

‘transverse’ Clifford conjugated **direction null** 1-vector $\tilde{0}\bar{n}$, $(\tilde{0}\bar{n})^2 = 0$ never vanish. We always have the invariant result (5.314) $n\bar{n} = (\tilde{\infty}n)(\tilde{0}\bar{n}) = 1 + \mathcal{B}$ of this hyperbola transformation.

5.7.1.3. The \mathcal{B} -bivector as an Information Signal

Of course, the **non-directional** scalar unit 1 is always preserved as the neutral element. The defined pseudoscalar $\mathcal{B} := \gamma_1\gamma_0 \Leftrightarrow n\bar{n} = \mathcal{B}$ as an \mathcal{B} -bivector supporting the Minkowski \mathcal{B} -plane is also always preserved by a hyperbola transformation. This pseudoscalar \mathcal{B} -bivector is defined as the product of the extension unit γ_1 acting on the information development unit γ_0 . The foundation idea of this is, that the propagating information *signal* about the extension has to be in balance. The primary *isometric measure* for this is the quadratic forms in balance (5.302)

(5.325) $\gamma_0^2 + \gamma_1^2 = 0 \Leftrightarrow \gamma_0^2 = -\gamma_1^2,$

that demand the Clifford geometric algebra $\mathcal{G}_{1,1}(\mathbb{R})$ with signatures $(+, -)$.

In the natural geometric plane, we separate two points A, B by defining an ordinary unit 1-vector $\sigma_1 := \overline{AB}$, with $\sigma_1^2 = 1$ as an ordinary metric signature $(+)$ in Euclidean line $\mathcal{G}_{1,0}(\mathbb{R})$, plane $\mathcal{G}_{2,0}(\mathbb{R})$ or space $\mathcal{G}_{3,0}(\mathbb{R})$. We settle out with the simple algebra of $\mathcal{G}_{1,0}(\mathbb{R})$ with the basis $\{1 = \sigma_1^2, \sigma_1\}$.

Here there is no pseudoscalar as $\lambda_1\sigma_1 \wedge \sigma_1 = 0$ only the scalar auto-product $\sigma_1 \cdot \sigma_1 = \sigma_1\sigma_1 = \sigma_1^2 = 1$, as the quadratic measure. The extension magnitude of this is $|\sigma_1| = \sqrt{\sigma_1 \cdot \sigma_1} = 1$.

In the tradition, this represents a ruler line **direction** drawn on a plane surface through A and B. As an external view, we let the abstract Minkowski \mathcal{B} -plane substance be projected into the plane surface, where we draw the 1-vector object $\sigma_1 := \overline{AB}$ one-to-one analogue to the physical extension **direction** in natural space. The idea is, that the \mathcal{B} -bivector is projection mapped onto σ_1 Figure 5.52:

(5.326) $\mathcal{B} \rightarrow \sigma_1 := \gamma_1\gamma_0 = \mathcal{B}_1 \rightarrow \sigma_1$

The defined \mathcal{B} -bivector supports the Minkowski \mathcal{B} -plane **direction** governed by the stable invariant *null line directions* (5.310) of the *null basis* $\{n, \bar{n}\}$ with $n\bar{n} = \mathcal{B}$. The 1-vector σ_1 support the objective **direction** of a physical line. The abstraction is the **direction map**:

(5.327) $\sigma_1 \leftrightarrow n\bar{n}.$

We interpret the \mathcal{B} -bivector $\mathcal{B} = n\bar{n}$ as the *isotropic*²⁷² *signal* of the objective unit **direction** σ_1 . The force of Geometric Algebra is, that the maps are replaced by multiplication operations.

