$\mathrm{P} = \mathcal{P}_{\mathrm{P}}(\mathbf{0})$, i.e., all points P has no extension, and all can be represented by the $\mathbf{0}$ vector ($|\mathbf{0}| = 0$), see (4.59), where we write $\mathbf{0} = \overrightarrow{\mathrm{PP}}$. [200]
Claim the first point of focus is always a priori assigned zero-vector as the initial location start vector, representing this arbitrary point as an origin (starting point).

We see that translation $\vec{\mathrm{t}}$ with the 1-vector $\mathbf{t}$ (just like the translation $\vec{\mathrm{x}}$ with $\mathbf{x}$) make no change to the point properties, therefore we say that *the points are translation invariant*.

4.4.2.13. The Translation of a Geometric 1-vector

We now look at an arbitrary geometric 1-vector called $\mathbf{x}$, which line segment object $\mathbf{x} = \overrightarrow{\mathrm{OX}}$ as subject ***direction*** is independent of all starting points. Here we let the zero-vector $\mathbf{0} = \overrightarrow{\mathrm{OO}}$ represents the starting point O.
A priori we have e.g., $\mathbf{x} = \mathbf{x}\text{-}\mathbf{0}$ and $\mathbf{t} = \mathbf{0}+\mathbf{t}$.
From the starting point of intuition, the origin O, we map $\mathcal{P}_{\mathrm{O}}$: $\mathbf{x} \to \mathrm{X}$ and plot 1-vectors $\mathbf{x}$ to the points $\mathrm{X} = \mathcal{P}_{\mathrm{O}}(\mathbf{x})$.

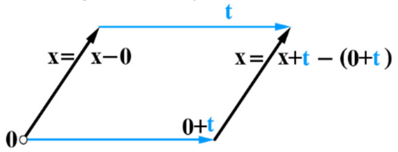

Figure 4.6 Translation of 1-vectors **x**.

The inverse mapping $\mathcal{P}_{\mathrm{O}}^{-1}$: $\mathrm{X} \to \mathbf{x}$ links to each point X a 1-vector $\mathbf{x}$ object from an origin O.
In Figure 4.6 the points are implicitly hidden, and the 1-vector objects show the structure.

Any linear translation $\vec{\mathrm{t}}$ represented by any arbitrary 1-vector $\mathbf{t}$ translates every point P through space $\vec{\mathrm{t}}$: $\mathcal{P}_{\mathrm{O}}(\mathbf{x}) \to \mathcal{P}_{\mathrm{O}}(\mathbf{x}+\mathbf{t})$, in particular $\vec{\mathrm{t}}$: $\mathcal{P}_{\mathrm{O}}(\mathbf{0}) \to \mathcal{P}_{\mathrm{O}}(\mathbf{0}+\mathbf{t})$.
Now we let the translation $\vec{\mathrm{t}} \leftarrow \mathbf{t}$ operate on all points of the line segment $\underline{\mathrm{OX}} \to \mathbf{x}$ as an object so that $\vec{\mathrm{t}}(\underline{\mathrm{OX}}) = \underline{\mathrm{O_tX_t}}$ and thus on the vector object $\mathbf{x}$. In Figure 4.6 is illustrated what happens for a vector $\mathbf{x}$ object translation, $\vec{\mathrm{t}}$: $\mathbf{x} \to \mathbf{x}+\mathbf{t} - (\mathbf{0}+\mathbf{t}) = \mathbf{x}$, to a new object representing the same vector. We have $\vec{\mathrm{t}}(\mathbf{x}) = \mathbf{x}$ for $\forall \vec{\mathrm{t}} \leftarrow \mathbf{t}$ or $\vec{\mathrm{t}}(\overrightarrow{\mathrm{OX}}) = \overrightarrow{\mathrm{OX}}$.
1-vectors as subjects are in this way invariant to translation.

A ***pqg***-1-vector subject $\mathbf{x} = \overrightarrow{\mathcal{P}_{\mathrm{O}}(\mathbf{0})\mathcal{P}_{\mathrm{O}}(\mathbf{x})} = \overrightarrow{\mathrm{OX}}$ is as object a line segment with ***direction*** and ***magnitude***, $\underline{\mathcal{P}_{\mathrm{O}}(\mathbf{0})\mathcal{P}_{\mathrm{O}}(\mathbf{x})} = \underline{\mathrm{OX}}$, that is invariant under translations through space

$$\vec{\mathrm{t}}\left(\underline{\mathcal{P}_{\mathrm{O}}(\mathbf{0})\mathcal{P}_{\mathrm{O}}(\mathbf{x})}\right) = \underline{\mathcal{P}_{\mathrm{O}}(\mathbf{0}+\mathbf{t})\mathcal{P}_{\mathrm{O}}(\mathbf{x}+\mathbf{t})} = \underline{\mathrm{O_tX_t}}. \tag{4.69}$$

This ***pqg***-1-vector substance is that the ***magnitude*** and ***direction*** are uniform across space.
The intuition of two 1-vectors as objects is equivalent if and only if they can be obtained by translation from one another.

