Multiplication of 1-vector with negative scalar is allowed $\beta = -\lambda$.
Remember that the 1-vector itself as a subject is independent of a start point, then a negative 1-vector is pointing towards the origin O along the ray r_{OE} with a negative orientation $-1\mathbf{e}$, caused by its positive orientated half line designated by the direction $\mathbf{e}=\overrightarrow{OE}$. E.g., $|-1\mathbf{e}| = |\mathbf{e}|$.
If the 1-vector object $\overrightarrow{OA} = \mathbf{a} = \lambda\mathbf{e}$ is pointing away from O for $\lambda > 0$, then the 1-vector subject $\mathbf{b} = -\lambda\mathbf{e}$ points A toward O, as the 1-vector object $\overrightarrow{AO} = \mathbf{b}$ pointing from A to O. The point B is located opposite to A relative to the origin O, for the 1-vector object $\overrightarrow{OB} = \mathbf{b} = -\lambda\mathbf{e}$ pointing away from O in a negative orientation opposite the positive of the ***direction*** $\mathbf{e}$.

(4.61) $$\mathbf{a} + \mathbf{b} = \mathbf{0} \quad \Leftrightarrow \quad \mathbf{b} = -\mathbf{a} \quad \text{or} \quad \overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AO} = \overrightarrow{OA} - \overrightarrow{OA} = \mathbf{0}.$$

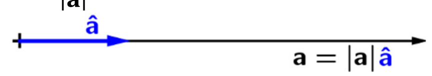
The negative 1-vector is a geometric 1-vector multiplied by the scaling -1.
Multiplication of a geometric 1-vector with the scalar -1 is a *line segment reversion* or *parity inversion* ***of first grade pqg-1*** concerning the Descartes *extension* of Euclidian space. This -1 multiplication of 1-vectors is sometimes just called a *parity operation*.
The negative line ***direction*** is inversely orientated to the positive ***direction*** *(vii)*.
The magnitude is retained $|\mathbf{a}|=|\mathbf{b}|$ but the orientation of ***direction*** is opposite for a ***grade one parity inversion*** in Euclidean space.
Division of 1-vector with a scalar $\lambda \neq 0$ is equivalent to multiplication by the reciprocal

(4.62) $$\lambda = \tfrac{1}{\alpha}, \qquad \text{that is} \quad \mathbf{d} = \tfrac{\mathbf{a}}{\lambda} = \tfrac{1}{\lambda}\mathbf{a} = \alpha\mathbf{a} \quad \Rightarrow \quad \mathbf{a} = \lambda\mathbf{d} = \tfrac{\mathbf{d}}{\alpha}.$$

Multiplication of a 1-vector with the scalar 0 (zero) is allowed, $0\mathbf{a} = \mathbf{0} = 0$. When a ***pqg***-1-vector is multiplied by the scalar 0 it loses its ***direction*** and turns into a ***pqg***-0 quality.
We see that the zero vector is a 0-vector, and hence the scalar 0.
Here we allow $\mathbf{0}= 0$. It may be advantageous to regard the zero vector as a scalar.

4.4.2.6. The unit object for a linear *direction*

Giving a 1-vector $\mathbf{a}$, we can define the unit ***direction*** vector $\hat{\mathbf{a}} \coloneqq \frac{\mathbf{a}}{|\mathbf{a}|} \Rightarrow \mathbf{a}=|\mathbf{a}|\hat{\mathbf{a}}$,
which is colinear with $\mathbf{a}$ and has the magnitude $|\hat{\mathbf{a}}| \equiv 1$.

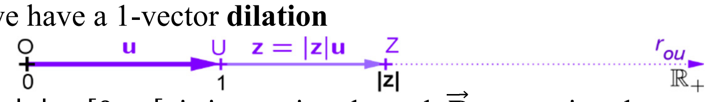


The unit vector $\hat{\mathbf{a}}$ sets a linear ***direction*** in space.
In addition to the indication hat^ on the vector of unit generating ***directions***, e.g., $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$, the following names for basis unit 1-vectors are often used $\mathbf{e}_i$ or $\boldsymbol{\sigma}_i$. Here the indices i indicate that there may be a basis set of several linear ***directions*** in space, i=0,1,2,3 … $n \in \mathbb{N}$
For an arbitrary variable ***direction*** in space, we now use the term $\mathbf{u}$, which not only is a unit 1-vector, but we also called it an **unitary** 1-vector, (to emphasise an arbitrary variable ***direction***).
In all these cases, we expect a unit $|\mathbf{u}| = |\mathbf{e}_i| = |\boldsymbol{\sigma}_i| = |\hat{\mathbf{x}}| = |\hat{\mathbf{y}}| = |\hat{\mathbf{z}}| = |\hat{\mathbf{a}}| = 1$

