5.1.1.4. The Concept of Different Angles

A ***category***: 1. Angles may be the same or different. 2. An individual angle can be divided into several angles, and 3. Different angles can be combined into one angle. These are the three criteria for a ***quantity category*** by Immanuel Kant. Then we can intuit the concept of a ***quality*** called angles in space 𝔖 as a ***category*** of angular ***quantity*** in physics.

5.1.1.5. The Concept Circular Arc of Angle

An angle β is formed by two half-line rays r_1 and r_2 with different ***directions*** from a point O. Given a point A on r_1 that the 1-vector $\mathbf{e}_1 = \overrightarrow{OA}$ represent the primary ***direction*** that we assign the magnitude $|\mathbf{e}_1| = |\overrightarrow{OA}| = 1 \in \mathbb{R}$. This magnitude is selected as the radius of a circle with a center in O with the ray r_1 intersecting the circle at point A, and the ray r_2 intersecting at point B, See Figure 5.2. This circle is called a unit circle. We assign the angle a scalar ***quantity*** corresponding to the unit circle arc_{AB},

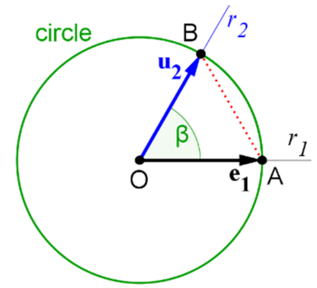

Figure 5.2 The circle of an angle.

(5.3) $$\beta_{1,2} = \text{arc}_{AB} = \frac{|\widehat{AB}|}{|\overrightarrow{OA}|} = \quad |\widehat{AB}| \quad > |AB| \quad > 0.$$

This scalar radian arc measure is relative to $|\mathbf{e}_1| = |\overrightarrow{OA}| = 1$.

The secondary ***directional*** 1-vector $\mathbf{u}_2 = \overrightarrow{OB}$ along r_2, $\mathbf{u}_2 \neq \mathbf{e}_1$ has as a result of the circle also the magnitude $|\mathbf{u}_2| = |\overrightarrow{OB}| = |\mathbf{e}_1| = 1$.

The circle is in the plane γ_{OAB} defined by the three points O, A, B. –

All straight lines ℓ_{OP} through O intersect the unit circle in a point P is in the plane γ_{OAB} like the ray $r_u = r_{OP} \subset \ell_{OP}$ is in the plane γ_{OAB} of the circle. An arbitrary ray r_{OP} gives an angle ∠AOP concerning the ray r_1 of the primary given ***directional*** 1-vector

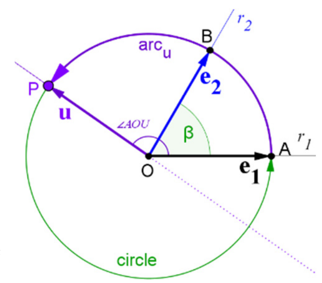

Figure 5.3 The angular ($\widehat{\text{arc}}$) in the unit circle.

$\mathbf{e}_1 = \overrightarrow{OA}$. The idea of the plane $\gamma_{\mathbf{e}_1,\mathbf{e}_2} = \gamma_{OAB}$ for all of these angles ∠$(\mathbf{e}_1, \mathbf{u})$ is given as a span from two unit ***pqg***-1-vectors $\mathbf{e}_1 = \overrightarrow{OA}$ for the primary ***direction*** and $\mathbf{e}_2 = \mathbf{u}_2 = \overrightarrow{OB}$ for the second ***direction*** from point O.

5.1.1.6. The Concept of the Primary Quality of Second Grade (*pqg*-2)

The triangles △ABC ⊂𝔖 from §o. to the triangle △AOB ⊂𝔖 with the angle ∠AOB from §p. (E III.Pr.20.) constitute figures in space.

We now look specifically at the angle of object ∠AOB in space 𝔖.

We claim that the lines OA and OB are not parallel and intersect each other at point O. The negation *non-parallel* opens the definition of a ***quality*** we call the ***direction*** of an angle.

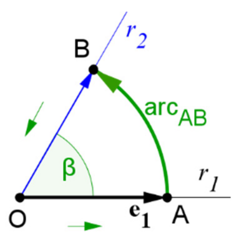

Figure 5.4 Angular ***direction*** expressed sequentially as O,A,B ~ A,B,O ~ B,O,A.

In Figure 5.4 the ***direction*** from the point O through the 1-vector $\mathbf{e}_1 = \overrightarrow{OA}$ over the point A through the arc_{AB} to point B (designated by the 1-vector $\mathbf{e}_2 = \overrightarrow{OB}$, Figure 5.3), where the cyclic orientation of ***direction*** is indicated as the *sequences* O,A,B,O ~ A,B,O,A ~ B,O,A,B.

In postulate 5.1.1.2,k. the ***direction*** is given by the triangle △ABC, etc. With center O.

This triangle may be circumscribed by a circle, as shown in Figure 5.1,n. and o. This circle defines a rotation around the center O, this circle rotation ***direction*** is determined by the points A, B, C, in this drawing progressive ↺ anti-clockwise in orientation. The angular ***direction*** ∡O=∡AOB is defined as the same as in circle rotation ↺ABC shown in Figure 5.1,o., that is, from A to B as seen from the center O around the circle ○ABC, note the arrows in Figure 5.4. ↖,↙,→.

