5.7.2.3. Minkowski Space with Display of Three Extension Dimensions

We follow the display with the development ***direction*** γ_0 vertical and the *null* ***directions*** turned $\pm 45°$ from this in the $\mathcal{B}_1$-bivector-plane. In Figure 5.54 we display the projection of the extension ***directions*** γ_1 and γ_2 with the hyperbola transformation parameter λ close to one in the defining formula (5.319) as $\gamma_1(\lambda\cong 1)$ and $\gamma_2(\lambda\cong 1)$. The same we do in Figure 5.55 where we further introduce the third extension ***direction*** γ_3, with λ→0; projection of $\gamma_3(\lambda=\tilde{0})$ displayed *antiparallel* ⇃↾ to γ_0, $\gamma_3 \| \gamma_0$. Chosen with the opposite orientation of the development ***direction***. But attention; $\gamma_0 \cdot \gamma_3 = 0$; there are all algebraically *orthogonal* $\gamma_0 \cdot \gamma_k = 0$, for $k=1,2,3$, (5.303), and we too presume $\gamma_1 \cdot \gamma_2 = \gamma_2 \cdot \gamma_3 = \gamma_3 \cdot \gamma_1 = 0$.

The projection mapping of the Minkowski basis $\{\gamma_0, \gamma_1, \gamma_2, \gamma_3\}$ with signatures $(+,-,-,-)$ into the Euclidean basis $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3\}$ with signatures $(+,+,+)$ is performed by the multiplication operations (5.328)

(5.341)
$$\begin{array}{llll} \gamma_1 \longrightarrow \boldsymbol{\sigma}_1 \overset{\leftarrow}{\equiv} \gamma_1\gamma_0, & & \boldsymbol{\sigma}_1 \longrightarrow \gamma_1 \overset{\leftarrow}{\equiv} \boldsymbol{\sigma}_1\gamma_0 \\ \gamma_2 \longrightarrow \boldsymbol{\sigma}_2 \overset{\leftarrow}{\equiv} \gamma_2\gamma_0, & \text{convers} & \boldsymbol{\sigma}_2 \longrightarrow \gamma_2 \overset{\leftarrow}{\equiv} \boldsymbol{\sigma}_2\gamma_0 \\ \gamma_3 \longrightarrow \boldsymbol{\sigma}_3 \overset{\leftarrow}{\equiv} \gamma_3\gamma_0, & & \boldsymbol{\sigma}_3 \longrightarrow \gamma_3 \overset{\leftarrow}{\equiv} \boldsymbol{\sigma}_3\gamma_0 \end{array}$$

In Figure 5.55 we will imagine that the third Minkowski $\mathcal{B}_3$-bivector-plane $\mathcal{B}_3^2=1$, $\mathcal{B}_3 \coloneqq \gamma_3\gamma_0$ has the same start ***direction*** as $\mathcal{B}_1 \coloneqq \gamma_1\gamma_0 \to \boldsymbol{\sigma}_1$. ($\mathcal{B}_3$ as $\mathcal{B}_2$ is un-displayed) The invariant nilpotent *null* ***directions*** n and $\overline{n}$ in the $\mathcal{B}_1$-bivector-plane is a translation invariant moved one unit along the $\pm\gamma_2$ ***direction*** of the Figure 5.55 display. The idea is that the ***entity*** represented by this frame is rotating with an oscillation in the extension plane $\gamma_1\gamma_2$ $\bigcirc = e^{\gamma_1\gamma_2\theta} = e^{i\theta}$, relative to an external laboratory.

By this oscillating rotation of both $\mathcal{B}_3$ and $\mathcal{B}_1$-bivector ***directions*** the nilpotent isotropic ***directions*** are twisted up in two opposite equal-orientated *null helixes* stretched out in the past as the oscillator is propagating into the future. It is the nilpotence of the *null basis* $\{n, \overline{n}\}$ (5.310) $n^2 = \overline{n}^2 = 0$ (5.311) that makes this invariant stretching possible carrying an information signal through the development from the past we call the *subton memory space of extension*.

This signal of information through space is what we in chapter I. 3.4 called a subton, which was displayed in Figure 3.13. where we did not have the nilpotent concept idea, but implicit had, that carrier time always is $t_{\overrightarrow{c^\infty}} = 0$ throughout the propagation as described in section I. 3.5.2, and which transfer of information in principle is displayed in Figure 3.14. The subton idea is the a priori foundation concept of a $U(1)$ spin-1-rotor oscillator propagating with the speed of information, which we set isometric equal to one, where the autonomous measure is one count of ***quantum*** in the oscillation of the subton ***entity*** development. The primitive subton idea is the foundation for the concept of a photon (and properly also gluons).

