The operator $\boldsymbol{i}$ acts on 1-vectors that is rotated anti-clockwise to its perpendicular:

(5.77)
$$\begin{aligned}
\boldsymbol{i}\boldsymbol{\sigma}_1 &= +\boldsymbol{\sigma}_2, & \boldsymbol{i}^1 &= +\boldsymbol{i} = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\\
\boldsymbol{i}\,\boldsymbol{i}\boldsymbol{\sigma}_1 &= -\boldsymbol{\sigma}_1, & \boldsymbol{i}^2 &= -1\cdot = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 = -\boldsymbol{\sigma}_2\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\boldsymbol{\sigma}_1\\
\boldsymbol{i}\,\boldsymbol{i}\boldsymbol{i}\boldsymbol{\sigma}_1 &= -\boldsymbol{\sigma}_2, & \boldsymbol{i}^3 &= -\boldsymbol{i} = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 = -\boldsymbol{\sigma}_2\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\\
\boldsymbol{i}\,\boldsymbol{i}\boldsymbol{i}\boldsymbol{i}\boldsymbol{\sigma}_1 &= +\boldsymbol{\sigma}_1, & \boldsymbol{i}^4 &= +1\cdot = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_2\boldsymbol{\sigma}_2\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1\boldsymbol{\sigma}_1\boldsymbol{\sigma}_1\boldsymbol{\sigma}_1
\end{aligned}$$

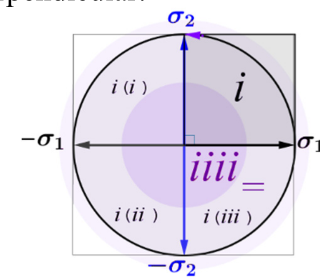

Figure 5.13. Four operations with $\boldsymbol{i}$ in the co-plane trough the four 1-vector objects give one cycle in the unit circle.

Using the unit bivector operation $\boldsymbol{i}$ four times in the same plane the 1-vector is turned once around the unit circle to itself.[225]

— Modulo[226] for the unit circle is $(\boldsymbol{iiii})$.

Here, I define the cyclic counting operator $\mathbf{1}=(\boldsymbol{iiii})$ for each time there has been a full rotation of the circle, the result is identical
$\mathbf{1}^n\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_1 \;\Leftrightarrow\; \boldsymbol{i}^{4n}\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_1,\; n\in\mathbb{N}$.
– Something happens in space. – One recognized the 1-vector once again n times. –[227]
The operator $\boldsymbol{ii} = \boldsymbol{i}^2 = -1\cdot$ is an inversion (reverse) operator for geometric 1-vectors just as scalar multiplication by $(-1)\in\mathbb{R}$. We have $\mathbf{a}\to\boldsymbol{ii}\mathbf{a} = \boldsymbol{i}^2\mathbf{a} = -1\cdot\mathbf{a} = -\mathbf{a}$. The operation switches the orientation of ***pqg***-1 ***direction***, also called an additive parity inversion or negation.
In the cyclical process; the negation of the negation $(\boldsymbol{ii})(\boldsymbol{ii}) = \boldsymbol{iiii} = \mathbf{1}$ is an involution and leads back to the same $\mathbf{a}\to\boldsymbol{iiii}\mathbf{a} = \mathbf{a}$, but the result is one cyclic count of times.
The cyclic process hereby enables a timing process.[228]

5.2.6.4. The Form Structure of the Plane Subject $\boldsymbol{i}$ has Arbitrary Shaped Objects

As a concept, the unit-bivector $\boldsymbol{i}$ is the generating plane segment for the plane concept. The ***direction*** of $\boldsymbol{i}$ determines the plane ***direction***, and when $\boldsymbol{i}$ operates on a 1-vector ***direction*** $\boldsymbol{\sigma}_1$ in the plane of $\boldsymbol{i}$, this 1-vector is rotated into a second 1-vector ***direction*** $\boldsymbol{\sigma}_2 = \boldsymbol{i}\boldsymbol{\sigma}_1$, whereby the basis set $\{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2\}$ of 1-vectors gives the ***direction*** of the plane following $\boldsymbol{\sigma}_2\boldsymbol{\sigma}_1 = \boldsymbol{i}$. The intuition of this object is synonymous with a ***directional*** *circle* object $\boldsymbol{i}$ with radius $\frac{1}{\sqrt{\pi}}$ and surface area 1 as shown in Figure 5.14, and the amoeba object $\boldsymbol{i}$:

- $\boldsymbol{i}$ is synonymous with any ***directional*** plane *amoeba* area, which has ***direction as the primary quality of second grade*** (***pqg***-2).
- The generating plane segment, unit bivector $\boldsymbol{i}$ as subject, has always the objective ***quantitative*** magnitude $|\boldsymbol{i}| = 1$.