We make the direct connection $\gamma_1\gamma_0 \leftrightarrow \sigma_1$ and replace it with $\sigma_1 \equiv \gamma_1\gamma_0$. We generalise γ_1 and σ_1 with more dimensions as γ_k and σ_k , for $k=1,2,3$, and get the following multiplication rules

(5.328)

$\sigma_k \equiv \gamma_k\gamma_0,$	$\sigma_k\gamma_0 \equiv \gamma_k,$	$\sigma_k\gamma_k \equiv \gamma_0,$	$\sigma_k \leftarrow \mathcal{B}_k := \gamma_k\gamma_0.$
$-\sigma_k \equiv \gamma_0\gamma_k,$	$\gamma_0\sigma_k \equiv -\gamma_k,$	$\gamma_k\sigma_k \equiv -\gamma_0.$	

By this we have introduced a Minkowski space foundation basis $\{\gamma_0, \gamma_1, \gamma_2, \gamma_3\}$ for the metric Clifford algebra $\mathcal{G}_{1,3}(\mathbb{R})$ with signatures $(+, -, -, -)$, we will call it a Dirac-Clifford algebra.²⁷³

5.7.2. The Traditional Display of the Minkowski \mathcal{B} -plane

In the tradition, the time axis is displayed vertically, and the spatial extension is displayed horizontally as in Figure 5.50. Here we try to make an autonomous measure of the development seen from the *entity* for our intuition. Therefore, we choose the cyclic *quantum oscillator*²⁷⁴ of the *entity* as an autonomous reference and make that unit count from this oscillator the measure of *one quantum* count of development γ_0 . A space extension γ_k shall as a priori presumption be measured relative to the development as a signal of information about extension in a founding balance as (5.325)

(5.329) $\gamma_0^2 + \gamma_k^2 = 0 \Leftrightarrow \gamma_0^2 = -\gamma_k^2,$ for each $k = 1,2,3,$ as the *primary quality of isometry*.

²⁷² The expression *isotropic signal* is the modern way to say Einstein’s presumption of constant light speed in all space *directions*.
²⁷³ We have chosen the basis names γ_μ as Hestenes [6] who mention the isomorphism with the group of Dirac matrices: called γ_μ . We will look further into this structure of *isotropic information signals* relative to *space extension* below in chapter 7.1. etc...
²⁷⁴ The *quantum oscillator* is introduced and described in chapter I. 3 section 3.6

This is equivalently expressed as the nilpotent *null basis* $\{n, \bar{n}\}$ by (5.311) $n^2 = \bar{n}^2 = 0$, and causes that the isotropic speed of information is one:

(5.330) $c = \frac{|\gamma_k|}{|\gamma_0|} = 1,$ because $|\gamma_0| = |\gamma_k| = 1,$ (5.303), (5.300)-(5.301).

This is an a priori analytic judgment for the founding measurement of extension by an information signal travelling along the extension. These balanced units are displayed in Figure 5.53 as a S^1 unit circle \circ for the autonomous *entity* for the unit basis $\{1, \gamma_0, \gamma_k, \mathcal{B}_k := \gamma_k\gamma_0\}$ from *one quantum* count measure γ_0 of development for a Minkowski \mathcal{B}_k -plane.

