4.4.3.5. The 1-vector Concept as the *Primary Quality of First Grade* (*pqg*-1)

As we have seen the substance for a 1-vector is a ***quality*** that can generate the idea of a straight line through space. The geometric 1-vector is the simplest ***primary quality*** of space 𝔖 consisting of a ***direction*** and an extension that forms magnitude with a relative ***quantity*** $[\mathbb{R}^1_{+\text{pqg-1}}] \to \mathbb{R}_+$.
Especially, any arbitrary given 1-vector ***direction*** $\mathbf{u}$ we assign the real scalar quantity $|\mathbf{u}|=1 \in\mathbb{R}$ as a reference. This 1-vector can then be dilated to the 1-vector $\mathbf{z} = \lambda\mathbf{u}$ along the straight line $\ell_{\mathbf{u}}$ to any magnitude $|\mathbf{z}| = \lambda \in \mathbb{R}_+$.
The magnitude is thus a real scalar $\mathbb{R}_+$ for the relative ***quantity*** $[\mathbb{R}^1_{+\text{pqg-1}}]$ for a concept the ***primary quality of the first grade*** (***pqg***-1), namely a 1-vector concept.
With the term $[\mathbb{R}^1_{+\text{pqg-1}}]$, we understand not only a positive real number $\lambda \in \mathbb{R}_+$ representing the magnitude, but also the concept of extension in space with ***direction***.
A 1-vector is thus a $[\mathbb{R}^1_{+\text{pqg-1}}]$ ***quantity*** of the ***quality*** (***pqg***-1).
We call a designated geometric 1-vector for *a **first grade** object*. – A 1-vector object for us.

4.4.3.6. The Scalar as a Simple Pure *Quantity*

We have seen that the real numbers ℝ express the relative proportions of 1-vectors along a straight line. In itself, the real numbers ℝ do not express geometric ***quality***. Therefore, the pure real scalars ℝ have a ***primary quality of zero grade*** (***pqg***-0). ~ (0-vector real scalar) –
The real scalar has no extension in the geometric space of physics!
Just as a point has no spatial extension ***(i)***. Interpreted as a vector
But given a 1-vector $\mathbf{u}$, the real numbers $\lambda \in\mathbb{R}$ designates points out from a point O along the line $\ell_{\mathbf{u}}$ through O, see § 4.4.2.7, so that the 1-vectors $\overrightarrow{\mathrm{OZ}} = \mathbf{z} = |\mathbf{z}|\mathbf{u}$ are a ***pqg***-1 ***quality***, where $|\mathbf{z}| \in \mathbb{R}_{\text{pqg-0}}$ is the pure **scalar *quantity*** intuited as a ***zero grade*** *subject*.

The scalar $\lambda \in\mathbb{R}$ can be attached as substantia to a geometric point $\mathrm{P} \in \ell_{\mathbf{u}}$ by a map $\mathfrak{p}: \mathbb{R}\to \ell_{\mathbf{u}}$, as indices λ, or an argument $\lambda \in\mathbb{R}$ for a representation $\mathrm{P}= \mathrm{P}_\lambda = \mathfrak{p}(\lambda)$.
The scalar accident for points appears as contingent to the reverse map $\lambda = \mathfrak{p}^{-1}(\mathrm{P}) \in\mathbb{R}$.
In general, this inverse mapping does not assign a proper geometric ***quality*** to the real scalars $\mathbb{R}_{pqg\text{-}0}$ but expresses only the magnitude differences between geometric *objects* of the same ***grade of primary quality pqg***-*k*. The magnitude is an objective ratio.
In this special case with the 1-vector $\mathbf{u}$ as a reference, it is the relation to the ***pqg***-1 objects $\mathbf{z}$.
As has been the tradition in physics, it is possible to assign a scalar potential to geometric points for different substances (e.g., temperature, electric- and gravitational potentials).

4.4.4. Relationship Between the Concepts of the 0-vector Scalar and the 1-vector

4.4.4.1. The scalar product between the co-linear 1-vectors

We have seen that the real scalar is a relative measure between two geometric collinear 1-vectors $\mathbf{a}$ and $\mathbf{u}$ along a straight line $\ell_{\mathbf{u}}$. We choose the 1-vector $\mathbf{u}$ as a norm for the scalar.
From this, the scalar distance $\lambda = |\mathbf{a}|/|\mathbf{u}| \geq 0$ is the relative ratio between $\mathbf{a}$ and $\mathbf{u} \neq 0$.
$\mathbf{a} = \lambda\mathbf{u}$ is a scalar dilation, whence the real scalar equation $|\mathbf{a}|= \lambda|\mathbf{u}|$ must apply.
How can we construct a scalar ratio of 1-vectors $\mathbf{a}= \lambda\mathbf{u}$ and $\mathbf{b}= \beta\mathbf{u}$?
We try a product between 1-vectors. Since scalars commute by multiplication, we synthetic judge that scalars also commute with 1-vectors and products between 1-vectors, so here in the co-linear case we have

(4.72) $$\mathbf{a}\cdot\mathbf{b} = (\lambda\mathbf{u})\cdot(\beta\mathbf{u}) = \lambda\beta(\mathbf{u}\cdot\mathbf{u}) = \lambda\beta(\mathbf{u})^2.$$

Since $\mathbf{u}$ is the norm for a scalar relationship we must judge $(\mathbf{u})^2 = 1$, thus the a priori idea

