In the circular plane, the third 1-vector is linearly dependent on the other two 1-vectors.
In the symmetrical case Figure 5.33 $\mathbf{u_a} = -(\mathbf{u_b}+\mathbf{u_c})$, $\mathbf{u_b} = -(\mathbf{u_c}+\mathbf{u_a})$, $\mathbf{u_c} = -(\mathbf{u_a}+\mathbf{u_b})$,
and generally as in the asymmetric case in Figure 5.32 $\mathbf{u_c}$ can be written, $\mathbf{u_c} = \alpha\mathbf{u_a} + \beta\mathbf{u_b}$, etc.
The rotation is defined as positively orientated in the alphabetical order[237] A,B,C sequential 1,2,3, ...
The first point; a 1-vector object, or *statement*, then
A. The second point; a 1-vector object, or *statement*, which relates to A, from which follows
B. The third point; a 1-vector object, or *statement*.
These three together form a cyclic unit! A syllogism of the two to a third, a conclusion through an inference. C is made by a combination of A and B in this situation.
The center of the circle is the origo of an object with the icon[238] ⦿, a locus situs (locality) or a context for the statements. This scheme is fundamental for all actions, in our intuition.
We see that when we relate to the rotation, is a process from A over B to C and back to A. The rotation process provides a closed cyclic unit, whose concept ⦿ we must take seriously as a physical ***entity***.[239] The ability to perform a syllogism is a natural physical substance as a reality for us, and the thought of a circular movement is possible. The circle is still noumenon, a Platonic idea for us. The rotation in the circle is a ***primary quality of second grade*** (***pqg***-2).

5.3.1.2. Two Points Define the *Primary Quality of First Grade* (*pqg*-1)

The central two-symmetry is obtained by finding the center between two points A and B, as the midpoint of the straight-line segment AB. In Figure 5.31 in thought, we omit the point C of the triangle and the circle collapses to the two anti-co-linear 1-vectors $\mathbf{u_a}+\mathbf{u_b} = \mathbf{0}$ shown in Figure 5.34 since ($\mathbf{u_c} = \mathbf{0}$). Thus, we here have

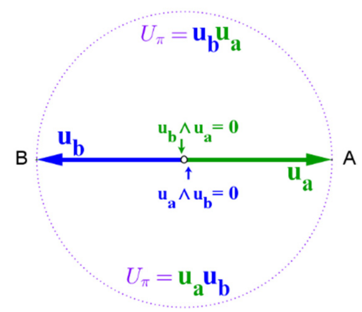

Figure 5.34 The anti-co-linear collapse of the circle plane to a straight line segment. Remark here that $U_{-\pi} = U_\pi = -1$.

(5.119) $\mathbf{u_b} \wedge \mathbf{u_a} = \mathbf{u_a} \wedge \mathbf{u_b} = 0$, and hereby

(5.120) $\left.\begin{aligned}\mathbf{u_b u_a} &= \mathbf{u_b}\cdot\mathbf{u_a} + \mathbf{u_b}\wedge\mathbf{u_a} = \mathbf{u_b}\cdot\mathbf{u_a} + 0 = -1\\ \mathbf{u_a u_b} &= \mathbf{u_a}\cdot\mathbf{u_b} + \mathbf{u_a}\wedge\mathbf{u_b} = \mathbf{u_a}\cdot\mathbf{u_b} + 0 = -1\end{aligned}\right\}$ and

Therefore

(5.121) $U_\pi = \mathbf{u_a u_b} = \mathbf{u_a u_b} = -1$, further $U_\pi \mathbf{u_a} = -\mathbf{u_a} = \mathbf{u_b}$,

which does not define a plane but the anti-co-linear reflection at a point, it is not a reflection along the line but internal in the line, a line ***pqg***-1 orientation inversion operation on 1-vectors, that corresponds to $\mathbf{u_b} = (-1)\mathbf{u_a} = -\mathbf{u_a}$. This is just the multiplication of a 1-vector with the scalar -1, as in § 4.4.2.5 (4.61). The rotor $U_\pi = -1$, which is completely invariant independently of any
rotation plan[240], is the operator which multiplies all geometric 1-vector subjects in space with -1.
Refer also to Figure 5.13 by formula (5.74) and (5.77) we get the *parity inversion operator*

(5.122) $\boldsymbol{ii} = \boldsymbol{i}^2 = -1 = U_\pi$.

