– Two angles may be compared relative to each other, but the normalization has its cause in radius. E.g., the scalar for a straight angle is in all cases *absolute* π (or 180°).
– The *absolute norm* for the idea of a unit circle is its radius.
Whence, *phase angles are absolute autonomous* in a cyclic development.

5.1.1.8. Addition of angular quantities

We have seen angle ***quantity*** $[\mathbb{R}_{pqg\text{-}2}]$ represented by real numbers $\phi \in \mathbb{R}_{pqg\text{-}2}$.
However, the angle ***pqg*-2 *quality*** is cyclical singular the same as the arc of the unit circle centred at the angle origo O. The three arcs shown in Figure 5.1,o. are added to one turn in the circle $\angle AOB + \angle BOC + \angle COA = ○ABC = ○$,
whose real scalar ***quantities*** are added to $∡AOB + ∡BOC + ∡COA = ∡○ = 2\pi$,
and the subtend for planar triangles is $∡CAB + ∡ABC + ∡BCA = ∡\triangle ABC = \pi$.
The a priory π per se for the plane idea. The addition of angles in the same plane is *absolute*.
The real scalar of the angular measure is periodic modulo 2π, as it represents an angle around the circle, as shown in Figure 5.3.
The secondary point P of the angle is represented by $\phi \in \mathbb{R}$ in this way

(5.6) $$\phi \to P(\phi) = P(\phi_0) \quad \text{for} \quad \forall\phi = \phi_0 + 2\pi n, \quad \text{where} \quad ∡AOP = \phi_0 \in [0,2\pi[\;\wedge\; \forall n \in \mathbb{Z}$$

From the angle primary object 1-vector $\mathbf{e}_1 = \overrightarrow{OA}$ (from starting points O and A) the scalar $\phi \in \mathbb{R}_{pqg\text{-}2}$ ***quantity*** designates a point P on the circle ○, as we remember that the circle-specific ***pqg*-2 *quality*** is given in the plane given by ○ABC in our intuition. Since the real numbers are additive the angle ***quantity*** of compound angles will be the sum of the individual angular ***quantities*** in *one and the same* plane of the angular ***direction quality***.

5.1.1.9. The Angular Quantity as a Sector Area

Together with the unit circle, two different unit 1-vectors, $\mathbf{u}_1 \neq \mathbf{u}_2$, where $|\mathbf{u}_1| = |\mathbf{u}_2| = 1$, forming an angle $\angle(\mathbf{u}_1, \mathbf{u}_2)$, whose ***quantity*** is $\text{arc}(\mathbf{u}_1, \mathbf{u}_2) = \text{arc}_\theta = \theta = ∡(\mathbf{u}_1, \mathbf{u}_2)$
an arcus-measure of the unit circle. As known for the unit circle r=1,
the circumference is $2\pi r = 2\pi$, and the area is $\pi r^2 = \pi 1^2 = \pi$.
Angular sector area starting from 1-vector $\mathbf{u}_1$ to $\mathbf{u}_2$ relate to the total area for the unit circle as the ratio of the angle of the arc to the entire circumference.
This ratio is multiplied by the entire area of the circle

(5.7) $$A_{\angle(\mathbf{u}_1,\mathbf{u}_2)} = A_\theta = \frac{\text{arc}_\theta}{2\pi}\pi 1^2 = \frac{\theta}{2\pi}\pi = \frac{1}{2}\theta\,. \quad \text{Thus,}$$

the angular ***quantity*** is a double measure of the sector area

(5.8) $$A_{\angle(\mathbf{u}_1,\mathbf{u}_2)} = \frac{\theta}{2} = \frac{1}{2}∡(\mathbf{u}_1,\mathbf{u}_2), \qquad \text{see Figure 5.5}$$

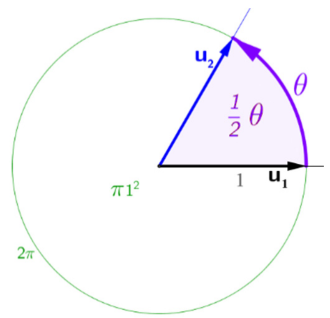

Figure 5.5 The sector area in the unit circle as a half measure for the angular ***quantity***.

