The closed algebra for this basis fulfils the multiplication scheme shown in Table 5.2a. below.

Besides this basis, we further form the non-zero *null basis* $\{n, \bar{n}\}$ for the $\mathcal{B}$-plane in $\mathcal{G}_{1,1}(\mathbb{R})$ defined by:

(5.310) $$n := \sqrt{½}(\gamma_0 + \gamma_1), \quad \text{and} \quad \bar{n} := \sqrt{½}(\gamma_0 - \gamma_1),$$

with the nilpotent spectral basis properties

(5.311) $$n^2 = \bar{n}^2 = 0, \quad \text{and} \quad \bar{n}\cdot n = n\cdot\bar{n} = 1.$$

This basis idea does not possess orthogonality, though it displays perpendicular ***directions*** in Figure 5.50.
The pseudoscalar for $\{n, \bar{n}\}$ is the same as (5.305)[270]

(5.312) $$n \wedge \bar{n} = \mathcal{B},$$

still with $\mathcal{B}^2=1$, and reversed like (5.306) as a bivector

(5.313) $$\bar{\mathcal{B}} = \tilde{\mathcal{B}} = -\mathcal{B} = \bar{n} \wedge n,$$

We have the product of the two *null-basis*-1-vectors

(5.314) $$n\bar{n} = n\cdot\bar{n} + n\wedge\bar{n} = \underbrace{1 + \mathcal{B}}, \quad \text{and} \quad \bar{n}n = \underbrace{1 - \mathcal{B}},$$

The full mixed *null-basis* for the $\mathcal{B}$-plane is

(5.315) $$\{1,\ n,\ \bar{n},\ \mathcal{B} = n\wedge\bar{n}\}$$

The operation of $\mathcal{B}$ gives the ***direction*** *absorptions*

(5.316) $$\boxed{\begin{array}{ll} \mathcal{B}n = n, & \mathcal{B}\bar{n} = -\bar{n}, \\ n\mathcal{B} = -n, & \bar{n}\mathcal{B} = \bar{n}\ . \end{array}}$$

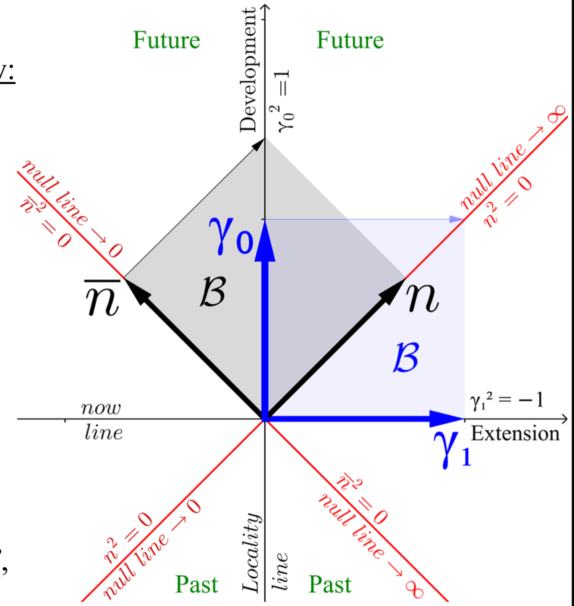

Figure 5.50 The *null basis* $\{n, \bar{n}\}$ with the pseudoscalar $\mathcal{B}$-bivector unit $\mathcal{B} = n \wedge \bar{n} = \gamma_1\gamma_0$. Multivectors expressed as $M = \tau\gamma_0 \pm \tau\gamma_1, \forall\tau\in\mathbb{R}$ fulfils the balance (5.302) $(\tau\gamma_0)^2 + (\tau\gamma_1)^2 = 0$ and falls on the *null lines* $M = \tau\sqrt{2}n$ or $M = \tau\sqrt{2}\bar{n}$.

The closed algebra for this basis fulfils the multiplication scheme shown in Table 5.2b.

Table 5.2 Multiplication basis for the $\mathcal{B}$-plane algebra $\mathcal{G}_{1,1}(\mathbb{R})$ with the pseudoscalar unit $\mathcal{B}^2 = 1$, $\mathcal{B} \equiv \gamma_1\gamma_0$.

a: $\{1, \gamma_0, \gamma_1, \mathcal{B}\}$ orthonormal

left \ right	1	γ_0	γ_1	$\mathcal{B}$
1	1	γ_0	γ_1	$\mathcal{B}$
γ_0	γ_0	+1	$-\mathcal{B}$	$-\gamma_1$
γ_1	γ_1	$\mathcal{B}$	−1	$-\gamma_0$
$\mathcal{B}$	$\mathcal{B}$	γ_1	γ_0	1