When this acts on Euclidean ***pqg***-1-vectors

(5.123) $\bar{\mathbf{a}} = \boldsymbol{ii}\mathbf{a} = U_\pi\mathbf{a} = \boldsymbol{ii}\,\mathbf{a} = -\mathbf{a}$, for $\forall\mathbf{a}$, where $\mathbf{a}^2 \geq 0$ (Euclidean).

This is the case for all 1-vectors not only in the same plane due to the plane collapse for U_π illustrated in Figure 5.34 but in all space 𝔊 in the concept of physics. Therefore, we define

(5.124) $\overline{A} = \overline{A(\mathrm{a,b,c},\dots)} = A(\bar{\mathrm{a}}, \bar{\mathrm{b}}, \bar{\mathrm{c}},\dots) = A(-\mathrm{a},-\mathrm{b},-\mathrm{c},\dots)$ for ∀a,b,c,..., for all 1-vectors in A as a

[237] This refers to the causal sequence as it has been used since Aristotle introduced his syllogisms.
If you assign A,B,C clockwise seen from the front you must simply take the positive orientation seen from the back of the plane.
[238] The Mercedes ⦿ icon is turned −90° ⦿.
[239] If we doubt the physical existence of a cyclical process, the whole basis of scientific developments since Aristotle is impossible. Our ontology is based on the idea of the possibility of a cyclic ***entity***.
[240] that contains the 1-vector it operates on, that is inverted by the operation.

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multi-vector constructed on a basis of 1-vectors. More on this Parity inversion issue below.

5.3.1.3. The Orthonormal Basis for Circular Plane Symmetry

With association to parity inversion of a geometric 1-vector, we return to the eigenvalue concept for a unit bivector from formula (5.75)

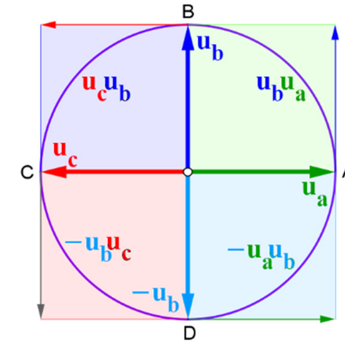

Figure 5.35 Unit circle with four successive orthonormal 1-vectors.

(5.125) $\widehat{\mathrm{B}}^2 = \boldsymbol{i}^2 = -|\boldsymbol{i}|^2 = -|\widehat{\mathrm{B}}|^2 = -1 \quad\Rightarrow\quad \widehat{\mathrm{B}} = \pm\boldsymbol{i} = \pm 1\boldsymbol{i}$

Point C in Figure 5.32 is moved so that it is opposite to the center origo O, so $\mathbf{u_c} = -\mathbf{u_a}$ and point C∼−A .
And B is moved so the two 1-vectors $\mathbf{u_a}$ and $\mathbf{u_b}$ are perpendicular and orthogonal $\mathbf{u_b}\cdot\mathbf{u_a} = 0$, and thereby $\mathbf{u_b u_a} = \mathbf{u_b}\wedge\mathbf{u_a}$.
We introduce a fourth 1-vector $-\mathbf{u_b}$ designating a fourth point D∼−B, all shown in Figure 5.35. The cyclical cause in the circle is then A, B, C, D or A, B, −A, −B.
The two 1-vectors $\mathbf{u_a}$ and $\mathbf{u_b}$ we rename as the standard designation for orthogonal geometric unit basis vectors $\boldsymbol{\sigma}_1 = \mathbf{u_a}$ and $\boldsymbol{\sigma}_2 = \mathbf{u_b}$, in accordance with section 5.2.6.
Hereby we get two inversed vectors $\mathbf{u_c} = -\boldsymbol{\sigma}_1$ and $-\mathbf{u_b} = -\boldsymbol{\sigma}_2$.
The plane ***direction*** is defined from the plane segment $\boldsymbol{i} = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ and thus, from the orthonormal basis set $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$.
Is origo O known, the set $\{\mathrm{O}, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$ as object determines the plane in space where the orthonormal basis set $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$ implies the parity inverse vectors $-\boldsymbol{\sigma}_1$ and $-\boldsymbol{\sigma}_2$ then the unit circle is spanned by the 1-vector set $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, -\boldsymbol{\sigma}_1, -\boldsymbol{\sigma}_2\}$ from the implicit origo O as the center is shown in Figure 5.36.[241]

