5.2.9.2. The 1-Spinor as a Generator Radius-vector Multiplied to a Basis 1-vector

From the product of any two 1-vectors **a** and **b**, both of which can be written as a linear dilation of two unitary 1-vectors $\mathbf{u}_1, \mathbf{u}_2$, wherein $\mathbf{u}_2^2 = \mathbf{u}_1^2 = 1$, so that $\mathbf{a} = \alpha\mathbf{u}_1$ and $\mathbf{b} = \beta\mathbf{u}_2$, we can form the 2-multi-vector

(5.104) $\mathcal{Z} = \mathbf{ba} = \alpha\beta\mathbf{u}_2\mathbf{u}_1 = \alpha\beta U_\theta = \alpha\beta e^{\boldsymbol{i}\theta}$, $(\rho = \alpha\beta)$

We now dare to use the first 1-vector as a norm and thus set $\mathbf{u}_1 = \mathbf{a}$, and thereby $\alpha = 1$. Instead of **b** we use $\mathbf{r} = \mathbf{b} = \rho\mathbf{u}_2$, with $\rho = |\mathbf{r}| = |\mathbf{b}| = \beta$, we then write

(5.105) $\mathcal{Z} = \mathbf{ba} = \mathbf{r}\mathbf{u}_1 = \alpha\beta\mathbf{u}_2\mathbf{u}_1 = \rho\mathbf{u}_2\mathbf{u}_1 = \rho U_\theta = \rho e^{\boldsymbol{i}\theta}$

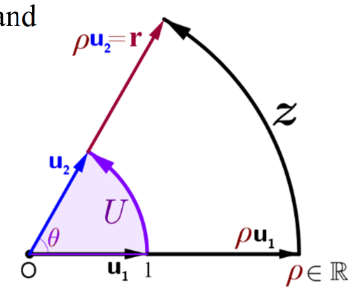

Figure 5.29 The complex *quantity* as a 2-vector $\mathcal{Z} = \mathbf{r}\mathbf{u}_1$.

The product of the two 1-vectors $\mathbf{r}\mathbf{u}_1$ expressing the rotation through space by the rotor U_θ, along the unit circle ○ ***direction***[235] as a base for that plane $\gamma_○$ joined with a dilation $\rho = |\mathbf{r}|$,

(5.106) $\mathbf{r}\mathbf{u}_1 = \mathcal{Z} = \rho U_\theta = \rho e^{\boldsymbol{i}\theta}$ $(= \mathbf{r}\cdot\mathbf{u}_1 + \mathbf{r}\wedge\mathbf{u}_1)$.

This expression (5.106) is only dependent on the relative angular relationship between the 1-vectors $\mathbf{u}_1$ and **r**, together with the relative magnitude ratio $\rho = |\mathbf{r}|/|\mathbf{u}_1|$.

A. Here $\mathcal{Z}$ can be interpreted as a circle $\overset{\frown}{\text{arc}}$ with ***direction*** at the radius ρ, or simply a **2-vector** $\mathbf{r}\mathbf{u}_1 = \rho U_\theta = \mathcal{Z}$, as a 1-*spinor* ***quantity*** of a plane substance in geometric space. To this, the traditional complex scalar $z = \rho e^{i\theta} \in \mathbb{C}$ for the ***quantity*** is just expressing an angle θ of rotation in a circle of radius ρ, in an abstract subject called the complex plane.

B. Alternative expressed $\mathbf{r}\mathbf{u}_1 = \mathbf{r}\cdot\mathbf{u}_1 + \mathbf{r}\wedge\mathbf{u}_1 = \langle\mathcal{Z}\rangle_0 + \langle\mathcal{Z}\rangle_2$ separated in a scalar part $\langle\mathcal{Z}\rangle_0 = ½(\mathcal{Z} + \mathcal{Z}^\dagger) = \mathbf{r}\cdot\mathbf{u}_1$ and a bivector part $\langle\mathcal{Z}\rangle_2 = ½(\mathcal{Z} - \mathcal{Z}^\dagger) = \mathbf{r}\wedge\mathbf{u}_1$ for the spinor. This encourages us unfortunate to transform to the real cartesian plane with two real coordinates $x = \text{Re}(\mathcal{Z}) = \langle\mathcal{Z}\rangle_0$ and $y = \text{Im}(\mathcal{Z}) = -\boldsymbol{i}\langle\mathcal{Z}\rangle_2$, from where $\mathcal{Z} \to (x, y) \to \text{Z}$ but this gives us only coordinates for a point Z in the plane, and not the spin operation. Here the corresponding complex number is $z = x + iy \in \mathbb{C}$.

Both formulations of the plane spinor have a ***quality*** structure created from the product of two ***primary qualities of first grade*** (***pqg***-1)⊗(***pqg***-1) ↔ $\mathbf{r}\mathbf{u}_1$ gives ***primary qualities of zero plus second grades*** (***pqg***-0)⊕(***pqg***-2) ↔ $\mathbf{r}\cdot\mathbf{u}_1 + \mathbf{r}\wedge\mathbf{u}_1$,

Here, the reader can compare the imaginary unit $i \in \mathbb{C}$ of complex numbers with the unit bivector $\boldsymbol{i}$ as the unit pseudoscalar of the plane geometric algebra.
I contend that both concepts $\boldsymbol{i} \leftrightarrows i \in \mathbb{C}$ can be interpreted as subjects, that for us to generate that substance of physics we call the plane concept.