In this case, all the intuit objects of a ***pqg***-1-vector are allocated to one and the same symbol, e.g., $\mathbf{x}$.
The retained identity of the 1-vector subject is $\vec{\mathrm{t}}(\mathbf{x}) = \mathbf{x}$ for $\forall(\vec{\mathrm{t}} \leftarrow \mathbf{t})$.
We have: <u>All 1-vectors subjects in $\mathfrak{G}$ space is translation invariant.</u>
We must note, that the bundle of all line segments (4.69) for $\forall \mathbf{t} \in \mathfrak{G}$ is synonymous with the 1-vector concept $\mathbf{x}$. Therefore, the concept of 1-vectors is invariant under translation.
Note that the translation is generated by the same ***pqg***-1 concept, namely:
The 1-vector $\mathbf{t}$ is a generator for the translation.

———

[200] If you have trouble accepting the $\mathbf{0}$ vectors for point O, just in mind replace the position vector $\mathbf{s} \leftarrow \mathbf{0}$ to the desired point from an arbitrary origin. Then point X is represented by the position vector $\mathbf{s}+\mathbf{p}$.
A priori $\mathbf{p}+\mathbf{s} = \mathbf{p}+\mathbf{s}$. This is the philosophical problem I call the scholastic TAQ problem of the difference between esse (translation) and essence (background) for the point space through which the translation is made.
The reason is transcendental for our intuition. More about this later below [203]

4.4.3. The Straight Line Idealism

The concept of a straight line is a simple platonic idea (noumenon) that does not have the existence of itself. Intuition appears to us when we include the surrounding space and our recital has several dimensions. In this way, spatial ***direction*** for all lines makes sense from the concept of geometric 1-vectors.

4.4.3.1. The Objective Reality of a Difference

The line segment finality allows the two objective locations in space to be different.
The distinguishability of points as objective locations provides that the ***primary quality of first grade*** has subjects,[201] that enables ***quantitative*** judgments.[202]
A ***pqg***-1 subject can be assigned a ***quantitative*** object magnitude $[\mathbb{R}^{+1}_{\mathrm{pqg}\text{-}1}]$ by the map:

$$\text{Magnitude: } \mathrm{AB} \to |\mathrm{AB}| \qquad \text{or} \qquad \text{1-vector Magnitude: } \overrightarrow{\mathrm{AB}} \to |\overrightarrow{\mathrm{AB}}| \tag{4.70}$$

From this, we can derive a preliminary but necessary concept of a simple memory.

4.4.3.2. The Concept of Simple Memory

To recognize a difference in space the memory must be able to remember the difference.
Claim: memory $(\overrightarrow{\ \ })$ must in itself at least contain a ***direction*** extension distance in space.
The idea of the physical *object*, the memory, contains a subject $(\overrightarrow{\mathrm{AB}})$ (A line segment).
The memory of a difference in space $(\overrightarrow{\mathrm{AB}})$ has a second difference AB in space as a cause.
Memory is a ***quality*** at least a ***primary quality of the first grade*** (***pqg***-1). (+higher grades).
The content of a simple memory is one ***quantity*** $[\mathbb{R}^{1}_{+\mathrm{pqg}\text{-}1}]$.

4.4.3.3. The Simplest Subject

The ***primary quality*** of a simple memory must necessarily have ***existence*** *in reality*.
(*Cogito ergo sum*). The simple memory ***essence quality*** is its real ***quantitative*** content.[203]

4.4.3.4. The Simplest Object

The difference in space can be recognized by an observer when there is established a map of the difference in space to the memory; a subject\object → interpretant → subject\object relation that results in the map

$$\text{Interpretant: } \overrightarrow{\mathrm{AB}} \to \mathrm{AB} \to (\mathrm{AB}) \to (\overrightarrow{\mathrm{AB}}) \to |\mathrm{AB}|, \tag{4.71}$$

that results in a ***quantity*** $[\mathbb{R}^{1}_{+\mathrm{pqg}\text{-}1}]$ from the subject $\overrightarrow{\mathrm{AB}}$, which in principle must be relative measurable, or at least be intuited as an objective ***quantity*** in our interpretation.
The simplest subject is thus a distance as an object which can be represented relatively by a real number for a difference in space to the observer.[204]
– Be aware that the relations always are given relative from a ***direction*** in space.
The simplest subject a ***direction*** as the ***primary quality of first grade*** (***pqg***-1) achieves first an objective ***direction*** from an interpretant of higher grades.[205]

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[201] The concepts terms: The *subject* is used here in the scholastic meaning as the underlying, as a *subject* belongs to a *substance* which is characterised by the ***primary quality***, and the *object* is the idea of something for the intuition (das Ding für uns).
[202] The real numbers are not numerable, therefore spaced points along a line could not be counted, therefor the point concept itself is **not *quantitative***. A single point has no size. The real numbers, however, have an order and represent the relative differences between points and apply ***quantitative*** magnitudes.
[203] The division into ***existence*** (esse) and ***essence*** concepts can, as far as I can ascertain traced back to scholastic by Thomas Aquinas & Co. In my conscience of the world, something that exists has ***quality*** and its ***essence*** is an associated ***quantity***.
In short, there must be ***quality*** prior to a concept of a ***quantity*** to be extracted and measured.
[204] The issue § 4.4.3.2-4.4.3.4 is a simple linear picture and requires an adequate spatial full treatment later below.
[205] A simple objective example of a memory $(\overrightarrow{\ \ })$ is the text in this book, with an implicit reading direction. The reader is the interpretant, and you will realise booths you and the book text possess higher grades than ***pqg***-1.