Two cases of mixed basis

$\gamma_1\gamma_0 = \mathcal{B} = \mathcal{B} = n\wedge\bar{n}$,

$\mathcal{B}^2 = +1, \quad n^2 = \bar{n}^2 = 0,$

$\gamma_0\gamma_1 = \bar{\mathcal{B}} = -\mathcal{B} = \bar{n}\wedge n$

b: $\{1, n, \bar{n}, \mathcal{B}\}$ the *null*-basis

left \ right	1	n	$\bar{n}$	$\mathcal{B}$
1	1	n	$\bar{n}$	$\mathcal{B}$
n	n	0	$1+\mathcal{B}$	$-n$
$\bar{n}$	$\bar{n}$	$1-\mathcal{B}$	0	$\bar{n}$
$\mathcal{B}$	$\mathcal{B}$	n	$-\bar{n}$	1

Conversely to (5.310), we get the $\{\gamma_0, \gamma_1\}$ basis from the $\{n, \bar{n}\}$ null basis

(5.317) $$\gamma_0 = \sqrt{½}(n + \bar{n}), \quad \text{and} \quad \gamma_1 = \sqrt{½}(n - \bar{n}).$$

The nilpotence (5.311) of the null basis $\{n, \bar{n}\}$ makes it possible to scale the two bases as $\left\{\lambda n, \frac{1}{\lambda}\bar{n}\right\}$ in the way that we preserve the unit pseudoscalar $\mathcal{B}$-bivector and the inner product

(5.318) $$\mathcal{B} = \lambda n \wedge \frac{1}{\lambda}\bar{n}, \quad \text{and} \quad \frac{1}{\lambda}\bar{n}\cdot\lambda n = \lambda n\cdot\frac{1}{\lambda}\bar{n} = 1, \quad \text{for } \forall\lambda\in\mathbb{R},\ \lambda \neq 0.$$

We see that the null basis 1-vectors have no specific magnitudes, it is the ***direction*** $\mathcal{B}$-bivector pseudoscalar and the inner product scalar that is preserved in the multivector (5.314) $n\bar{n} = \underbrace{1 + \mathcal{B}}$.
From this, we form the hyperbola transformed of (5.317) that is displayed in Figure 5.51

(5.319) $$\gamma_0(\lambda) = \sqrt{½}\left(\lambda n + \frac{1}{\lambda}\bar{n}\right), \quad \text{and} \quad \gamma_1(\lambda) = \sqrt{½}\left(\lambda n - \frac{1}{\lambda}\bar{n}\right), \quad \text{for } \forall\lambda\in\mathbb{R},\ \lambda\neq 0.$$

Further from this we also have the preserved unit pseudoscalar area as (5.305)

(5.320) $$\mathcal{B} = \gamma_1(\lambda)\gamma_0(\lambda),$$

[270] The reason is: $n\wedge\bar{n} = ½(n\bar{n} - \bar{n}n) = ½\left(½(\gamma_0+\gamma_1)(\gamma_0-\gamma_1) - ½(\gamma_0-\gamma_1)(\gamma_0+\gamma_1)\right) = ¼(4\gamma_1\gamma_0) = \gamma_1\gamma_0 = \mathcal{B}$.

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and still for the basis $\{1, \gamma_0(\lambda), \gamma_1(\lambda), \mathcal{B}\}$ algebraically orthonormality is preserved as (5.303)[271]

(5.321) $$\gamma_0(\lambda)\cdot\gamma_1(\lambda) = 0, \quad \text{and} \quad |\gamma_0(\lambda)| = |\gamma_1(\lambda)| = 1 \quad \Leftarrow \left|(\gamma_0(\lambda))^2\right| = \left|(\gamma_1(\lambda))^2\right| = 1.$$

The basis $\{\gamma_0(\lambda), \gamma_1(\lambda)\}$ is the hyperbola transformed from the null basis $\{n, \bar{n}\}$. The former is orthonormal but remark that its display is certainly not perpendicular nor normal in Figure 5.51. We display this transformation invariant structure from the a priory ***direction*** $\mathcal{B} = n\wedge\bar{n}$ basis $\{n, \bar{n}\}$ displayed as perpendicular 1-vectors $\bar{n}\perp n$, but as the null-basis not orthogonal $n\cdot\bar{n}=1$. $\{n, \bar{n}\}$ in Figure 5.51 turned by –45° from the defined ***directions*** displayed in Figure 5.50 above:

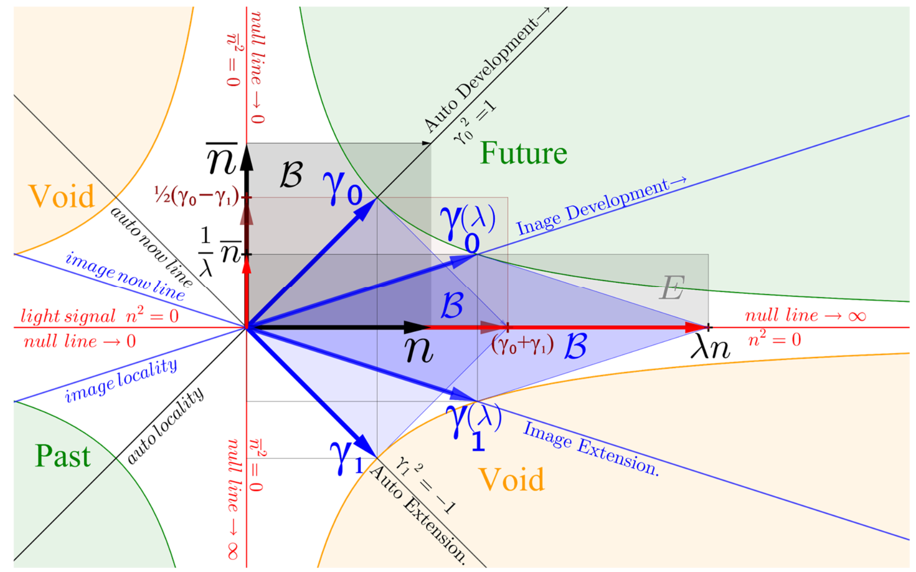
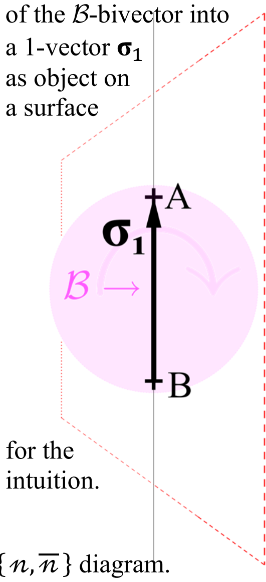


Figure 5.52 Projection of the $\mathcal{B}$-bivector into a 1-vector $\boldsymbol{\sigma}_1$ as object on a surface



for the intuition.

Figure 5.51 The invariant hyperbola transformed $\gamma_0(\lambda)$ and $\gamma_1(\lambda)$ is displayed in the null basis $\{n, \bar{n}\}$ diagram.
We have chosen a perpendicular display of the null basis 1-vectors $\{n, \bar{n}\}$ representing the founding $\mathcal{B}$-bivector $\mathcal{B}=n\wedge\bar{n}$ defining the ***direction*** of the $\mathcal{B}$-plane of this figure. The *null line* $n^2=0$ representing a possible light signal of information is displayed from left to right in the reading direction. The Clifford conjugated *null line* $\bar{n}^2=0$ is chosen displayed vertically perpendicular $\bar{n}\perp n$. Due to the ***quality*** of the full mixed null basis $\{1,\ n, \bar{n},\ \mathcal{B}=n\wedge\bar{n}\}$ of the $\mathcal{B}$-plane (5.315) these are not orthogonal $n\cdot\bar{n}=1$, (5.311) but multiplied invariant constant $n\bar{n} = n\cdot\bar{n} + n\wedge\bar{n} = 1 + \mathcal{B}$, (5.314).
The word *auto* refers to an autonomous ***entity*** that has a locality-now center, performed by the intersection of the locality development line ***direction*** γ_0 and the *now* extension *line* ***direction*** γ_1, with the autonomous full mixed basis $\{1, \gamma_0, \gamma_1, \mathcal{B}\}$, (5.308). The invariant hyperbola transformed (5.319) $\gamma_0(\lambda)$ and $\gamma_1(\lambda)$ are displayed for the argument, e.g. $\lambda\sim 2.5$.
The product $\mathcal{B} = \gamma_1(\lambda)\gamma_0(\lambda)$ (5.320) and orthonormality (5.321) are invariant preserved for all $\forall\lambda\in\mathbb{R},\ \lambda\neq 0$.
$\lambda<0$ change orientation. $\lambda\to\infty$ gives the external fare away parallel units, and $\lambda\to 0$ is the local anti-parallel unit balance.

5.7.1.2. An *Entity* Seen from the External Far Distant as a Null Signal

We have seen that the unit ***direction*** $\mathcal{B}$-bivector is invariant preserved for all $\forall\lambda\in\mathbb{R},\ \lambda\neq 0$

(5.322) $$\mathcal{B} = \gamma_1(\lambda)\gamma_0(\lambda) \quad = n\wedge\bar{n},$$

We now look at $\lambda\to\infty$, and we note it as $\lambda = \tilde{\infty}$, and $\frac{1}{\lambda} = \tilde{0}$, whereby we from (5.320) and (5.318) write

(5.323) $$\mathcal{B} = \gamma_1(\tilde{\infty})\gamma_0(\tilde{\infty}) \quad = n\wedge\bar{n} = \quad \tilde{\infty}n\wedge\tilde{0}\bar{n}, \quad \text{and} \quad n\cdot\bar{n} = \tilde{\infty}n\cdot\tilde{0}\bar{n} = 1.$$

In our display practice, this form will result in

(5.324) $$\gamma_0(\tilde{\infty}) = \sqrt{½}(\tilde{\infty}n + \tilde{0}\bar{n}) \to \tilde{\infty}n, \quad \text{and} \quad \gamma_1(\tilde{\infty}) = \sqrt{½}(\tilde{\infty}n - \tilde{0}\bar{n}) \to \tilde{\infty}n,$$

convergent towards ***direction*** n for the null 1-vector $(\tilde{\infty}n)^2 = 0$, but the impact from the

[271] The reader is encouraged to verify this in detail, in that, it is fundamental to how one can perceive the physical world.

For quotation reference use: ISBN-13: 978-8797246931