5.2.10. The Circle Oscillating 1-rotor in Development Action as One Plane Substance

We presume we have an *angular development* following an angular parameter $\varphi \in \vec{\mathbb{R}}$. From (5.93) and (5.73) we then define a 1-rotor

(5.107) $U := e^{+\boldsymbol{i}\varphi} = \cos\varphi + \boldsymbol{i}\sin\varphi = \cos\varphi + \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\sin\varphi$,

that oscillates as a function $U_\varphi = U(\varphi)$ through a plane $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$.
Let this operate on $\boldsymbol{\sigma}_1$ by right multiplied by $\boldsymbol{\sigma}_1$, we have

(5.108) $U\boldsymbol{\sigma}_1 = (\cos\varphi + \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\sin\varphi)\boldsymbol{\sigma}_1 = (\cos\varphi)\boldsymbol{\sigma}_1 + (\sin\varphi)\boldsymbol{\sigma}_2 = \mathbf{u}$,

with the Cartesian coordinates $(\cos\varphi, \sin\varphi)$ from basis $\{\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2\}$ in Figure 5.30. Right multiplying once more by $\boldsymbol{\sigma}_1$ we achieve

(5.109) $U = U\boldsymbol{\sigma}_1\boldsymbol{\sigma}_1 = \mathbf{u}\boldsymbol{\sigma}_1 = \cos\varphi + \boldsymbol{i}\sin\varphi = e^{\boldsymbol{i}\varphi} \sim ⊙_{\boldsymbol{i}}^{\varphi}$.

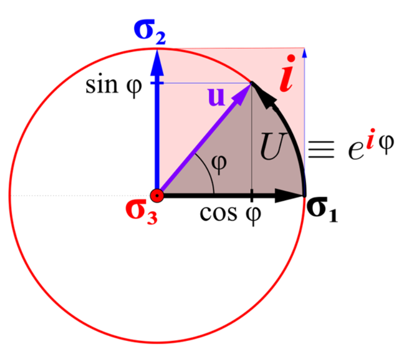

Figure 5.30 Circular oscillation plane $\boldsymbol{i} = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$.

Such multiplication operations teach us, what is intuitively going on in geometric algebra.

[235] The unit circle may itself define a plane in space or we can let a basis set $\{\text{O}, \mathbf{u}_1, \mathbf{u}_2\}$ define the circle plane ***direction***.

5.3. The Rotor Concept as the *Primary Quality of Even Grades* (*pqg*-0-2).

We have now seen that we intuit the plane concept from a rotation. By a *rotor* and a *dilation*. Previously page 154, § 5.1.1.2,k has alleged that the plane uniquely is defined by three points A, B, C which are not located on one straight line. The three points can as alleged in (n) and (o) shown in Figure 5.1 be circumscribed by a circle that has implicitly a center we call O. This circle can then be selected as a unit circle ○ for the plane, which forms the basis of the unitary rotor U.
The rotation around the circle ***direction*** is defined as positively orientated along the arc from A to B, that is, positively[236] around the triangle △OAB, or positively orientated around the triangle △CAB~△BCA~△ABC, circumscribed by the circle ○ABC through the three points A,B,C.
We make the circumscribed circle ○ABC to determine a locus situs, a place in a plane world.
I introduce an icon ⦿ as the symbol for this centred local interaction, in a circular defined plane γ_{ABC}. The three points A,B,C thus implies three unitary 1-vectors objects $\mathbf{u}_a = \overrightarrow{\text{OA}}$, $\mathbf{u}_b = \overrightarrow{\text{OB}}$ and $\mathbf{u}_c = \overrightarrow{\text{OC}}$, of form unit radius $\forall\mathbf{u}, |\mathbf{u}| = 1$ for the circumscribed unit circle wheel spanning the circle plane from the center O of the circle ○ABC, as shown in Figure 5.31.
The 1-rotor U from the arc $\overset{\frown}{\text{AB}}$ is defined as the product of the two 1-vectors $\overrightarrow{\text{OA}}$ and $\overrightarrow{\text{OB}}$,

(5.110) $U_{\overset{\frown}{\text{AB}}} = (\overrightarrow{\text{OB}})(\overrightarrow{\text{OA}}) = \mathbf{u}_b\mathbf{u}_a = e^{\boldsymbol{i}\theta} = U_\theta$, where $\theta = ∢\text{AOB} = \text{arc}(\overset{\frown}{\text{AB}})$,

and further through the two other rotation angles of the 1-rotors

(5.111) $U_{\overset{\frown}{\text{BC}}} = (\overrightarrow{\text{OC}})(\overrightarrow{\text{OB}}) = \mathbf{u}_c\mathbf{u}_b = e^{\boldsymbol{i}\varphi} = U_\varphi$

