We now have two different epistemological pictures of the physical space $\mathfrak{G}$ where plane surfaces $\mathfrak{P}$ by analogy can be drawn as two *object* 1-vectors $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ ***directions***, from which we form the standard basis $\{1, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{i} \coloneqq \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\}$ for the Euclidean plane, that allows an Euler rotation as (5.195) or oscillation as (5.196). The 1-rotor of this Euler rotation is

(5.381) $$U_\phi = e^{i½\phi} = e^{\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1½\phi}.$$

This is equivalent to the same rotation in the extension plane of the Space-Time-Algebra (STA)

(5.382) $$U_\phi = e^{\gamma_1\gamma_2½\phi}.$$

We see that the Euler rotation is the founding essence for the plane concept $\mathfrak{P}$, which we know for a local classic two-dimensional Euclidean picture surface internal in the physical space $\mathfrak{G}$. Both models of the STA basis and the traditional Euclid Cartesian basis are compared in Table 5.3 below.

When we include the measure of information development to our picture of the physical space $\mathfrak{G}$, where we are familiar with the Euclidean plane concept $\mathfrak{P}$ containing the Euler rotation idea, we endow our model with an enrichment of the concept of the Minkowski $\mathcal{B}_k$-bivectors supporting an information plane like the concept of balance (5.302):

(5.383) $\gamma_0^2 + \gamma_k^2 = 0$. When the measure γ_0 is the *action,* the isometric extension γ_k is the *reaction*!

This plane-like isometric balance is constructed orthogonal $\gamma_0 \cdot \gamma_k = 0$ as a Minkowski $\mathcal{B}_k$-plane supported by the $\mathcal{B}$-bivector $\mathcal{B}_k \coloneqq \gamma_k\gamma_0$. The choice $\gamma_0^2 \coloneqq 1$ demand $\gamma_k^2 = -1$ for the balance. This balance also gives the *null* ***directions*** by their sum $(\gamma_0 + \gamma_1)$, or difference $(\gamma_0 - \gamma_1)$, (5.310). This construction makes the Lorentz rotation from section 5.7.4 possible inside such a $\mathcal{B}_k$-plane by the Lorentz rotor (5.349), together with its full regular rotation by the canonical form (5.358),

(5.384) $$U_{\zeta,k} = e^{½\zeta\mathcal{B}_k} = \exp(½\zeta\mathcal{B}_k), \quad \text{where } \mathcal{B}_k^2{=}1 \quad \Rightarrow \quad \gamma_k' = e^{½\zeta\mathcal{B}_k}\gamma_k e^{-½\zeta\mathcal{B}_k} = e^{\zeta\mathcal{B}_k}\gamma_k.$$

This rotation is a parameter augmented by the rapidity $\zeta \in \mathbb{R}$, that is approaching infinity $\zeta \to \infty$ when the relativistic speed $\beta = \tanh\zeta \to 1$ is approaching the speed of information $c{=}1$, which is the case when we look at things by light, which it transmits towards us.
The foundation of the two structures is shown in Table 5.3 by the mapping of these two (5.380).

Table 5.3 Comparison of the two standard basis pictures of the plane concept: $\{\gamma_0, \gamma_1, \gamma_2, \gamma_1\gamma_2\} \leftrightarrow \{1, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{i} \coloneqq \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\}$

