5.7.4. Lorentz Rotation in the Minkowski $\mathcal{B}$-plane

In Figure 5.53 we display the hyperbola curves for invariant transformations by the scaling parameter λ of the nilpotent *null* ***directions*** (5.318) in a geometric Clifford algebra $\mathcal{G}_{1,1}(\mathbb{R})$. Using the idea from Kepler's second law, that the angular area of the curve is essential as a parameter for a transformation. The area unit element in Minkowski $\mathcal{B}$-plane is just the $\mathcal{B}$-bivector $\mathcal{B} := \gamma_1\gamma_0$, where $\mathcal{B}^2 := 1$. From this, we choose a real parameter $\zeta \in \mathbb{R}$ often called *rapidity* for the area argument $\frac{1}{2}\zeta\mathcal{B}$ in a hyperbolic multivector exponential function we call a Lorentz 1-rotor[280]

(5.349) $$U_\zeta = e^{\frac{1}{2}\zeta\mathcal{B}} = \exp(\tfrac{1}{2}\zeta\mathcal{B}) = \cosh(\tfrac{1}{2}\zeta\mathcal{B}) + \sinh(\tfrac{1}{2}\zeta\mathcal{B}).$$

The essential argument in this 1-rotor has a positive Clifford signature $(+)$: $\mathcal{B}^2 = +|\mathcal{B}| := 1$

(5.350) $$(\tfrac{1}{2}\zeta\mathcal{B})^2 = \tfrac{1}{4}\zeta^2 \geq 0$$

The multivector value of the exponential function (5.349) exists in the mixed algebra $\mathcal{G}_{1,1}(\mathbb{R})$ of the Minkowski $\mathcal{B}$-plane. The algebra for these functions is analysed in the next section 5.8, but here we will use that cosh is even in its power series of $\mathcal{B}^2=1$, causing cosh to be an even scalar, and that sinh is odd so that factor $\mathcal{B}$ can be separated from the scalar function, then (5.349) is

(5.351) $$U_\zeta = e^{\frac{1}{2}\zeta\mathcal{B}} = \cosh(\tfrac{1}{2}\zeta\mathcal{B}) + \sinh(\tfrac{1}{2}\zeta\mathcal{B}) = \cosh(\tfrac{1}{2}\zeta) + \mathcal{B}\sinh(\tfrac{1}{2}\zeta), \qquad \zeta \in \mathbb{R}.$$

To investigate the possible impact of such a Lorentz rotor we will intuit a Euclidean space $\mathcal{G}_2(\mathbb{R})$ or $\mathcal{G}_3(\mathbb{R})$ 1-vector unit ***direction*** *object* $\mathbf{u} = u_k\boldsymbol{\sigma}_k$ that has a measure $\gamma_0^2=1$ (5.348) mapping it to a *subject* 1-vector in a full Minkowski space $\mathcal{G}_{1,2}(\mathbb{R})$ or $\mathcal{G}_{1,3}(\mathbb{R})$ substance. The mapping is performed by right multiplying by the development measure ***direction unit quantum*** γ_0: $\gamma_\mathrm{u} := \mathbf{u}\gamma_0$, producing a unit Minkowski $\mathcal{B}$-plane *subject* $\mathcal{B}_\mathrm{u} := \gamma_\mathrm{u}\gamma_0$.
From the Euclidean ***direction*** *object* $\mathbf{u}$, we can dilate to an arbitrary 1-vector $\mathbf{p}_\mathrm{u} = \lambda_\mathrm{u}\mathbf{u}$, $\lambda_\mathrm{u} \in \mathbb{R}$ with magnitude $|\mathbf{p}_\mathrm{u}| = |\lambda_\mathrm{u}|$, that in the basis $\{\boldsymbol{\sigma}_k\}$ is $\mathbf{p}_\mathrm{u} = \sum_{k=1}^{2 \text{ or } 3}\lambda_k\boldsymbol{\sigma}_k$, where $\lambda_k = \lambda_\mathrm{u}\mathbf{u}\cdot\boldsymbol{\sigma}_k$.
From this, we construct a paravector like (5.342) with an arbitrary development parameter λ_0