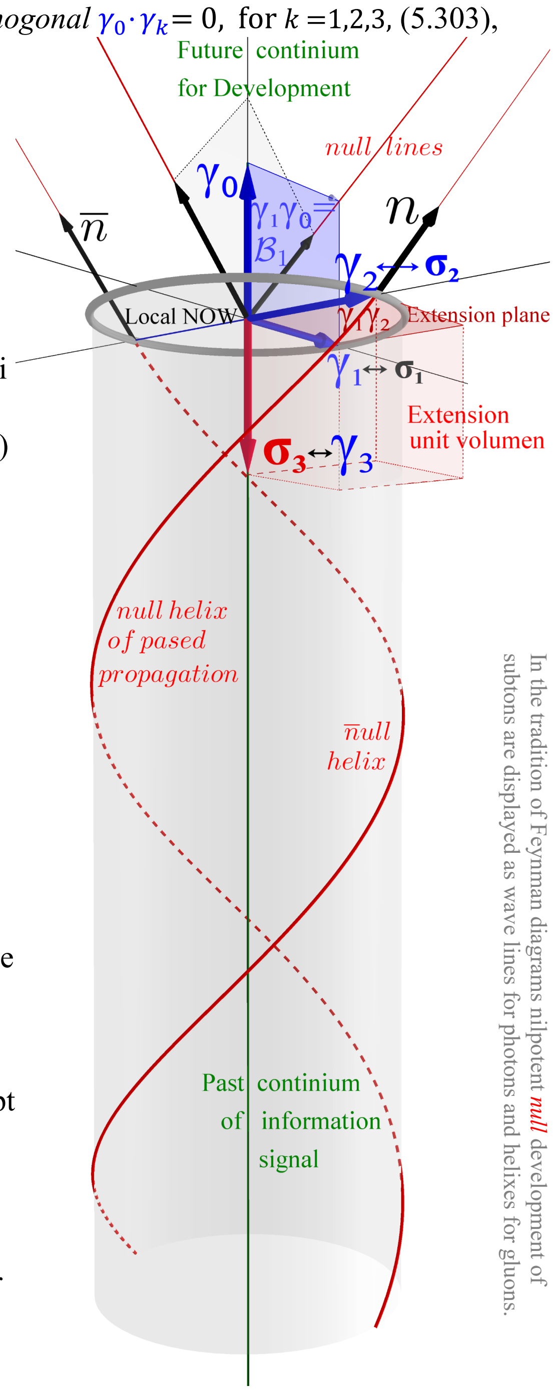

Figure 5.55 The four-dimensional Minkowski space projected into the figure plane. The extension space is generated by a plane cyclic oscillating rotation producing a helix-formed nilpotent *null* curve of development. The frame $\{\boldsymbol{\sigma}_k\}$ performs a rotating oscillation around

5.7.3. The Paravector Space and the Minkowski 1-vector in STA

In § 5.5.4.2 (5.240) we defined the foundation of a paravector as a scalar plus a 1-vector

(5.342) $$\wp = \lambda_0 + \mathbf{p} = \lambda_0 1 + \lambda_1 \mathbf{u}.$$

The 1-vector can have different directions $\mathbf{p} = \lambda_1\boldsymbol{\sigma}_1 + \lambda_2\boldsymbol{\sigma}_2$ in a Euclidean plane $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$ or in space $\{\boldsymbol{\sigma}_k\}$, $\mathbf{p} = \lambda_k\boldsymbol{\sigma}_k$, for $k=1,2,3$.[278] The para-multi-vectors of ***grade*** form $\langle A\rangle_0 + \langle A\rangle_1$ are then

(5.343) $$\wp = \lambda_0 + \mathbf{p} = \lambda_0 1 + \lambda_k \boldsymbol{\sigma}_\mathrm{k}.$$

The pure magnitude of the paravector is given from (5.253)-(5.255) as

(5.344) $$|\wp| = |\overline{\wp}| = \sqrt{|\wp\overline{\wp}|} = \sqrt{|\lambda_0^2 - \lambda_1^2 - \lambda_2^2 - \lambda_3^2|} \geq 0.$$

The paravector space is supported from the mixed basis $\{1, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3\}$ of form $\langle A\rangle_0 + \langle A\rangle_1$.