Lorentz - Minkowski - Hestenes, STA [6]				**Euclid - Descartes - Euler → Pauli**			
STA planes	$\mathcal{G}_{1,2}(\mathbb{R})$	Signature	**STA,** k=1,2	**Euclidean**	Signature	$\mathcal{G}_2(\mathbb{R})$	**Plane**
Development	1-vector	$\gamma_0^2 = +1$	$\gamma_0 \leftleftarrows \boldsymbol{\sigma}_k\gamma_k$	unknown reason	idea as cause	static existence	eternally immutable
Extension	1-vectors	$\gamma_k^2 = -1$	$\gamma_k \leftleftarrows \boldsymbol{\sigma}_k\gamma_0$				
Minkowski	$\mathcal{B}$-bivector	$\mathcal{B}_k^2 = +1$	$\mathcal{B}_k \coloneqq \gamma_k\gamma_0$	$\boldsymbol{\sigma}_k \leftleftarrows \gamma_k\gamma_0$	$\boldsymbol{\sigma}_k^2 = +1$	1-vectors	extension
Extension	bivector	$(\gamma_1\gamma_2)^2 = -1$	$\gamma_1\gamma_2 = \boldsymbol{i}$	$\boldsymbol{i} \coloneqq \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$	$\boldsymbol{i}^2 = -1$	bivector	extension
Oscillation	**rotor** $(,-,-,)$	$U_\phi U_\phi^\dagger{=}1$	$U_\phi{=}e^{\gamma_1\gamma_2½\phi}$	$U_\phi = e^{i½\phi}$	$U_\phi U_\phi^\dagger{=}1$	$(+,+)$ **rotor**	**oscillator**
Lorentz	**rotor** $(+,-)$	$U_{\zeta,k}\widetilde{U}_{\zeta,k}{=}1$	$U_{\zeta,k}{=}e^{½\zeta\mathcal{B}_k}$	Anny chance is transcendental to Cartesian space.			
The geometric Clifford algebra $\mathcal{G}_{1,2}(\mathbb{R})$ with a mixed signature basis $(+,-)$ or $(+,-,-,...)$ makes it possible to establish the fundamental idea of an isometric measurement concept for physics. We use the extended circular Euler rotating oscillator $(,-,-,)$ to produce the development ***direction*** γ_0 as an extra dimension of ***quantum*** timing counts. A Lorentz rotation $(+,-)$ express relative external speed.				We use the local Cartesian orthonormal basis, that by analogy, objects can be drawn on a surface and by that illustrate the plane concept for intuition. From this, we construct the geometric Clifford algebra $\mathcal{G}_2(\mathbb{R})$ with the simple quadratic form as signature $(+,+)$, and the bivector $\boldsymbol{i}^2 = -1$ from which we form the Euler rotor circle oscillator plane.			

The *null* ***direction*** in STA space can be interpreted as the ***direction*** where we see the Euclidean object of the Cartesian basis $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$ drawn on our practical analogy surface $\mathfrak{P}$ in our physical space $\mathfrak{G}$. Compare this object with Figure 5.14 as the intuition of the basis $\{1, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{i} \coloneqq \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\}$ for the plane concept, where we have the defining unit scalar measure $|\boldsymbol{\sigma}_1| = |\boldsymbol{\sigma}_2| = |\boldsymbol{i}| = 1 = \gamma_0^2$. The founding chronometric unit γ_0 is given as ***one quantum*** *radian* circumference phase angular measure $1 = \gamma_0^2$ of the reference oscillating Euler circular rotation that performs a Descartes extended plane for an ***entity*** in the physical space of one universal Nature.

5.7.5.3. Exponential Function in the Plane Concepts

We have now introduced the two forms of exponential functions with the bivector argument

(5.385) $$\exp(\boldsymbol{i}\varphi) = e^{\boldsymbol{i}\varphi} \in \mathcal{G}_2(\mathbb{R}), \quad \text{where} \quad \boldsymbol{i}^2 = -1,$$

for the Euler form for the rotation in a Euclidean plane supported from $\boldsymbol{i} \coloneqq \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 \in \mathcal{G}_2(\mathbb{R})$. And

(5.386) $$\exp(\mathcal{B}\zeta) = e^{\mathcal{B}\zeta} \in \mathcal{G}_{1,1}(\mathbb{R}), \quad \text{where} \quad \mathcal{B}^2 = 1$$

for the Lorentz rotation in a Minkowski $\mathcal{B}$-plane supported from $\mathcal{B}_k \coloneqq \gamma_k\gamma_0 \in \mathcal{G}_{1,1}(\mathbb{R})$.
These functions are both multivector valued and given by multivector augments.
Before we proceed with geometric algebra for higher ***grades*** of dimensions, e.g., for the natural 3-space, we will introduce a generalised concept of the exponential function in geometric algebra.

We benefit from a general knowledge of the exponential power expansions for multivectors.

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