(5.112) $U_{\overset{\frown}{\text{CA}}} = (\overrightarrow{\text{OA}})(\overrightarrow{\text{OC}}) = \mathbf{u}_a\mathbf{u}_c = e^{\boldsymbol{i}\phi} = U_\phi$

We have $\phi + \varphi + \theta = 2\pi$, once around one circle, hence

(5.113) $U_\phi U_\varphi U_\theta = e^{\boldsymbol{i}\phi}e^{\boldsymbol{i}\varphi}e^{\boldsymbol{i}\theta} = e^{\boldsymbol{i}(\phi+\varphi+\theta)} = U_{\phi+\varphi+\theta} = (\boldsymbol{iiii}) = \boldsymbol{i}^4 = 1$

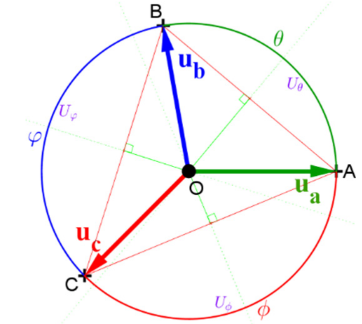

Figure 5.31 The three points of the circle.

The product of the three 1-rotor operators is the same as the product of the three 2-multi-vectors (six 2·3 1-vectors)

(5.114) $(\mathbf{u}_a\mathbf{u}_c)(\mathbf{u}_c\mathbf{u}_b)(\mathbf{u}_b\mathbf{u}_a) = \mathbf{u}_a\mathbf{u}_c\,\mathbf{u}_c\mathbf{u}_b\,\mathbf{u}_b\mathbf{u}_a = 1$

We simply use the product of the unitary multi-vectors, each consisting of two 1-vectors representing the rotors which are a great advantage, rather than splitting into scalars and bivector

(5.115)
$$\mathbf{u}_b\mathbf{u}_a = \mathbf{u}_b\cdot\mathbf{u}_a + \mathbf{u}_b\wedge\mathbf{u}_a$$
$$\mathbf{u}_c\mathbf{u}_b = \mathbf{u}_c\cdot\mathbf{u}_b + \mathbf{u}_c\wedge\mathbf{u}_b$$
$$\mathbf{u}_a\mathbf{u}_c = \mathbf{u}_a\cdot\mathbf{u}_c + \mathbf{u}_a\wedge\mathbf{u}_c$$

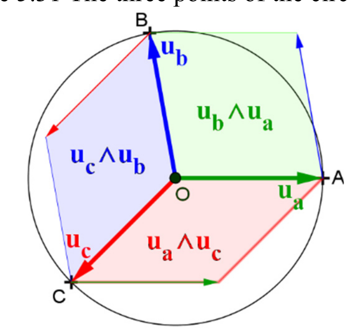

Figure 5.32 The three bivectors.

If we look at three sectors central symmetrical circle, we have

(5.116)
$$\mathbf{u}_b\mathbf{u}_a = \mathbf{u}_b\cdot\mathbf{u}_a + \mathbf{u}_b\wedge\mathbf{u}_a = -\tfrac{1}{2} + \mathbf{u}_b\wedge\mathbf{u}_a$$
$$\mathbf{u}_c\mathbf{u}_b = \mathbf{u}_c\cdot\mathbf{u}_b + \mathbf{u}_c\wedge\mathbf{u}_b = -\tfrac{1}{2} + \mathbf{u}_c\wedge\mathbf{u}_b$$
$$\mathbf{u}_a\mathbf{u}_c = \mathbf{u}_a\cdot\mathbf{u}_c + \mathbf{u}_a\wedge\mathbf{u}_c = -\tfrac{1}{2} + \mathbf{u}_a\wedge\mathbf{u}_c$$

In this symmetrical circle plane $\mathbf{u}_a + \mathbf{u}_b + \mathbf{u}_c = \mathbf{0}$, the sum of these three 1-vectors does not contribute to a translation. But the three identical 1-rotors

(5.117) $U_{\frac{2\pi}{3}} = \mathbf{u}_b\mathbf{u}_a = \mathbf{u}_c\mathbf{u}_b = \mathbf{u}_a\mathbf{u}_c$

causing a combined full turn of the circle and thereby

(5.118) $\left(U_{\frac{2\pi}{3}}\right)^3 = U_{2\pi} = U_0 \sim 1$, see Figure 5.33,

as modulo of the three central symmetrical rotations.

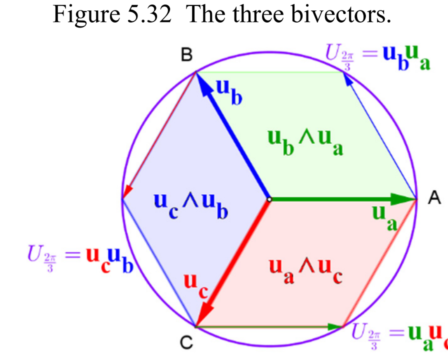

Figure 5.33 The three-sector central symmetrical circle combined of three equal rotations by the **rotor** $U_{\frac{2\pi}{3}}$.

[236] If the triangle ABC is not orientated counterclockwise simply change the view to the opposite side of the plane γ_{ABC}(paper).