(5.352) $$\wp_\mathrm{u} = \lambda_0 + \mathbf{p}_\mathrm{u} = \lambda_0 1 + \lambda_\mathrm{u}\mathbf{u}.$$

Letting this paravector left operate on the development ***direction unit quantum*** γ_0 we achieve by the invention of a generalised Minkowski 1-vector in a pure $\langle A\rangle_1$ ***grade*** form in STA as (5.345)

(5.353) $$p_\mathrm{u} = \wp\gamma_0 = \lambda_0\gamma_0 + \mathbf{p}_\mathrm{u}\gamma_0 = \lambda_0\gamma_0 + \lambda_\mathrm{u}\gamma_\mathrm{u} = \lambda_0\gamma_0 + \lambda_k\gamma_k = \lambda_\mu\gamma_\mu, \qquad \lambda_\mu \in \mathbb{R}, \qquad (\mu = 0, k).$$

The extension Clifford conjugated of this is

(5.354) $$\overline{p}_\mathrm{u} = \overline{\wp}\gamma_0 = \lambda_0\gamma_0 - \mathbf{p}_\mathrm{u}\gamma_0 = \lambda_0\gamma_0 - \lambda_\mathrm{u}\gamma_\mathrm{u} = \lambda_0\gamma_0 - \lambda_k\gamma_k = \lambda_\mu g_{\mu\mu}\gamma_\mu, \qquad g_{\mu\mu} = \begin{pmatrix}1&&&\\&-1&&\\&&-1&\\&&&-1\end{pmatrix}.$$

Now we have a *subject* p in the Minkowski space to act on by the $\mathcal{B}_\mathrm{u}$-plane Lorentz rotor

(5.355) $$U_{\zeta,\mathrm{u}} = e^{+\frac{1}{2}\zeta\mathcal{B}_\mathrm{u}} = \cosh(\tfrac{1}{2}\zeta) + \mathcal{B}_\mathrm{u}\sinh(\tfrac{1}{2}\zeta).$$

The conjugated of this is achieved by the reversed $\mathcal{B}$-bivector (5.306) $\overline{\mathcal{B}} = \widetilde{\mathcal{B}} = \mathcal{B}^\dagger = -\mathcal{B}$

(5.356) $$\widetilde{U}_{\zeta,\mathrm{u}} = e^{-\frac{1}{2}\zeta\mathcal{B}_\mathrm{u}} = \cosh(\tfrac{1}{2}\zeta) - \mathcal{B}_\mathrm{u}\sinh(\tfrac{1}{2}\zeta) \quad = U_{\zeta,\mathrm{u}}^\dagger.$$

The $\mathcal{B}_\mathrm{u}$-plane Lorentz rotor is unitary

(5.357) $$U_{\zeta,\mathrm{u}}\widetilde{U}_{\zeta,\mathrm{u}} = \widetilde{U}_{\zeta,\mathrm{u}}U_{\zeta,\mathrm{u}} = e^0 = \cosh^2(\tfrac{1}{2}\zeta) - \sinh^2(\tfrac{1}{2}\zeta) = 1.$$

We form the Lorentz rotation by the canonical form

(5.358) $$p'_\mathrm{u} = \underline{U}_\zeta p_\mathrm{u} = U_{\zeta,\mathrm{u}}p_\mathrm{u}\widetilde{U}_{\zeta,\mathrm{u}} = e^{\frac{1}{2}\zeta\mathcal{B}_\mathrm{u}}p_\mathrm{u}e^{-\frac{1}{2}\zeta\mathcal{B}_\mathrm{u}} = e^{\zeta\mathcal{B}_\mathrm{u}}p_\mathrm{u} = U_{\zeta,\mathrm{u}}^2 p_\mathrm{u} = (\cosh\zeta + \mathcal{B}_\mathrm{u}\sinh\zeta)p_\mathrm{u}.$$

We split this into two parts, first the development of hyperbolic rotation

(5.359) $$\lambda_0\gamma'_0 = U_{\zeta,\mathrm{u}}^2\lambda_0\gamma_0 = e^{\frac{1}{2}\zeta\mathcal{B}_\mathrm{u}}\lambda_0\gamma_0 = \lambda_0(\gamma_0\cosh\zeta + \mathcal{B}_\mathrm{u}\gamma_0\sinh\zeta) = \lambda_0(\gamma_0\cosh\zeta + \gamma_\mathrm{u}\sinh\zeta),$$

[280] This area argument differs from the cyclic angular form of (5.192) $e^{i\theta}$ where the argument has a negative signature $(-)$, $i^2 = -1$.