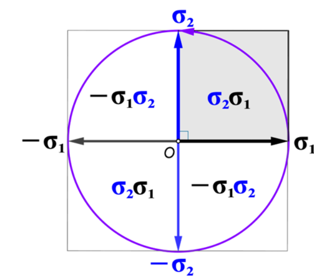

Figure 5.36 Four quadrants with the four intuit objects of identical indistinguishable unit bivectors $\boldsymbol{i}$ subjects. The bivector $\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 = -\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2$.

Once again, we remember that the unit basis bivector $\boldsymbol{i} = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ as a generator subject for the plane substance is rotational invariant in its plane, in addition, it is translation invariant throughout the natural geometric space 𝔊 of physics, just as it is in its own corresponding plane subject $\gamma_{\boldsymbol{i}} \in 𝔓$.

5.3.2. The Geometric Algebraic Complex Plane

The bivector subject, formed from an orthonormal basis set $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$ of 1-vectors objects Displayed in Figure 5.37, is the generator of the unit circle 1-rotor $U_\theta \coloneqq e^{\boldsymbol{i}\theta}$ from (5.89).
The angular scalar[242] θ multiply the bivector $\boldsymbol{i} = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ and act as a bivector argument in the exponential function, and thereby give the 1-rotor U_θ. Just as the operator $\boldsymbol{i}$ gives the operation $\boldsymbol{\sigma}_2 = \boldsymbol{i}\boldsymbol{\sigma}_1$, the operation $\mathbf{u}_\theta = e^{\boldsymbol{i}\theta}\boldsymbol{\sigma}_1$, or just $\mathbf{u} = U\boldsymbol{\sigma}_1$ gives a unit vector with a new ***pqg***-1 ***direction***, as a result from $\boldsymbol{\sigma}_1$.
This takes place in the plane object based on origo O with the basis $\{\mathrm{O}, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$ where the ***pqg***-2 ***direction*** for the plane is given by the object $\boldsymbol{i} = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 = (\overrightarrow{\mathrm{OB}}_{\mathrm{Im}})(\overrightarrow{\mathrm{OA}}_{\mathrm{Re}})$.
The 1-vector $\mathbf{u}_\theta$ designates a point P from O on the unit circle relative to the starting point $\mathrm{A_{Re}}$, where $\theta = \mathrm{arc}(\widetilde{\mathrm{A_{Re}P}})$.
By dilating the ***directional*** 1-vector $\mathbf{u}_\theta$ through multiplication with a real scalar ρ, we can form a colinear 1-vector $\mathbf{r} = \rho\mathbf{u}_\theta$, which from O designates

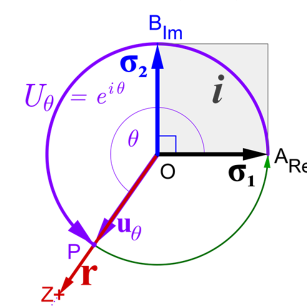

Figure 5.37 The complex plane spanned from the bivector $\boldsymbol{i} = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ by the polar real scalar coordinates (ρ, θ) by the 1-spinor $\rho U_\theta = \rho e^{\boldsymbol{i}\theta}$ of the S^1 circular plane symmetry or just by the abstract complex number $z = \rho e^{i\theta} \in \mathbb{C}$.

[241] The circle and its plane need only three points Figure 5.31 to be uniquely spanned. Here there is a surplus fourth point.
[242] Defined earlier above in § 5.1.1.5-5.1.1.8 and specified in formula (5.5).