The sequence of x, y in the coordinate set is lost in the commutative linear addition algebra. The addition of two linear independent ***pqg***-1-vectors $\mathbf{r} = x\boldsymbol{\sigma}_x + y\boldsymbol{\sigma}_y$, thus yields a ***pqg***-1-vector. From this, we can conclude that for the addition of ***quantities*** of type $\mathbb{R}^1_{\text{pqg-1}}$ applies

(5.157) $$\mathbb{R}^1_{\text{pqg-1}} \oplus \mathbb{R}^1_{\text{pqg-1}} \sim \mathbb{R}^2_{\text{pqg-1}} \longrightarrow \mathbb{R}^1_{\text{pqg-1}}$$

Thus, for linear 1-vectors, the additive algebra applies to vector spaces V_n=$(\mathbb{R}^n_1, \mathbb{R})$ independent of the dimension n, therefore $\mathbb{R}^n_{\text{pqg-1}} \longrightarrow \mathbb{R}^1_{\text{pqg-1}}$. In this way, the addition of 1-vectors is still a 1-vector, i.e., V_n vector spaces are ***pqg***-1-vector spaces. e.g., 2D, 3D ... nD ...

Do we imagine a physical ***entity*** Ψ with two linearly independent degrees of freedom Ψ_x, Ψ_y, and describe these as orthogonal 1-vector ***quantities*** $\mathrm{q}_x = q_x\boldsymbol{\sigma}_x$ and $\mathrm{q}_y = q_y\boldsymbol{\sigma}_y$ in the ***pqg***-2 ***direction*** $\boldsymbol{\sigma}_2 \wedge \boldsymbol{\sigma}_1$ of an orthonormal $\boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_1 = 0$ basis $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$, where $q_x, q_y \in \mathbb{R}$, will an arbitrary superposition of these still be a linear 1-vector ***quantity*** $\mathrm{q} = \kappa_x \mathrm{q}_x + \kappa_y \mathrm{q}_y$ where $\kappa_x, \kappa_y \in \mathbb{R}$. To consider the two linear independent degrees of coordinate axis given by 1-vectors $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2$ as generators for a spanned plane in space $\mathfrak{G}$ should be limited to space properties itself. I.e., the coordinates as distance ***quantities*** along the respective axes $\mathbf{r}_x = x\boldsymbol{\sigma}_x$ and $\mathbf{r}_y = y\boldsymbol{\sigma}_y$

It has been a tradition when it comes to a physical ***entity***, with two or more linear independent *degrees of freedom* $\Psi_1, \Psi_2, \ldots$ to regard them as linked to the spatial axes ***directions*** given by orthogonal basis of ***directional*** 1-vectors $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2 \ldots$ in geometric space $\mathfrak{G}$, and for the ***entities*** Ψ_j of the orthogonal degrees of freedom to write the separated ***directional*** vector field ***quantities*** as $\mathrm{q}_j = q_j\boldsymbol{\sigma}_j$ for j=1,2,… in the ***direction*** of the axes. E.g., special for electric and magnetic fields $\vec{E}$ and $\vec{B}$ as 1-vectors.
Here, and in particular, at multiple degrees of freedom, it is not appropriate to associate these quantities with the physical geometric space $\mathfrak{G}$, thus inventing new spatial dimensions whenever a new degree of freedom is found.
Of course, it may be possible to have abstract vector spaces, where each freedom object has its own dimension, and even have an orthogonal basis $\{\mathrm{e}_j\}$ through defining scalar products, that $j \neq k \Rightarrow \mathrm{e}_j \cdot \mathrm{e}_k = 0$ for that vector space. But then the angle concept has no longer any meaning for the intuition, therefore, such vector spaces cannot be perpendicular *Cartesian*, although it makes sense to judge them as orthogonal.