In Figure 5.53 green for $\lambda_0 \geq 1$. Then second the extension like hyperbolic rotation

(5.360) $$\lambda_\mathrm{u}\gamma_0 = U_{\zeta,\mathrm{u}}^2\lambda_\mathrm{u}\gamma_\mathrm{u} = e^{\frac{1}{2}\zeta\mathcal{B}_\mathrm{u}}\lambda_\mathrm{u}\gamma_\mathrm{u} = \lambda_\mathrm{u}(\gamma_\mathrm{u}\cosh\zeta + \mathcal{B}_\mathrm{u}\gamma_\mathrm{u}\sinh\zeta) = \lambda_\mathrm{u}(\gamma_\mathrm{u}\cosh\zeta + \gamma_0\sinh\zeta),$$

yellow for $\lambda_\mathrm{u} \geq 1$ in Figure 5.53. (The extended area where the autonomous information is void)
The white area $\lambda_0 \leq 1$ and $\lambda_\mathrm{u} \leq 1$ in Figure 5.53 is inside the limit of ***one quantum*** count in the cyclic oscillation that gives the development unit γ_0 of the ***entity*** that possesses the isometry

(5.361) $$|\gamma_0| = |\gamma_\mathrm{u}| = 1 \quad = |\gamma_1| = |\gamma_2| = |\gamma_3| = 1 \quad \Leftrightarrow \quad |\gamma_\mu| = 1.$$

The information of any Minkowski STA 1-vector p_u to and from the ***entity*** follows the Lorentz rotation invariant *null line* ***directions*** $\{n, \overline{n}\}$ in the $\mathcal{B}_\mathrm{u}$-plane. The ***directions*** of γ_0 and γ_u are altered by the Lorentz rotation, but their orthonormality (5.303) is invariant preserved just as (5.321)

(5.362) $$\gamma'_0\cdot\gamma'_\mathrm{u} = 0 \quad \text{and} \quad |\gamma'_0| = |\gamma'_\mathrm{u}| = 1.$$

As orthonormal *subjects*, these ***directions*** *are never perpendicular objects for us*! (Kein Ding für uns)
The drawn 1-vector ***direction*** of the development γ_0 in Figure 5.50, Figure 5.51 and Figure 5.53 as an intuition perpendicular *object* to the extension ***direction*** γ_1 is an illusion! They just display in a diagram an instrumentalization of a development measure substance as a *subject* of ***one quantum*** count γ_0. The magnitude as (5.346) of any STA 1-vector is invariant preserved by the unitary Lorentz rotation given by the canonical form (5.358), in the $\mathcal{B}_\mathrm{u}$-plane simply $p'_\mathrm{u} = e^{\zeta\mathcal{B}_\mathrm{u}}p_\mathrm{u}$

(5.363) $$|p'_\mathrm{u}| = |e^{\zeta\mathcal{B}_\mathrm{u}}p_\mathrm{u}| = |e^{\zeta\mathcal{B}_\mathrm{u}}||p_\mathrm{u}| = |p_\mathrm{u}|, \qquad \text{because of } |e^{\zeta\mathcal{B}_\mathrm{u}}|^2 = e^{\zeta\mathcal{B}_\mathrm{u}}e^{-\zeta\mathcal{B}_\mathrm{u}} = e^0 = 1.$$

