the linear additive algebra of geometric vector space, we form the bilinear Clifford algebraic quadratic form from which we develop a vector product space called *geometric algebra,* where the *geometric product* is classified in an *inner* 5.2.3 and an *outer* product 5.2.5, that in the plane concept is an *inner scalar* and an *outer bivector*. The geometric product of two 1-vectors results in a 1-rotor concept in plane 5.2.7, with a bivector argument exponential function 5.2.8, followed by the plane 1-spinor concept. In section 5.3 we synthesise some ***qualities*** of the plane concept in the view of geometric algebra with multivectors of ***grades*** 0,1, and 2, i.e., scalars, 1-vectors, and bivectors, together with 1-rotors as exponential functions. In section 5.4 we describe the transformations that geometric algebraic elements can do to the plane concept for physical entities as reflections 5.4.2 and rotations 5.4.5 with the introduction of the canonical form for sandwich operations. Section 5.5 inherit ***qualities*** of nilpotence 5.5.3 and idempotence 5.5.4, paravectors and mutual annihilating projection spectral basis. Followed 5.6 by the 2×2 real matrix concept representation of the Euclidean plane related to the geometric algebra $\mathcal{G}_2(\mathbb{R})$.

5.9.2. The Non-Euclidean Plane Concept

In section 5.7 definition of the Minkowski isometric measure balance in a $\mathcal{B}$-plane algebra $\mathcal{G}_{1,1}(\mathbb{R})$ with mixed signature $(+,-)$ resulting in *null basis* ***directions*** for the information propagation.
A possible length ***quantity*** for an extension ***direction*** shall inherit its measure from an isometric balance with a development measure. The Lorentz invariant rotations secure the conservations and the traditional relativistic Lorentz transformation makes an apparent deformation from boosts.

5.9.3. General Exponential Series

In section 5.8 we generalise the exponential power series to concern all types of geometric multivectors, where the product of two multivector valued exponential functions only commute if their two arguments commute. Simply if there are independent basic ***directions*** of the generating multivectors (1-vectors, bivectors, etc.) for the exponential arguments that support the acting rotations, the sequential order of the multiplication operations has a causal impact.

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