invariant by the change of radial speed of that galaxy. This point of view is primitive and may seems naïve, but it is a priory fundamental to a model of the extension of our spatial universe. Remember that the invariant extension ***direction*** $\mathbf{u}$ is a Cartesian and Euclidean ***pqg***-1-vector *object* that has only to do with the orthonormal *subject* 1-vectors ***directions*** units γ_0 and γ_u through their outer product bivector $\mathcal{B}_\mathrm{u} \coloneqq \gamma_\mathrm{u}\gamma_0 = \gamma_\mathrm{u}\wedge\gamma_0$, where the isomorphic map $\mathbf{u} \leftrightarrow \mathcal{B}_\mathrm{u}$ represent the same ***direction*** in our spatial universe. This invariant ***direction*** is equivalent to the invariant *null line* ***direction*** $\{\mathcal{n},\overline{\mathcal{n}}\}$ by the outer product $\mathcal{n}\wedge\overline{\mathcal{n}} = \mathcal{B}_\mathrm{u}$.
When you try to compare the magnitudes of two colinear Euclidean ***pqg***-1-vectors $\mathbf{x}$ and $\mathbf{u}$ you are forced to construct a measure by a development unit γ_0 and then you inherit a ***primary quality of second grade*** (***pqg***-2) for the isometry $\mathcal{B}_\mathrm{u}$-bivector for the measurement process.

5.7.4.3. The Lorentz boost

In the tradition of Lorentz transformations, we are looking at two equal but distinguishable ***entities*** Ψ_R and Ψ_S with relative velocities to each other. We presume the velocity of interest is in the ***direction*** from R to S. We name the speed $\beta \in \mathbb{R}$, where $\beta > 0$, velocity $\pm$magnitude orientations away from each other. We presume further that defining oscillators of each *entity* is equal and that therefore their local development unit γ_0 is comparable. We take the autonomous viewpoint of ***entity*** Ψ_R. The information received by Ψ_R about source Ψ_S stays in the *null line* ***directions***.
Orthonormal to the development unit γ_0 we have the extension unit γ_u. (not perpendicular)
We interpret their ***direction*** $\gamma_\mathrm{u}\gamma_0 = \mathcal{B}_\mathrm{u} \leftrightarrow \mathbf{u}$ as isomorph to the ***direction*** between R and S. This is the same as the ***direction*** of the *null lines* $\mathcal{n}\wedge\overline{\mathcal{n}} = \mathcal{B}_\mathrm{u}$ in the ***direction*** of the STA Minkowski $\mathcal{B}_\mathrm{u}$-bivector-plane.[281] In the traditional classical interpretation, of the ***direction*** is a Cartesian Euclidean 1-vector $\mathbf{u}$ starting in point R of ***entity*** Ψ_R pointing towards Ψ_S. The idea is, that $\mathbf{u}$ represents the autonomous *object* ***direction*** of ***entity*** Ψ_R receiving a signal from source Ψ_S.
For Ψ_S to send a signal towards Ψ_R it must use the ***direction*** of $\mathbf{u}$ with negative orientation.
We presume we can choose some perpendicular transverse 1-vector $\mathbf{u}_\perp \perp \mathbf{u}$ in Ψ_RS, $\mathbf{u}_\perp^2 \coloneqq 1$.
Then we can choose to imagine the display of the *null-basis* 1-vectors ***directions*** as $\mathcal{n}\|\mathbf{u}$ and $\overline{\mathcal{n}}\|\mathbf{u}_\perp$.
The information of the signal has ***direction*** represented by the *null lines* of $\mathcal{n}\wedge\overline{\mathcal{n}} = \mathcal{B}_\mathrm{u}$, i.e., the $\mathcal{B}_\mathrm{u}$-bivector. We conclude the demand $\mathbf{u}\,\|\mathcal{B}_\mathrm{u}$, $\mathbf{u}_\perp\|\mathcal{B}_\mathrm{u}$, $\mathbf{u}\wedge\mathbf{u}_\perp = \mathcal{B}_\mathrm{u}$, $\mathbf{u}\cdot\mathbf{u}_\perp = 0$, and $\mathbf{u}^2 = \mathbf{u}_\perp^2 = 1$.[282]
When it comes to the transmission of information (classically called 'forces'[283]) we gain knowledge from using STA 1-vectors with Minkowski metric that's generated by the $\mathcal{B}$-plane supported by the orthometric $\mathcal{B}$-bivector unit, e.g. $\mathcal{B}_\mathrm{u} \coloneqq \gamma_\mathrm{u}\gamma_0$.
We presume that the simplest information can be represented by the STA 1-vector like (5.353)

