Restricted to brief peruse for research, reviews, or scholarly analysis, © with required quotation reference: ISBN-13: 978-8797246931

© Copyrighted material from hardback: ISBN-13: 978-8797246931, paperback: ISBN-13: 978-8797246948, Kindle and PDF-file: ISBN-13: 978-8797246917

E I.Po.3. To describe a circle with any center and radius.
E I.Po.4. That all right angles equal one another.
E I.Po.5. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.
E I.Pr.11. To draw a straight line at right angles to a given straight line from a given point on it.
E I.Pr.12. To draw a straight line perpendicular to a given infinite straight line from a given point not on it.
E I.Pr.15. If two straight lines cut one another, then they make the vertical angles equal to one another.
Corollary. If two straight lines cut one another, then they will make the angles at the point of section equal to four right angles.
E I.Pr.16. In any triangle, if one of the sides is produced, then the exterior angle is greater than either of the interior and opposite angles.
E I.Pr.27. If a straight line falling on two straight lines makes the alternate angles equal to one another, then the straight lines are parallel to one another.
E III.De.1. *Equal circles* are those whose diameters are equal, or whose radii are equal.
E III.De.2. A straight line is said to *touch* a circle which, meeting the circle and being produced, does not cut the circle.
E III.Pr.1. To find the center of a given circle.
Corollary. If in a circle a straight line cuts a straight line into two equal parts and at right angles, then the center of the circle lies on the cutting straight line.
E III.Pr.2. If two points are taken at random on the circumference of a circle, then the straight line joining the points falls within the circle.
E III.Pr.3. If a straight line passing through the center of a circle bisects a straight line not passing through the center, then it also cuts it at right angles; and if it cuts it at right angles, then it also bisects it.
E III.Pr.4. If in a circle two straight lines which do not pass through the center cut one another, then they do not bisect one another.
E III.Pr.5. If two circles cut one another, then they do not have the same center.
E III.Pr.6. If two circles touch one another, then they do not have the same center.
E III.Pr.18. If a straight line touches a circle, and a straight line is joined from the center to the point of contact, the straight line so joined will be perpendicular to the tangent.
E III.Pr.19. If a straight line touches a circle, and from the point of contact a straight line is drawn at right angles to the tangent, the center of the circle will be on the straight line so drawn.
E III.Pr.20. In a circle the angle at the center is double the angle at the circumference when the angles have the same circumference as base.
E IV.Pr.2. To inscribe in a given circle a triangle equiangular with a given triangle.
E IV.Pr.5. To circumscribe a circle about a given triangle.
E XI.Pr.2. If two straight lines cut one another, then they lie in one plane; and every triangle lies in one plane.
E XI.Pr.7. If two straight lines are parallel and points are taken at random on each of them, then the straight line joining the points is in the same plane with the parallel straight lines.
E XI.De.8. *Parallel planes* are those which do not meet.
“

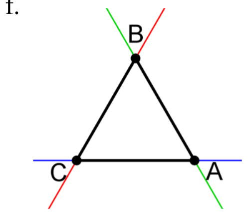
For quotation reference use: ISBN-13: 978-8797246931

Research on the a priori of Physics
Geometric Critique of Pure Mathematical Reasoning

Jens Erfurt Andresen
Edition 2, © 2020-22

5.1.1.2. Additional A Priori Judgments to the Euclidean Plane Geometry

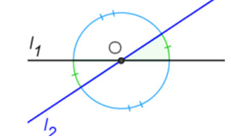
Figure 5.1 Concept of a plane. k- f.