(4.73) $$|\mathbf{u}|^2 = |\mathbf{u}| = 1.$$

Looking at $\mathbf{a}$ alone, we have $(\mathbf{a})^2 = \mathbf{a}\cdot\mathbf{a} = (\lambda\mathbf{u})\cdot(\lambda\mathbf{u}) = \lambda^2(\mathbf{u}\cdot\mathbf{u}) = \lambda^2(\mathbf{u})^2 = \lambda^2 \geq 0$, hence

(4.74) $$(\mathbf{a})^2 = |\mathbf{a}|^2 = \lambda^2 \quad\Rightarrow\quad |\mathrm{a}| = |\lambda| \in\mathbb{R}_+, \quad \text{we then define} \quad |\mathbf{a}| = \sqrt{\mathbf{a}\cdot\mathbf{a}} = \sqrt{(\mathbf{a})^2}$$

We see by (4.72)-(4.73) that the product between colinear 1-vectors $\mathbf{a}\cdot\mathbf{b}$ gives a scalar $\lambda\beta$, and in particular the 1-vector auto dot product $\mathbf{a}\cdot\mathbf{a}$ gives the square of the magnitude $|\mathbf{a}|$ of the geometric 1-vector $\mathbf{a}$.

4.4.4.2. The Unitary Co-Linear Direction Vector and the Inverse Geometric 1-vector

We define the normed ***directional*** 1-vector called a unit-vector from a 1-vector $\mathbf{a}$;

(4.75) $$\hat{\mathbf{a}} := \frac{\mathbf{a}}{|\mathbf{a}|} = \mathbf{u}.$$

Here we have that $\hat{\mathbf{a}}\cdot\hat{\mathbf{a}} = 1$, and $\hat{\mathbf{a}}$ indicate the normal ***direction*** as a unitary linear ***direction*** $\mathbf{u}$, both a symbol for a specific object of the pure ***pqg***-1 ***quality*** a linear ***direction***.

We now define the multiplicative inverse geometric 1-vector $\mathbf{a}^{-1}$, as co-linear to $\mathbf{a}$.
The requirement for this is that the scalar dot product must meet $\mathbf{a}\cdot\mathbf{a}^{-1} = 1$, and then

(4.76) $$\mathbf{a}^{-1} = \frac{\mathbf{a}}{\mathbf{a}^2},$$

because, when $\mathbf{a} = \lambda\hat{\mathbf{a}}$ we have $\mathbf{a}\cdot\mathbf{a} = \lambda^2$, then $1= \hat{\mathbf{a}}\cdot\hat{\mathbf{a}}= \frac{\lambda\hat{\mathbf{a}}\cdot\lambda\hat{\mathbf{a}}}{\lambda^2} = \frac{\mathbf{a}\cdot\mathbf{a}}{\lambda^2} = \mathbf{a}\cdot\frac{\mathbf{a}}{\lambda^2} = \mathbf{a}\cdot\frac{\mathbf{a}}{\mathbf{a}\cdot\mathbf{a}} = \mathbf{a}\cdot\mathbf{a}^{-1}$.

4.4.4.3. The Zero-vector Representing All Points

When $\mathbf{a}\cdot\mathbf{a} > 0$ we say that the 1-vector $\mathbf{a}$ is a finite and proper 1-vector. We introduce the concept of the zero-vector $\mathbf{0}$. If the scalar product $\mathbf{a}\cdot\mathbf{a} = 0$ then $\mathbf{a} = \mathbf{0}$. We can write

(4.77) $$\mathbf{0} = \overrightarrow{\mathrm{OO}} = \overrightarrow{\mathrm{AA}} = \overrightarrow{\mathrm{PP}} \quad \text{for } \forall\mathrm{P}.$$

The zero-vector represents all points, because, for all points ∀P, the zero-vector map the point to $\mathrm{P} \underset{0}{\to} \mathrm{P}=\mathrm{P}$. Refer to section 4.4.2, where 1-vector $\overrightarrow{\mathrm{OA}}$ leads O into A.
The zero-vector is neutral to 1-vector addition $\mathbf{a}+\mathbf{0} = \mathbf{a}$. (4.3)
In general, for Euclidean geometric 1-vectors, the auto scalar-product applies
$\mathbf{a}\cdot\mathbf{a} > 0$ or $\mathbf{a}\cdot\mathbf{a} = 0 \Leftrightarrow \mathbf{a} = \mathbf{0}.$

4.4.4.4. The *First Grade Object*, a Geometric 1-vector → a Subject in a Substance as Idealism

We understand that the straight lines in geometry are platonic ideal subjects therefore they cannot be recognized.

- We know immediately about space, we can distinguish two locations, and that we can judge a ***direction*** from one local site A to the second location B.

To make this ***quality*** of space to an *object*, we have invented the concept of a geometric 1-vector as an *object **of first grade***. For us, this object is a geometric idea (*noumenon*) that as a *subject* represents a substance of space, namely the ***primary quality of first grade*** (***pqg***-1).
This substance is given by a specific intelligible *subject* as a physical difference in space, which we experience. To intuit this experience, we form as an idea a 1-vector *object* that represents the difference with a corresponding ***direction***.

Das Ding an sich,	*subject*;	***pqg***-1, the invisible straight line segment of ***direction***.
Das Ding für uns,	*object*;	the spatial difference as ***direction*** and magnitude.

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