The conclusion is that the Cartesian coordinate system is suitable for geometric location determination in a 2D plane and with common 3D spaces, as well as for translations and descriptions of spatial translation invariance. In addition, the Cartesian system is suitable for the display of functional mapping in the well-known forms $y = y(x) = f(x)$, special simple as the equation of the straight line $y = ax + b$ displayed in a rectangular grid tabula.

5.3.7. The 1-vector Product Complex Quantity and Polar Coordinates of Plane Concept

When we apply the plane concept $\mathfrak{P}$ for the geometric space $\mathfrak{G}$ of a physical ***entity*** Ψ, it is just the rotation invariance that is interesting. Here, the plane substance enters the intuition of the ***primary quality of even grades*** (***pqg***-0-2) (***zero*** and ***second grade***) through objects such as 2-multi-vectors $\mathbf{r}\boldsymbol{\sigma}_1$, rotational angles θ, 1-rotors $U_\theta := e^{\boldsymbol{i}\theta}$ and plane complex ***quantity*** in form $\mathcal{Z} = \mathbf{r}\boldsymbol{\sigma}_1 = \mathbf{r}\cdot\boldsymbol{\sigma}_1 + \mathbf{r}\wedge\boldsymbol{\sigma}_1 = \rho U_\theta = \rho e^{\boldsymbol{i}\theta}$ to be the primary tools we can use in the meeting with the a priori substantial ***pqg***-2 ***directional quality*** in the natural space of physics. The a priori substantial concept ***pqg***-2 here is what Immanuel Kant called transcendental to intuition.
We can never come to a deeper understanding of the concept of a plane, and we neverexperience the plane idea directly. But we can experience a flat surface of material things.

We have seen above that the 2-vectors $A = \mathbf{ba}$ of a plane are rotational invariant in that plane ***direction***. While the 1-vectors $\mathbf{a}, \mathbf{b}$ have ***pqg***-1 ***directions*** in the plane where each 1-vector is rotated by a rotor U, but there 2-vector $A = \mathbf{ba}$ is rotation invariant and exists free in the plane. A has no ***direction*** in the plane, A is the ***direction*** of that plane in space $\mathfrak{G}$.
All ***pqg***-0 real scalars as products $\alpha = \mathbf{b}\cdot\mathbf{a}$, as well as all ***pqg***-2 ***direction*** bivectors $\mathbf{b}\wedge\mathbf{a} = \beta\boldsymbol{i}$, (plan-pseudo-scalars), and 1-rotors $U_\theta := e^{\boldsymbol{i}\theta}$, together with traditional complex scales $z \in \mathbb{C}$, are all rotational invariant inside the same plane ***direction***.
Angles are, in principle, also rotational invariant (congruent triangles), in particular, the real angular arc absolute measure $\theta = \measuredangle(\mathbf{a}, \mathbf{b})$ is retained as a ***quantity*** of type $\mathbb{R}_{\text{pqg-2}}$.

The ***pqg***-2 ***direction*** is implicitly given from $\boldsymbol{i}$. The factor $\rho \in \mathbb{R}$ acting, as dilation of the unit circle[253] of the 1-rotor $U_\theta := e^{\boldsymbol{i}\theta}$, is a ***pqg***-0 real scalar, which (ρ) does <u>not</u> itself have ***direction***. From this, we have a plane polar coordinate system with the coordinate set

(5.158) $$(\rho, \theta) \in \mathbb{R}^2 \;\; \leftrightharpoons \;\; \mathbb{R}_{\text{pqg-0}} \otimes \mathbb{R}_{\text{pqg-2}}.$$

The *substance* of a geometric plane has *subjects*, each of which has two ***grade*** parts for $\rho e^{\boldsymbol{i}\theta}$:

- A ***pqg***-0 part subject with the ***primary quality of zero grade***
- A ***pqg***-2 part subject with the ***primary quality of second grade***

The plane concept idea has a complex object structure:

The following objects operate a priori by judgment on the plane substance $\mathfrak{P}$ in space $\mathfrak{G}$:

Primary quality	***pqg***-0 + ***pqg***-2	***pqg***-0	***pqg***-2 ***direction***
Object category:	**1-vector multiplication**	**Real scalars**	**Bivector\\ *pqg*-2\\ plane-pseudoscalar**
2-multi-vector	$\mathbf{ba} = \mathbf{b}\cdot\mathbf{a} + \mathbf{b}\wedge\mathbf{a}$	$\mathbf{b}\cdot\mathbf{a} = \mathbf{a}\cdot\mathbf{b} \in \mathbb{R}$	$\mathbf{b}\wedge\mathbf{a} = -\mathbf{a}\wedge\mathbf{b}$, anticommute
Rotor (operator)	$U_\theta := e^{\boldsymbol{i}\theta} = \mathbf{u}_2\mathbf{u}_1$	$\cos\theta \in \mathbb{R}$	$\boldsymbol{i}\sin\theta \in \boldsymbol{i}\mathbb{R} \dashrightarrow \mathbb{C}^{\circlearrowleft}$
pqg-1 ⊗ ***pqg***-1⟶	***pqg***-0 ⊗ ***pqg***-2	***Dilation***	***Unitary Rotor operator***
The Complex ***spinor quantity***	$\mathcal{Z} = \rho e^{\boldsymbol{i}\theta} = \mathbf{ru}$	$\rho \in \mathbb{R}_+$	$\exp(\boldsymbol{i}\theta) \dashrightarrow \mathbb{C}^{\circlearrowleft}$, where $\theta \in \mathbb{R}$
Polar plane coordinates	$(\rho, \theta) \in \mathbb{R}^2$	$\rho \in \mathbb{R}_+$	$e^{(\boldsymbol{i}\theta)}$, where $\theta = \measuredangle(\mathbf{a}, \mathbf{b}) \in [0, 2\pi[\; +2\pi n$

The generating unit $\boldsymbol{i}$ of the plane is a complex multiplication product of two 1-vectors as a bivector object of a ***primary quality of second grade*** (***pqg***-2) ($\boldsymbol{i}$ is without scalar product).
When the plane segment object representative $\boldsymbol{i}$ operates on any geometric 1-vector $\mathbf{a}$, another possible 1-vector $\mathbf{b} = \boldsymbol{i}\mathbf{a}$ orthogonal to it is generated, then $\mathbf{b}\cdot\mathbf{a} = 0$, $\mathbf{b}^2 = \mathbf{a}^2$. (see § 5.2.6.5)
The two vectors then span the plane for this arbitrary orthogonal basis $\{\mathbf{a}, \mathbf{b}\}$.
Do we use as a norm, $\mathbf{a}^2 = 1$, and put $\boldsymbol{\sigma}_1 = \mathbf{a}$, and get $\boldsymbol{\sigma}_2 = \boldsymbol{i}\sigma_1$, we have the orthonormal basis $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$. – As follows, I would like to consider the plane concept of physics: Any arbitrary object $\boldsymbol{i}$ operates on any of its geometric 1-vector and generates the object of its plane.

5.3.7.2. A plane Rotation and Dilation added to a Translation

A general arbitrary 2-multi-vector constructed from three ***primary qualities of grades*** ***pqg***-0 + ***pqg***-1 + ***pqg***-2 can be specified from three 1-vectors

(5.159) $$A = \mathbf{r}\cdot\mathbf{u} + \mathbf{t} + \mathbf{r}\wedge\mathbf{u} = \mathbf{r}\cdot\mathbf{u} + \mathbf{r}\wedge\mathbf{u} + \mathbf{t} = \mathbf{ru} + \mathbf{t}.$$

[253] Above we have defined the unit circle object as $\left\{\forall \mathrm{P} \mid \overrightarrow{\mathrm{OP}} = \mathbf{u}_\theta = e^{\boldsymbol{i}\theta}\boldsymbol{\sigma}_1, \forall\theta \in [0, 2\pi[\right\}$ from a radius $\boldsymbol{\sigma}_1$ in a plane $\boldsymbol{i}$.