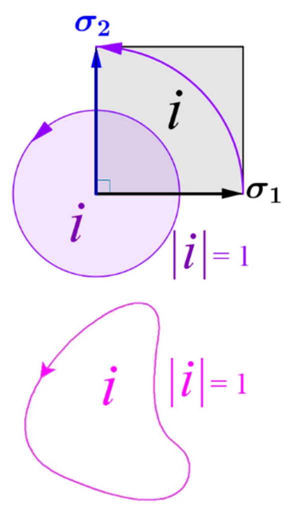

Figure 5.14 Example objects for the shapeless unit bivector $\boldsymbol{i}=\boldsymbol{i}=\boldsymbol{i}$ as a plane segment subject.

[225] Note the sequential left operation. Alternatively, a similar right operation with $i = -\boldsymbol{i} = \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2$ will perform the same.[223]
[226] Note that the modulo $(\boldsymbol{iiii})$ has the plane segment area $|\boldsymbol{iiii}| = 1$, while the area of the unit circle is π, and plane segment area count $4|\boldsymbol{i}|$, the four times unit − square area, i. e. 4. But any way, the result $\boldsymbol{iiii}\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_1$ is still a 1-vector, counted once.
[227] My opinion is: this is fundamental to catching any information.
[228] The negation of the negation is a philosophical expression by Hegel used by Frederick Engels in 1877 in Anti-Dühring: The negation of the negation is not necessarily a cancellation, this makes the development of history possible.

5.2.6.5. The Unit-bivector $\boldsymbol{i}$ Multiplied by a 1-vector

Given an arbitrary 1-vector $\mathbf{a}$, selecting a unit-base-vector $\boldsymbol{\sigma}_1 = \hat{\mathbf{a}}$ in the same ***direction***, that $\mathbf{a}$ can be expressed as a scalar $\alpha\in\mathbb{R}$ multiplied unit-base-vector

(5.78) $$\mathbf{a} = \alpha\boldsymbol{\sigma}_1$$

When the operator $\boldsymbol{i}$ acts on the 1-vector from left we get $\boxed{\boldsymbol{i}\mathbf{a} = \mathbf{b}}$,
where $\mathbf{b} = \alpha\boldsymbol{\sigma}_2$ and $\mathbf{b}^2 = \mathbf{a}^2$. But when it acts from right, we get $\boxed{\mathbf{a}\boldsymbol{i} = -\mathbf{b}}$.
When we multiply these equations by the inverse 1-vector $\mathbf{a}^{-1}$ defined by $\mathbf{a}^{-1}\mathbf{a} = \mathbf{a}\cdot\mathbf{a}^{-1} = 1$, where $\mathbf{a}^{-1}$ is co-linear to $\mathbf{a}$, as follows from the right $\boldsymbol{i}\mathbf{a}\mathbf{a}^{-1} = \mathbf{b}\mathbf{a}^{-1}$, and the left $\mathbf{a}^{-1}\mathbf{a}\boldsymbol{i} = -\mathbf{a}^{-1}\mathbf{b}$, we get

(5.79) $$\boxed{\boldsymbol{i} = \mathbf{b}\mathbf{a}^{-1}} \quad\text{and}\quad \boxed{-\boldsymbol{i} = \mathbf{a}^{-1}\mathbf{b}}$$ as illustrated in Figure 5.15

Based on the premise for this we have $\mathbf{b}^2\mathbf{a}^{-2} = 1$, in that $\alpha^2\alpha^{-2} = 1$
See also Figure 5.12 at formula (5.73) for rotation orientation.

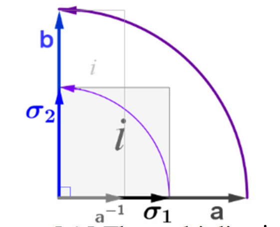

Figure 5.15 The multiplicative inverse (reciprocal) 1-vector in (5.79) for the unit area $\boldsymbol{i}$.