The possibility of the concept of an angular ***direction*** ∡O=∡AOB is a ***category*** called:

A ***primary quality of second grade*** (***pqg***-2). Angles shape the plane substance 𝔓 idea.

– The ***categorial pqg***-2 ***quality*** has as consequences the possibility of the angular objects in Figure 5.2, Figure 5.3, Figure 5.4 and all other angular objects are to be shown for intuition.

Research on the a priori of Physics

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Jens Erfurt Andresen

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5.1.1.7. The Quantity of an Angle

Angles have different sizes, e.g., an *acute*, *obtuse, or right angle*. (E I.De.21.)

The angular relative ***quantity*** is defined from a circular arc measured by the radius defined as 1.

The scalar radian arc measure (5.3) $\beta = \text{arc}_{AB} = |\widehat{AB}| \in \mathbb{R}_+$ is formed by the concept of the positive real numbers from the arc length $|\widehat{AB}|$ $[radius^{-1}]$ of the arc $\widehat{AB}$.

The ***quantitative*** order of the real numbers is coordinated with the order of points along the arc, thus also a ***quantitative*** order of points. In this way the concept of the real numbers ℝ is associated with the angle concept. Like the straight linear ray had a parametric equation in Section 4.4.2.8, the relative radian arc we endow with a real scalar parameter representation, that in its whole constitute the unit circle, as shown in Figure 5.3

(5.4) $$\circ ABC = \left\{\forall P \,\middle|\, |\widehat{AP}| = \text{arc}_{AP} = \phi|OA| = \phi|\mathbf{e}_1| = \phi \in \mathbb{R}_+,\ \text{radius} = |OP| = |OA| = |\mathbf{e}_1| = 1, (\exists P_B = B)\right\}$$

This parameter ϕ represents real scalar angle ***quantity*** and produces thus the development of the angle ∠AOP as a linear function of the parameter scalar $\phi \in \mathbb{R}$.

All points $P(\phi)$ develops the circle $P(\phi) \in \circ ABC \subset \gamma_{ABC}$. This development is called an *angle of rotation* or just a *circle rotation* in the plane $\gamma_{ABC} = \gamma_{AOB}$ roundabout the point O.

The scalar ***quantity*** of an angle ∠AOB is expressed as ∡AOB $= \beta \in \mathbb{R}$.

The ***direction*** of the angle ∡AOB is considered *positive orientated* here in the alphabetic order O,$\widehat{AB}$ → ∡AOB $= \beta = \text{arc}_{AB} = |\widehat{AB}| \geq 0$, around the center O. The reverse order is considered as a *negative orientation* ∡BOA $= \text{arc}_{BA} = -\text{arc}_{AB} = -|\widehat{AB}| \leq 0$, that is, ∡AOB = −∡BOA for a scalar associated with the relative angular arc.

The ***pqg***-2 ***direction quality*** of the angular concept in a plane has the relative real ***quantity*** $\beta \in \mathbb{R}$ where the sign represents the *orientation* of the *rotation* ***direction***.

For the spatial ***direction*** of an angle object ∠AOB we have the reverse *orientation* $(∡AOB)^\dagger =$ ∡BOA . Cyclic we expressed this iconic as ↺†= ↻ .

– Looking at the face of a clock, we know immediately which way the hands shall move, but when we imagine we see the clock face from the back (seen through a transparent face) we know that the display hands move the same way around, but the rotation orientation is reversed seen from our alternative viewpoint behind the clock. Is this the so-called 'time reversal' mirror?

The ***quality*** of angular ***direction*** has fundamentally only relative meaning in forming its plane.

The relationship between line segments OA and OB express the angle ∠AOB.

The ***quality direction*** in the plane can be ***quantified*** relatively as an angular real scalar, where positive values indicate one orientation of the rotation, while the negative gives the reverse.

For further understanding of the angle concept please refer to the literature on plane geometry, trigonometry, and mathematics. Here we only emphasise that the concept ***direction*** of a plane substance for the intuition a priori in its idea is singular, as a tabula object: Its defining angle starting from one given point O as the origo, and two other points A and B are necessary.

The order of real numbers also implies an order of angles in one plane; however, the angular arc size has a periodicity along the circle circumference. The arc is just not straight lines and has **no *primary quality of the first grade***, but only a ***category of second grade*** (***pqg***-2).

Assuming that the measuring unit is $|\mathbf{e}_1| = |\mathbf{e}_2| = |OA| = |OB| = 1 \in \mathbb{R}$ etc. $|\mathbf{u}| = 1$, we apply:

- The circular arc $\widehat{AP}$ belongs to the ***primary quality of second grade*** (***pqg***-2).
- The circular arc $\widehat{AP}$ is a scalar ***quantity*** ∢AOP $= \arg(\widehat{AP})/|OA| = \phi \in \mathbb{R}$.

This is a measure relative to the ***pqg***-1 ***quality***, which is the $[\mathbb{R}^1_{+\text{pqg-1}}]$ ***quantity***, that we can always assume normalized |OA|=1 to the unit circle radius. Thereby the angle measure

(5.5) $$\beta = ∢(\mathbf{e}_1, \mathbf{e}_2) = ∢(r_1, r_2) = ∢(r_{OA}, r_{OB})$$ is an *absolute scalar **quantity*** $[\mathbb{R}_{\text{pqg-2}}]$.