This agrees with the component coordinates $(\lambda_0, \lambda_\mathrm{u})$ from (5.359) and (5.360) that is preserved.
It is the orthonormal basis 1-vectors that change the ***directions*** $\{\gamma_0, \gamma_\mathrm{u}\} \leftrightarrow \{\gamma'_0, \gamma'_\mathrm{u}\}$ inside the $\mathcal{B}_\mathrm{u}$-plane $\mathcal{B}'_\mathrm{u} = \mathcal{B}_\mathrm{u} = \gamma_0\gamma_\mathrm{u} = \gamma'_0\gamma'_\mathrm{u}$.
What implication does this have on the physical world? We view intuit at two identical ***entities*** that are distinguishable Ψ and Ψ', each has their individual frames $\Psi\sim\{\gamma_0, \gamma_\mathrm{u}\}$ and $\Psi'\sim\{\gamma'_0, \gamma'_\mathrm{u}\}$.
These identical ***entities*** have the same ***primary qualities*** possessing equal ***quantities***: λ_0 and λ_u.
It is the orthonormal frame ***directions*** inside the Minkowski $\mathcal{B}_\mathrm{u}$-plane that the Lorentz rotation change in the information transmission boost. An example is two Hydrogen spectrums ${}^1_1$H and ${}^1_1$H' that for each local autonomy is the same but look different due to the boost of frame reference ${}^1_1$H'.

5.7.4.2. The Lorentz Transformation of a paravector

The Lorentz rotor (5.355) is written with implicit knowledge of the development ***unit quantum*** γ_0, when multiplying with $\gamma_0\gamma_0 = \gamma_0^2 = 1$ we can by (5.348) write (for details see below section 5.8)

(5.364) $$U_{\zeta,\mathrm{u}} = \cosh(\tfrac{1}{2}\zeta) + \mathbf{u}\sinh(\tfrac{1}{2}\zeta) = \cosh(\tfrac{1}{2}\zeta\mathbf{u}) + \sinh(\tfrac{1}{2}\zeta\mathbf{u}) = e^{\frac{1}{2}\zeta\mathbf{u}} = \exp(\tfrac{1}{2}\zeta\mathbf{u})$$

Going back to the paravector form (5.352) $\wp_\mathrm{u} = \lambda_0 + \mathbf{p}_\mathrm{u} = \lambda_0 1 + \lambda_\mathrm{u}\mathbf{u} \quad = p_\mathrm{u}\gamma_0$.
We find that the Lorentz rotation by the canonical form (5.358) can be written

(5.365) $$\wp'_\mathrm{u} = e^{\frac{1}{2}\zeta\mathbf{u}}\wp_\mathrm{u}e^{-\frac{1}{2}\zeta\mathbf{u}} = e^{\zeta\mathbf{u}}\wp_\mathrm{u} = (\cosh\zeta + \mathbf{u}\sinh\zeta)\wp_\mathrm{u} = (\cosh\zeta + \mathbf{u}\sinh\zeta)(\lambda_0 + \lambda_\mathrm{u}\mathbf{u}).$$

Here we lose knowledge, the rotation plane is undefined; we only have one 1-vector ***direction*** $\mathbf{u}$.
We need a $\mathcal{B}$-bivector to define a rotation plane ***direction***. This problem is obvious when we split it into two parts, first, the hyperbolic rotation of the development scalar is not a scalar $e^{\frac{1}{2}\zeta\mathbf{u}}\lambda_0 = \lambda_0(\cosh\zeta + \mathbf{u}\sinh\zeta)$, but we get a new Lorentz transformed scalar

(5.366) $$\lambda'_0 = \lambda_0\cosh\zeta + \lambda_\mathrm{u}\sinh\zeta,$$

and an invariant Euclidean 1-vector ***direction*** $\mathbf{u}$ with a Lorentz transformed magnitude

(5.367) $$\lambda'_\mathrm{u}\mathbf{u} = (\lambda_\mathrm{u}\cosh\zeta + \lambda_0\sinh\zeta)\mathbf{u}.$$

The magnitude of the paravector $\wp'_\mathrm{u} = \lambda'_0 1 + \lambda'_\mathrm{u}\mathbf{u}$ is as (5.363) preserved $|\wp'_\mathrm{u}| = |\wp_\mathrm{u}|$.
We conclude that a unit Euclidean 1-vector $\mathbf{u}^2 := 1$ defining the extension ***direction*** $\mathbf{u}$ of a local ***entity*** is invariant preserved by the Lorentz rotation transformation for the autonomous ***entity***.
In traditional language, when you see a galaxy, your looking ***direction*** towards that galaxy is

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