To see how the paravector concept looks like in Minkowski space we map by multiplication operation from the right with the ***quantum*** *count* of development expressed as the 1-vector unit γ_0

(5.345) $$p = \wp\gamma_0 = \lambda_0\gamma_0 + \mathbf{p}\gamma_0 = \lambda_0\gamma_0 + \lambda_k\boldsymbol{\sigma}_\mathrm{k}\gamma_0 = \lambda_0\gamma_0 + \lambda_k\gamma_k = \lambda_\mu\gamma_\mu. \quad \text{Use (5.328) } \boldsymbol{\sigma}_k\gamma_0 = \gamma_k.$$

This is a 1-vector in a four-dimensional Minkowski space supported as a span from the orthonormal 1-vector basis $\{\gamma_\mu\}$, for $\mu = 0,1,2,3$.[279] Here the multivector form is a pure $\langle A\rangle_1$, i.e., a ***primary quality of first grade*** (***pqg***-1) with the *development count* as the ***quantity*** unit. The geometric algebra founded on this basis $\{\gamma_\mu\}$ we as Hestenes [6] 1966 call Space-Time-Algebra (STA).

The 1-vector basis $\{\gamma_\mu\}$ is sometimes called a Dirac basis of STA -or just- a Minkowski basis.

The magnitude of such a 1-vector $p = \lambda_\mu\gamma_\mu$ in the Minkowski metric is given by the quadratic form

(5.346) $$p^2 = (\lambda_\mu\gamma_\mu)^2 = \lambda_0^2 - \lambda_1^2 - \lambda_2^2 - \lambda_3^2 \quad \Rightarrow \quad |p| = \sqrt{|p^2|} = \sqrt{|\lambda_0^2 - \lambda_1^2 - \lambda_2^2 - \lambda_3^2|} \geq 0.$$

The two pictures describe the same structure of a physical ***entity*** therefore the magnitude $|p| = |\wp|$.

Expressed from the supporting 1-vector basis $\{\gamma_\mu\}$ the Cartesian basis $\{\boldsymbol{\sigma}_k\} = \{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3\}$ looks like a Minkowski $\mathcal{B}$-bivector basis $\{\mathcal{B}_k\}$ by the map (5.326) $\mathcal{B}_k \coloneqq \gamma_k\gamma_0 \leftrightarrow \boldsymbol{\sigma}_k$

The Cartesian basis $\{\boldsymbol{\sigma}_k\}$ supports the Euclidean space by 1-vectors describing the Cartesian extension of locality for an ***entity*** in physics. Adding a scalar dimension to the Cartesian frame gives us a paravector (5.343) with a mixed basis $\{1, \mathbf{u}\}$, $\{1, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$ or $\{1, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3\}$.

This paravector picture split the picture of STA developing space with four ***direction*** dimensions into a scalar part without ***direction*** and one, two or three Cartesian extension ***directions*** $\boldsymbol{\sigma}_k$.

This agrees with the classical way to interpret space extension with a distance measure separated from the development of time as a pure scalar measure from a clock without extensive ***direction***.

Contrary if the local cyclic oscillating rotation clock produces the ***direction*** of ***quantum*** *counts* γ_0 and we can right multiply it to the paravector basis using (5.328) and get the STA basis

(5.347) $$\{1\gamma_0, \boldsymbol{\sigma}_1\gamma_0, \boldsymbol{\sigma}_2\gamma_0, \boldsymbol{\sigma}_3\gamma_0\} = \{\gamma_0, \gamma_1, \gamma_2, \gamma_3\},$$

then γ_0 by ***inheritance*** to the common measure of extensions gives the performed development.

Having a single 1-vector ***direction*** $\mathbf{u}$ in a Euclidean space $\mathbf{u}^2=1$, we can choose it as a frame basis 1-vector e.g., $\boldsymbol{\sigma}_1 = \mathbf{u}$. We multiply this with $\gamma_0\gamma_0 = \gamma_0^2 = 1$ and get the mapping operation

(5.348) $$\mathbf{u} = \mathbf{u}\gamma_0\gamma_0 = \boldsymbol{\sigma}_1\gamma_0\gamma_0 = \gamma_1\gamma_0 = \mathcal{B}_1 = \mathcal{B}$$

We hereby see that the Euclidean 1-vector ***direction*** $\mathbf{u}$ is synonymous with a Minkowski $\mathcal{B}$-bivector including an autonomous development ***quantum*** measure count of its own unit extension.

We note that these two concepts have the same Clifford signature $(+)$: $\mathbf{u}^2 = 1$ and $\mathcal{B}^2=1$.

Physical ***entities*** that we represent by Euclidean 1-vectors are in Space-Time Algebra (STA) represented by $\mathcal{B}$-bivectors with positive signature $(+)$.

[278] Here we use Einstein sum convention $\lambda_k\boldsymbol{\sigma}_k = \sum_{k=1}^{2 \text{ or } 3} \lambda_k\boldsymbol{\sigma}_k$ over double indices, and low indices for the covariant coordinates in the orthonormal frame of the Euclidian space, to keep intuition simple. More about $k=3, \{\boldsymbol{\sigma}_k\}$ in chapter 6.

[279] Note we commonly use Greek letter indices for four-dimensional Minkowski space and Latin indices for Euclidian 1,2,3 support.