5.2.7. The Unitary Rotor Operator as a Concept

We have seen that the operator $\boldsymbol{i}$ rotates a geometric 1-vector $\boldsymbol{\sigma}_1$ to the orthogonal $\boldsymbol{\sigma}_2$.
We associate this with the operator concept called a rotor U_θ for a rotation through a plane ***direction*** with a dedicated angle θ.
Especially the orthogonal rotation $U_\perp = \boldsymbol{i} = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_1$ through the plane-segment as we intuit with the square object $\boldsymbol{\sigma}_2\perp\boldsymbol{\sigma}_1$ which we see as the generator unit for a possible rotation in the plane or even the plane concept itself.

5.2.7.1. The Geometric Rotor in the Euclidean Plane

We are now looking at two unit-1-vectors $\mathbf{u}_1, \mathbf{u}_2$ with a mutual angle θ, spanning a plane from an origo point O, like in § 5.2.2.3.
For the unitary 1-vectors applying $\mathbf{u}_2{}^2 = \mathbf{u}_1{}^2 = 1 \Rightarrow |\mathbf{u}_2| = |\mathbf{u}_1| = 1$, as well as $\boldsymbol{i}^2 = -1$. See an object in Figure 5.17. From (5.49) we have

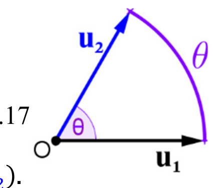

Figure 5.17 Angle $\angle(\mathbf{u}_1,\mathbf{u}_2)$.

(5.80) $$\cos\theta \coloneqq \mathbf{u}_2\cdot\mathbf{u}_1 \in \mathbb{R}_{\text{pqg-0}},$$ and further

(5.81) $$\boldsymbol{i}\sin\theta \coloneqq \mathbf{u}_2\wedge\mathbf{u}_1 \in \boldsymbol{i}\,\mathbb{R}^1_{\boldsymbol{i},\text{pqg-2}},$$

which by squaring $-\sin^2\theta \coloneqq (\mathbf{u}_2\wedge\mathbf{u}_1)^2$ justified by (5.62).
See Figure 5.20 for an illustration of $\cos\theta$ and $\sin\theta$.

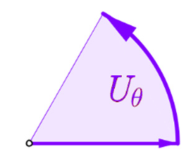

Figure 5.18 **Rotor** U_θ.

The geometric product (5.59) is defined for the two unitary 1-vectors

(5.82) $$\mathbf{u}_2\mathbf{u}_1 = \mathbf{u}_2\cdot\mathbf{u}_1 + \mathbf{u}_2\wedge\mathbf{u}_1$$

We now for $\forall\theta\in\mathbb{R}$ define a **rotor** from this geometric product

(5.83) $$U_\theta = \mathbf{u}_2\mathbf{u}_1 = \mathbf{u}_2\cdot\mathbf{u}_1 + \mathbf{u}_2\wedge\mathbf{u}_1 = \cos\theta + \boldsymbol{i}\sin\theta \coloneqq e^{+\boldsymbol{i}\theta}$$

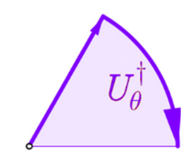

Figure 5.19 *Revers* **rotor** $U_\theta^\dagger$.

The *reverse rotor* is then

(5.84) $$U_\theta^\dagger = \mathbf{u}_1\mathbf{u}_2 = \mathbf{u}_1\cdot\mathbf{u}_2 - \mathbf{u}_2\wedge\mathbf{u}_1 = \cos\theta - \boldsymbol{i}\sin\theta \coloneqq e^{-\boldsymbol{i}\theta}$$

Together, the rotor and its reverse are the *identical* operators

(5.85) $$U^\dagger U = UU^\dagger = 1 \Leftrightarrow \mathbf{u}_2\mathbf{u}_1\mathbf{u}_1\mathbf{u}_2 = U_\theta U_\theta^\dagger = e^{\boldsymbol{i}\theta}e^{-\boldsymbol{i}\theta} = e^{\boldsymbol{i}0} = 1.$$

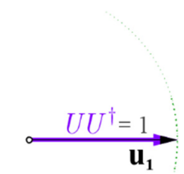

Figure 5.16 *Identity* **rotor**.

Condition $U^\dagger U = UU^\dagger = 1$ is the requirement for a unitary operator.
Thus, we have the identical operation $UU^\dagger\mathbf{u}_1 = \mathbf{u}_1$, displayed in Figure 5.16
As the rotor U_θ is unitary $U_\theta U_\theta^\dagger = 1$, and the magnitude of the rotor is $|U| = 1$.
Such simple rotors in one plane are called 1-rotors for only one plane ***grade***-2 ***direction***.