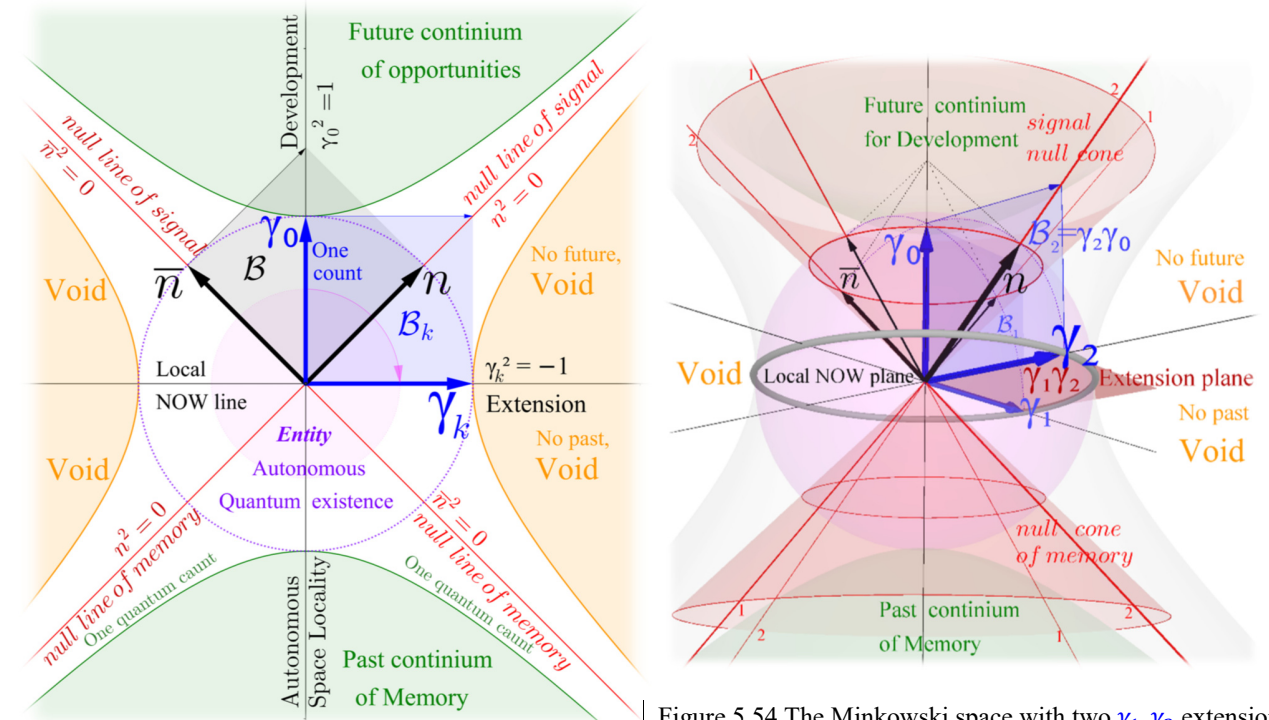


Figure 5.53 The Minkowski \mathcal{B}_k -plane in one γ_k extension **direction** relative to information development unit count γ_0 . The local origo for the *entity* is displayed as the intersection of the Space Locality line for the development **direction** and the Local NOW line for the extension **direction**.

The isotropic *null lines* are an indication of the balance (5.330) $|\tau\gamma_0| = |\tau\gamma_k| = \tau \in \mathbb{R}$ where the speed of information is one. The invariant hyperbolic transformation (5.322) makes a signal of *one quantum* information possible inside the white areas of this display figure. This Minkowski \mathcal{B} -plane display makes it possible for YOU (as a Thomas Aquinas GOD) to ‘look’ into areas of **Void**, where the *autonomous entity* neither can experience any information signals by light nor gravitation.

The white areas along the *null lines* in Figure 5.53 are where *one quantum* of development is an isometric measure of one unit of extension. This unit measure $n\bar{n} = 1 + \mathcal{B}$, (5.314) or just the \mathcal{B} -bivector is invariant by hyperbola transformation (5.319) as (5.320) $\mathcal{B}_k = \gamma_k(\lambda)\gamma_0(\lambda)$, for $\forall \lambda \in \mathbb{R}, \lambda \neq 0$, in its **direction** and magnitude, $\mathcal{B}_k^2 = 1$. Here in one dimension of the extension direction γ_k , the invariant \mathcal{B}_k -bivector seems to have no restriction against development by $x^0\gamma_0, x^0 \in \mathbb{R}$ of the extension $x^k\gamma_k, x^k \in \mathbb{R}$, as long as the memory knowledge of the past, giving

(5.331) $|x^k| \leq |x^0|,$ for $k=1,2,3.$

Note we use upper indices x^k for the contravariant coordinates due to the mixed metric (5.300)-(5.301).

Research on the a priori of Physics

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Geometric Critique of Pure Mathematical Reasoning