(5.368) $$p_\mathrm{u} = \lambda_0\gamma_0 + \lambda_\mathrm{u}\gamma_\mathrm{u}$$

Now we are ready to look at the communication that due to the relativistic speed β has to include the Lorentz boost transformation as the rotation in the $\mathcal{B}_\mathrm{u}$-plane (5.358)

(5.369) $$p'_\mathrm{u} = e^{\zeta\mathcal{B}_\mathrm{u}}p_\mathrm{u} = (\cosh\zeta + \mathcal{B}_\mathrm{u}\sinh\zeta)p_\mathrm{u}.$$

We presume the speed of information is set as (5.330) $c = |\gamma_\mathrm{u}|/|\gamma_0| = 1$ and the relative speed is

(5.370) $$\beta = \tanh\zeta = \sinh\zeta/\cosh\zeta \quad \in\mathbb{R}.$$

From the literature we have for $\cosh\zeta$ the Lorentz factor

(5.371) $$\gamma = \cosh\zeta = (1-\beta^2)^{-1/2} \quad \in\mathbb{R}.$$

$\beta = \pm\frac{|v|}{c}$, the traditional relative speed.
$\gamma = \frac{1}{\sqrt{1-\beta^2}}$, the Lorentz factor. $\in\mathbb{R}$.

Then we write the Lorentz rotation (5.358), (5.369) as

[281] The two *null line* ***directions*** $\{\mathcal{n},\overline{\mathcal{n}}\}$ is invariant and parallel to the $\mathcal{B}_\mathrm{u}$-plane in the $\mathcal{B}_\mathrm{u}$-bivector ***direction***. When they display for us, they appear advantageously perpendicular $\overline{\mathcal{n}}\perp\mathcal{n}$, but they are neither orthogonal nor normal units (5.311), note .
[282] The object idea $\mathbf{u}_\perp$ from an origo R, perpendicular to RS direction $\mathbf{u}$, is in principle superfluous here, but it helps us to intuit that there is something perpendicular transverse to the transmission ***direction*** of information along the *null* ***direction*** $\mathcal{n}$, i.e. $\overline{\mathcal{n}}$.
Later we will consider that the extension unit $\mathbf{u}_\perp$ represents an oscillating rotation in a transversal plane to $\mathbf{u}$. (background in I. 3.4).
[283] E.g., gravitation, strong, weak, and electromagnetic forces, and of course memory knowledge as modulation (e.g., OFDM).

(5.372) $$p'_\mathrm{u} = \gamma(1+\beta\mathcal{B}_\mathrm{u})p_\mathrm{u}.$$

The relativistic radial speed factor β can take the ***quantitative*** real values $-1 \le \beta \le 1$.
The radial velocity of Ψ_S is then $\mathbf{v}_\mathrm{S} = \beta\mathbf{u}$ seen from Ψ_R, where the information from Ψ_S is Lorentz transformed by (5.369), (5.372). Consulting (5.359)-(5.363) we have for the separated ***directions***:
First for the *development* like Lorentz rotation

(5.373) $$\gamma'_0 = e^{\frac{1}{2}\zeta\mathcal{B}_\mathrm{u}}\gamma_0 = \gamma_0\cosh\zeta + \gamma_\mathrm{u}\sinh\zeta = \gamma(1+\beta\mathcal{B}_\mathrm{u})\gamma_0, \qquad |\gamma'_0| = |\gamma_0| = 1$$

Secondly for the *extension* like Lorentz rotation

(5.374) $$\gamma'_\mathrm{u} = e^{\frac{1}{2}\zeta\mathcal{B}_\mathrm{u}}\gamma_\mathrm{u} = \gamma_\mathrm{u}\cosh\zeta + \gamma_0\sinh\zeta = \gamma(1+\beta\mathcal{B}_\mathrm{u})\gamma_\mathrm{u}, \qquad |\gamma'_\mathrm{u}| = |\gamma_\mathrm{u}| = 1$$

The magnitudes of the rotated ***direction*** units are invariant preserved just as the dilation coordinates $(\lambda_0,\lambda_\mathrm{u})$ for the information content for both the transmitter Ψ_S and the receiver Ψ_R represented by the STA 1-vector $p_\mathrm{u} = \lambda_0\gamma_0 + \lambda_\mathrm{u}\gamma_\mathrm{u}$ autonomous for both the ***entities*** Ψ_S and Ψ_R.
The transformation seen by R is

(5.375) $$p'_\mathrm{u} = \gamma(1+\beta\mathcal{B}_\mathrm{u})p_\mathrm{u} = \lambda_0\gamma'_0 + \lambda_\mathrm{u}\gamma'_\mathrm{u}$$

