In addition, we can conclude that for us the plane complex unit bivector $\boldsymbol{i} \coloneqq \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ is a generator

for the plane and its plane rotation. Here we clarify (again) that the subject $\boldsymbol{i}$ as a ***direction*** of the plane is independent of the specific ***pqg***-1 objects $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ as we prefer $\boldsymbol{\sigma}_2 \perp \boldsymbol{\sigma}_1$ to the intuition of the plane γ_i symmetry. We say that $\boldsymbol{i} \coloneqq \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ is rotation invariant in its own plane. We also remember that $\boldsymbol{i}$ is a variable amoeba shape invariant ***plane direction*** unit $|\boldsymbol{i}| = 1$.

A 1-rotor $U_\theta = \mathbf{u}\boldsymbol{\sigma}_1$ is unitary, as $\mathbf{u}\boldsymbol{\sigma}_1\boldsymbol{\sigma}_1\mathbf{u} = 1$, and thus $U_\theta U_\theta^\dagger = 1$, refer (5.86)-(5.90), and we remember (5.95) that the rotor U_θ as an operator does not commute with 1-vectors.

The 1-rotor $U_\theta \coloneqq e^{\boldsymbol{i}\theta}$ around the circle generates the complex plane, as an object for us, by dilation with ρ, as in the formulas (5.127) or (5.129) through the polar coordinates (ρ, θ).

On the other hand, the subject of a plane is not a recognizable substance for us, but we can judge that it (das Ding an sich) has a ***primary quality of second grade*** (***pqg***-2) that inherits the option for an orthogonal basis set and the corresponding rotation ***direction*** displayed as an object for the intuition $\boldsymbol{\sigma}_2 \perp \boldsymbol{\sigma}_1$.

5.3.5. A *Grade*-2 Object, 2-blade Bivector → a Subject in a Substance as Idealism

Throughout history, we have learned and assumed that geometric planes are platonic ideal subjects that cannot be recognized. But:

- We immediately know about space that we can perceive a surface in front of us, for example. A piece of paper, a book, a screen, a wall, the ground or just a stone, and that we can judge A: '*right* ', B: '*left up* ' and C: '*left down* ' for such a surface Corresponding to three points A, B, C in order: first A, then B, then through C, back to A.
- I have introduced the icon ⟲ as we by marking (drawing) on the surface can see that a rotation is possible. We can rotate from A to B to C... around the wheel whose locus situs is centred in between the cycling points A,B,C. In practice, we rotate a stone by hand.

To make this ***quality*** of space into an algebraic *object*, we have invented a concept of a 2-blade geometric bivector *object*, as a ***grade***-2 *subject*.

For us, this *subject* is a geometric idea representing a substance in space, namely the ***primary quality of second grade*** (***pqg***-2).

This substance appears by a specific physical subject as a plane rotation in space, which we experience as a *surface*.[250]

If we cannot experience a possible rotation, the concept of a plane has no meaning as a ***quality***. To intuit this experience, we form the idea of a 2-blade bivector as an *object*.

If we are skilled, the plane *subject* can, by the idea of a bivector $\boldsymbol{i} \coloneqq \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$, as an orthonormal basis $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$ be made into an *object*. Determine *locus situs* ⟲ for the physical ***entity***, the center O can be considered as *object* origo for a rotation, and the localised orthonormal basis set $\{\mathrm{O}, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$ as *tangent vectors* to O can be considered as intuit *object* for the physical ***entity*** described.

This locates the center of object by determining possible rotation symmetry through a plane ***direction*** given by a 2-multi-vector $\mathbf{b}\boldsymbol{\sigma}_1$ ideal transformed to a bivector $\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ as a 2-blade basis $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$. To be a *tangent* plane segment object for us we shall be able to point out a center O.

The Bivector 2-blade, 2-multi-vector represents

Das Ding an sich	Subject	***pqg***-2, a plane segment idea $\boldsymbol{i}$ with ***direction*** in space substance 𝔖
Das Ding für uns	Object	a rotationally plane surface in space with ***direction*** and magnitude

Thus, the subject exists in the unrecognisable plane substance, but we can judge a priori, that it has in itself a ***primary quality of second grade*** (***pqg***-2) with the orthonormal basis set $\{\mathrm{O}, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$

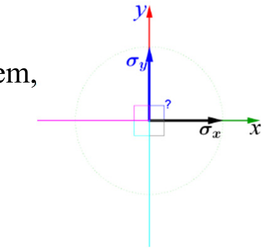
[250] I use the word surface to indicate a front and a backside of the plane. If the surface is curved there is a convex and concave side or an outer surface and an inside. More about this at higher dimensions later below. You are not inside a stone, but yourself.

as a symmetric object ⟲ for intuition.

To be able to intuitive view a plane directly, it is a physical object (a rotatable planar surface) shall be placed transversely across the (***pqg***-1) view ***direction*** in the field of view.

We know that the surface must emit light to be seen (from the tabula). There is thus a physical space 𝔖 beyond the plane we consider, more about this in chapter 6.