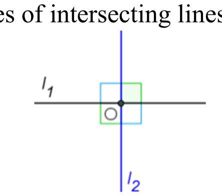
k. Three points→ in one plane.
m. Three points→ one triangle.

k. Postulate: A plane is uniquely defined by three points A,B,C, which do not lie on one straight line. The plane through A,B,C is named γ_{ABC}. A plane is spanned by at least two straight lines ℓ_{CA}, ℓ_{CB} or ℓ_{AB}, which intersect in pairs at the C, A or B, referring to E I.De.7. When one of the points C, A or B is displaced arbitrarily along one of its lines, its other line is displaced in the plane.

l. Two, not parallel straight lines that meet, form a finite angle from a point of intersection O, refer to E I.De.8.
Four angles in total, and equal in pairs (vertexes) E I.Pr.15.
These straight lines ℓ_1, ℓ_2 span a plane as above in k.
A special case of this is two perpendicular lines, forming four identical right angles.

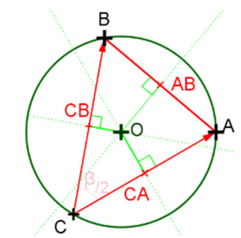


l. Angles of intersecting lines.



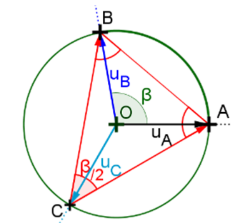
m. The simplest figure in the plane formed from three points A, B, C, is classic a triangle △ABC bounded by straight lines.

n. It is possible between the three points A, B, C that defining the plane γ_{ABC} as postulated in k. to draw a circle ○ABC (according to E I.De.15.) implicitly from a center point O, (E I.De.16) with a uniform radius (E I.Po.3.). The center O is determined by the intersection between two perpendicular bisectors of the line segments CB, CA or AB (E III.Pr.3.). That center of the circle is in the plane and given a priori at the bisectors m_{CA}, m_{CB}, and m_{AB} are a priori in that plane too, that we call the plane of the circle because each bisector cut the circle in two points, just as each side of the triangle intersects the circle at two points. The center O of the triangle △ABC and the circle ○ABC has by a priory judgment to be in the plane of the circle γ_{ABC}, which is also the plane of the triangle.



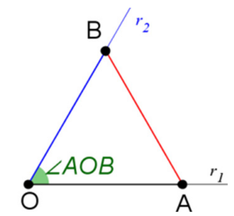
n. The circumscribed circle.

o. The ***entity*** of three points A, B, C form a triangle △ABC with three corner angles ∠ACB, ∠BAC, ∠CBA . The center O of the triangle and the circumscribed circle form three new angles ∠AOB, ∠BOC, ∠COA from the center O as the origo of plane γ_{ABC}.
We have for one turn in the circle circumference
$○ABC = ∡AOB + ∡BOC + ∡COA \;=\; 2(\,∡ACB + ∡BAC + ∡CBA\,)$
We know from E III.Pr.20. the center angle is twice the corner angle.
$∡AOB = 2∡ACB \quad \Leftrightarrow \quad \breve{\beta} = 2\frac{\beta}{2}$. Similar for the two other angles.



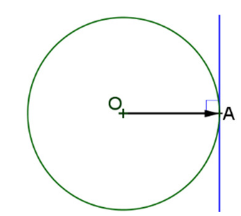
o. Angles of the triangle.

p. From a starting point O, the three points A, O, B form an angle ∠AOB, defined by the two half-lines $r_1 = r_{0A}$ and $r_2 = r_{0B}$.



p. *A priori* of a plane angle.

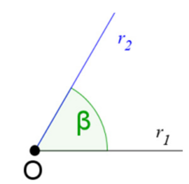
q. The circle circumference is perpendicular to the radius, in that a tangent straight line according to E III.Pr.18 is perpendicular to a straight line through the contact point and the center of the circle.



q. radius ⊥ tangent.

5.1.1.3. The Concept of an Angle

The fact that the two lines r_1 and r_2 meet at a point, form an angle as the ***category***: the plane ***quality*** of the geometric space 𝔊 for physics.



An angle in 𝔊

For quotation reference use: ISBN-13: 978-8797246931

© Copyrighted material from hardback: ISBN-13: 978-8797246931, paperback: ISBN-13: 978-8797246948, Kindle and PDF-file: ISBN-13: 978-8797246917