The magnitude of the STA 1-vector as well as its *development* and *extension* coordinates are preserved $|p'_\mathrm{u}| = |p_\mathrm{u}| = \lambda_0^2 - \lambda_\mathrm{u}^2$. – The ***quantities*** of the information are invariant preserved!
It is the STA frame orthonormal basis $\{\gamma_0,\gamma_\mathrm{u}\} \to \{\gamma'_0,\gamma'_\mathrm{u}\}$ that is distorted in the mixed basis $\{1,\gamma_0,\gamma_\mathrm{u},\mathcal{B}_\mathrm{u} \coloneqq \gamma_\mathrm{u}\gamma_0\}$, where part $\{1,\mathcal{B}_\mathrm{u}\}$, $\mathcal{B}_\mathrm{u}^2 = (\gamma_\mathrm{u}\gamma_0)^2 = 1$ is invariant. We remember that the subjects γ'_0 and γ'_u indeed do not fulfil the idea of perpendicular squareness but are anyway orthogonal. What we find is the frame distortion of the STA frame $\{\gamma'_0,\gamma'_\mathrm{u}\}$ in the $\mathcal{B}_\mathrm{u}$-plane by the Lorentz rotation boost (5.368)-(5.375) is displayed in Figure 5.51.[284]
Dialectic opposite, the *null basis* $\{\mathcal{n},\overline{\mathcal{n}}\}$ has the ***quality*** in its perceivable objective display the opportunity to represent the physical perpendicular ***directions*** of information, even though not orthonormal (5.310) $\overline{\mathcal{n}}\cdot\mathcal{n} = \mathcal{n}\cdot\overline{\mathcal{n}} = 1$, and $\mathcal{n}^2 = \overline{\mathcal{n}}^2 = 0$.[285]
All four components of the full mixed *null-basis* $\{1,\mathcal{n},\overline{\mathcal{n}},\mathcal{B}_\mathrm{u} = \mathcal{n}\wedge\overline{\mathcal{n}}\}$ is invariant in the $\mathcal{B}_\mathrm{u}$-plane.

The traditional apparently contraction is an illusion $(\gamma\lambda_0)|\gamma_0| = \lambda_0(\gamma|\gamma'_0|)$ and $(\gamma\lambda_\mathrm{u})|\gamma_\mathrm{u}| = \lambda_\mathrm{u}(\gamma|\gamma'_\mathrm{u}|)$.
All this invariance in the structure of the transformed ***entities*** and in all information that is exchanged, leads to the question, what is it that is changed in a Lorentz boost rotation?
The answer is the foundation of the measurement reference is inevitably justified locally. (everywhere)

5.7.4.4. The Doppler Effect of the Lorentz Boost

We have through this book tried to justify that an a priory measure is founded on counting ***quanta*** of radian development in an circular oscillating ***entity***. The task is to define a reference ***entity quality*** that oscillates as a reference *chronometer* clock, which defines frequency energies ω of other identical ***entities*** in consideration. A local R known ***entity*** Ψ_R has a known ***quality*** oscillating frequency energy $\omega_0 = \omega_{0,\mathrm{R}}$ ***quantity*** measured stationary in the local observing laboratory.
A far away boosted ***entity*** Ψ_S identical to Ψ_R has in its local autonomy the same oscillating frequency energy $\omega_{0,\mathrm{S}} = \omega_0$ as Ψ_R. What measured difference in received frequency will we perceive at R from the distant boosted ***entity*** Ψ_S? To answer this, we use the relativistic Doppler formula for the longitudinal speed $\beta_\| \in\mathbb{R}$ in the ***direction*** $\mathbf{u}$ of the boost in the $\mathcal{B}_\mathrm{u}$-bivector-plane

(5.376) $$\omega_{\text{received from S}} = \gamma\left(1-\beta_\|\right)\omega_0 = \frac{(1-\beta_\|)\omega_0}{\sqrt{1-\beta^2}} \xrightarrow{\beta_\|=\beta} \frac{(1-\beta)\omega_0}{\sqrt{(1-\beta)(1+\beta)}} = \omega_0\sqrt{\frac{1-\beta}{1+\beta}}.$$

For $0 < \beta \le 1$ we have the redshift, which we interpret as galaxies boosting further away. –

[284] Sorry to tell you that I have tried to find an analytic expression between the arguments λ and ζ in my formula books and Wikipedia etc. without success, some reader may find this. – I decide that I do not have the lifetime capacity for this.
[285] We see the double *null helixes* structure ***direction*** in Figure 5.55 perform physical perpendicular in its opposition in that the unit circular speed measure propagation speed from the objective extension ***direction*** $\mathbf{u} = \sigma_3$.

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