5.3.6. The Inadequate *Cartesian* (x, y) Coordinate System



A *Cartesian* coordinate system (x, y) usually means a right-angled system, which is generated from a basis of two 1-vectors $\{\boldsymbol{\sigma}_x, \boldsymbol{\sigma}_y\}$, which in an a priori synthetic judgment must be linearly independent

(5.154) $$x\boldsymbol{\sigma}_x + y\boldsymbol{\sigma}_y = \mathbf{0} \;\Rightarrow\; x = 0 \wedge y = 0.$$

The *canonical* requirement of the *Cartesian* system is that the spatial relationship between the two axes, the x-axis and the y-axis must consist of four right angles in symmetry. Have you first adopted the *Cartesian* coordinate system, the impact of the four right angle symmetry disappears in the a priori transcendental for our intuition. The four central symmetry immediately gives the naïve[251] opinion that the *Cartesian* system is the simplest primitive intuition of the plane as a ***quantity***.

It is unclear in which way the *Cartesian* plane endows substance ***quality***.

We have two coordinates $x, y \in \mathbb{R}$, each generating straight lines

(5.155) $$\{\forall \mathrm{x} |\; \mathbf{x} = x\boldsymbol{\sigma}_x,\ \text{for}\ \forall x \in \mathbb{R}\} \quad \text{and} \quad \{\forall \mathrm{y} |\; \mathbf{y} = y\boldsymbol{\sigma}_y,\ \text{for}\ \forall y \in \mathbb{R}\}.$$

These coordinate axes each constitute one ***primary quality of first grade*** (***pqg***-1), whose ***quantities*** are objects of the form $\mathbf{x} = x\boldsymbol{\sigma}_x$ and $\mathbf{y} = y\boldsymbol{\sigma}_y$, each of type $\mathbb{R}^1_{\text{pqg-1}}$, corresponding to the two 1-vector spaces $\mathbf{x} \in (\mathbb{R}^1_{\mathrm{x}}, \mathbb{R})$ and $\mathbf{y} \in (\mathbb{R}^1_{\mathrm{y}}, \mathbb{R})$.

It is usual to denote the set $(x, y) \in \mathbb{R}^2$, as representing a point X in the plane or a 1-vector $\mathbf{r}$ in the plane. When we combine the two linearly independent ***pqg***-1 ***qualities*** from (5.155), we get a ***quantity*** of the type $\mathbb{R}^1_{\text{pqg-1}} \oplus \mathbb{R}^1_{\text{pqg-1}}$ that we rename to $\mathbb{R}^2_{\text{pqg-1}}$ ***quantity*** corresponding to $\mathbf{x} + \mathbf{y} \in \mathbb{R}^2_{\mathrm{xy}} = \mathbb{R}^1_{\mathrm{x}} \oplus \mathbb{R}^1_{\mathrm{y}}$ which is the *Cartesian* 2D concept for a 2-dimensional plane. Hereby we construct a plane by ***pqg***-1-vectors in a linear combination

(5.156) $$\overrightarrow{\mathrm{OX}} = \mathbf{r} = x\boldsymbol{\sigma}_x + y\boldsymbol{\sigma}_y\,.$$

This linear 1-vector can usually designate a point in the plane as an object for a location place relative to an origo. In addition to this concept, it can generate a translation of all points in the plane.[252] This 1-vector is also called a geometric linear concept as it is in its one substance a ***pqg***-1 ***quality*** , that by dilation in its line can generate new 1-vectors with the same ***direction*** in the plane $\overrightarrow{\mathrm{OL}} = \lambda\mathbf{r} = \lambda x\boldsymbol{\sigma}_x + \lambda y\boldsymbol{\sigma}_y$, compare with (5.153).

The only thing that is variable within this ***pqg***-1 substance is the magnitude of the 1-vector in the usual way $|\overrightarrow{\mathrm{OL}}| = \lambda|\mathbf{r}| = \lambda\sqrt{x^2 + y^2}$.

In the ***quantitative*** designation of a location $(x, y) \rightarrow \mathrm{P}$ in the 2D plane $(x, y) \in \mathbb{R}^2_{\text{pqg-1}} \sim \mathbb{R}^2$, you first go along the x-axis as an object then go along the y-axis as an object (or vice versa) to the location P as an object. This is the traditional 2D location concept, where the idea of the right-angled coordinate system is pushed into the a priori transcendental, and thus the plane angle concept as a substance for a rotation has also gone transcendental to the intuition.

[251] It is unclear which of the four angles in command concerning whichever comes first, and mirror symmetry of axes if it is positive or negative coordinates is the earlier. We can ask, is the alternation of the axes or basis vectors $\{\boldsymbol{\sigma}_x, \boldsymbol{\sigma}_y\}$ allowed? You can just watch them from the back of the plane and be confused. First 1768 Immanuel Kant contra Leibniz [11]p.361ff.

[252] These rules apply of course also in three dimensions etc. $\overrightarrow{\mathrm{OP}} = \mathbf{r} = x\boldsymbol{\sigma}_x + y\boldsymbol{\sigma}_y + z\boldsymbol{\sigma}_z$, for a ***pqg***-1-vector.