We see that the intuition of angular ***quantity*** $[\mathbb{R}_{pqg\text{-}2}]$ can be viewed both as an arc measure and an area measure.
By this, we deduced that both the angle concept and the area concept are primary ***qualities of second grade*** (***pqg***-2).

Two linearly independent 1-vectors form an angle, i.e., two 1-vectors $\mathbf{a} \neq \mathbf{b}$ form an angle $\angle(\mathbf{a},\mathbf{b})$, where $∡(\mathbf{a},\mathbf{b}) \neq n\pi$ for $\forall n \in \mathbb{Z}$, when $\mathbf{a} \neq \lambda\mathbf{b}$, for $\forall\lambda \in \mathbb{R}$.
When $\hat{\mathbf{a}} \| \hat{\mathbf{b}}$ area $A_{\angle(\hat{\mathbf{a}},\hat{\mathbf{b}})}$ is a priori undefined as that pure ***pqg*-1 *quality***.

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5.1.1.10. The perpendicular tangent to the circle in the plan

We look at the plane making the foundation for the circle with a radius given by the two points O and A, shown in Figure 5.6.
This primary specific 1-vector $\overrightarrow{OA}$ is a ***pqg*-1 *quality***.
The circle magnitude $\alpha = |\overrightarrow{OA}| \in \mathbb{R}_+$ is a scalar.
Therefore, the magnitude is a ***pqg*-0 *quality***.
Looking at Figure 5.1,q. and Figure 5.6 from the propositions E III.Pr.18.-19. we get the tangent to the circle at point A.

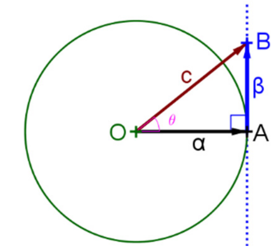

Figure 5.6 Circle with a tangent coordinate.

We can form a second 1-vector $\overrightarrow{AB}$ along the tangent of the circle perpendicular to the radius 1-vector $\overrightarrow{AB} \perp \overrightarrow{OA}$.
The magnitude of this $\beta = |\overrightarrow{AB}| \in \mathbb{R}_+$ is too a ***pqg*-0 scalar *quality***.
We are imaging implicit a unit 1-vector ***direction*** $\mathbf{e}_\beta = \overrightarrow{AB}/|\overrightarrow{AB}|$ from which we span the tangent $\{B|\overrightarrow{AB} = \beta\mathbf{e}_\beta, \text{ for } \forall\beta \in \mathbb{R}\}$. The trigonometric coordinate $\beta = \alpha \tan\theta$ for the tangent form a second 1-vector $\beta\mathbf{e}_\beta$ that has a ***pqg*-1 *quality*** for the tangent.
The circular arc θ along the tangent has a ***pqg*-2 *quality***, therefore a physical tangent inherits a ***primary quality of second grade***.
By that, the idea of a second coordinate β a priori is dependent on ***pqg***-2. For the right-angled triangle, we have $c^2 = \alpha^2 + \beta^2$, where $c = |\overrightarrow{OB}|$ is the magnitude of the indirect vector $\overrightarrow{OB}$.
The ***quality*** of the three 1-vectors $\overrightarrow{OA}$, $\overrightarrow{AB}$ and $\overrightarrow{OB}$ is ***pqg***-1, while the ***quality*** of the magnitude α of the circle is ***pqg***-0 as the first polar coordinate $\rho = \alpha$, while the second polar co-ordinate along the arcus tangent has a ***pqg*-2 *quality*** in its angular cause $\theta = \tan^{-1}(\beta/\alpha)$.

– An a priori ***quality*** principal ***category*** of the plane concept and a ***quantitative*** coordination, ρ from a center, and θ around that center. –