4.4.2.7. The Linear Extension from a 1-vector

An arbitrary given segment OU can be extended by multiplying with a positive real number from O to any arbitrary point Z on the half-line ray r_{OU}. For this, we use the concept of a unitary 1-vector $\mathbf{u} = \overrightarrow{OU}$, where we choose the a priori judgment that $|\mathbf{u}|=|\overrightarrow{OU}|=1$, and its ***direction*** is arbitrarily given by the 1-vector $\mathbf{u}$ along the ray $r_{\mathbf{u}}= r_{OU}$.
For the arbitrary point $Z \in r_{OU}$, we have a 1-vector **dilation**

(4.63) $$\overrightarrow{OZ} = \mathbf{z} = |\mathbf{z}|\mathbf{u} = |\mathbf{z}|\overrightarrow{OU},$$



when the positive real number $\lambda = |\mathbf{z}| \in [0, \infty[$ is increasing through $\vec{\mathbb{R}}_+$ spanning the ray r_{OU} …
By a positive scalar $\lambda \in \mathbb{R}_+$, the vector $\mathbf{z} = \lambda\mathbf{u}$ designates a point $Z \in r_{OU}$ from the unitary 1-vector $\mathbf{u} = \overrightarrow{OU}$ ***direction quality*** from the local origo O. $|\mathbf{z}| = \lambda = |\mathbf{z}|/|\mathbf{u}| = |\overrightarrow{OZ}|\,[\mathbf{u}^{-1}]$.
From the ***pqg***-1 ***direction*** $\mathbf{u} = \overrightarrow{OU}$ with the joint quantity $[\mathbb{R}^1_{+pqg-1}]$ of the scalar $\lambda = |\mathbf{z}| \geq 0$, we have the substance of the linear line idea and thus the idea of a straight half line as a ray equivalent to the idea of a beam of light from O.

4.4.2.8. The Parametric Development of a Straight Linear Ray

We introduce the linear 1-vector function $\mathbf{z} = \mathbf{z}(\lambda) = \lambda\mathbf{u}$, which we call the parameterization of ray r_{OU}. The real scalar argument $\lambda \in [0, \infty[\, \cong \mathbb{R}_+$ is referred to as the parameter that produces the idea of a **ray**, as a straight half-line from O

(4.64) $$r_{OU} = \{ Z \,|\, \overrightarrow{OZ} = \mathbf{z} = \mathbf{z}(\lambda) = \lambda\mathbf{u} = \overrightarrow{OU}, \ \lambda \in \mathbb{R}_+ \}.$$

4.4.2.9. The Parametric Span of a Straight Line

The linear 1-vector function $\mathbf{a} = \alpha\mathbf{e}$, $\alpha \in \mathbb{R}$ for the ***direction*** $\mathbf{e} = \overrightarrow{OE}$, where we include all real numbers (also negative) we call the parameterisation of the straight line

(4.65) $$\ell_{OE} = \{ A \,|\, \overrightarrow{OA} = \mathbf{a}\,(\alpha) = \alpha\mathbf{e} = \alpha\overrightarrow{OE}, \ \alpha \in \mathbb{R} \}$$

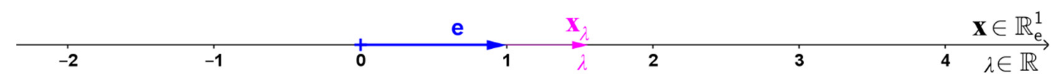
This real scalar argument $\alpha \in \mathbb{R}$, we call for the parameter in the production of the line.
This is not only a scalar for the dilation scaling ratio of a 1-vector, but the negative real scalars $\alpha < 0$ gives a parity inverse orientated ***direction*** to the basis vector $\overrightarrow{OE}$.
The scalar $\alpha \in \mathbb{R}$ is often also called the coordinate of point A along line ℓ_{OE}.

4.4.2.10. Co-linear 1-vectors

All geometric 1-vectors obtained by multiplying a particular 1-vector $\mathbf{e}$ with different scalars $\alpha_1, \alpha_2 \ldots \in \mathbb{R}$ are colinear $\mathbf{a}_1 = \alpha_1\mathbf{e}$, $\mathbf{a}_2 = \alpha_2\mathbf{e}$, …, lay in line.[199] We can also mark the vectors by indexing directly with the real numbers $\mathbf{x}_\lambda = \mathbf{x}(\lambda) = \lambda\mathbf{e}$, the index $\lambda \in \mathbb{R}$.

4.4.2.11. The Spatial Line as a Real Linear Vector Space $\mathbb{R}^1_{\mathbf{e}}$

Given the geometrical basis vector $\mathbf{e} = \hat{1}$, see § 4.1.1.7 (4.18) and section 4.1.2 (4.26), we get the number line as a 1-vector $\mathbf{x}_\lambda = \lambda_1\hat{1}_1 \in \mathbb{R}^1_{\mathbf{e}}$ produced by the real scalars $\lambda_1 \in \mathbb{R}$, shown in Figure 4.1, and here:



Thus, we have the straight line as a graphic representation of a 1-dimensional 1-vector space.

4.4.2.12. A 1-vector Intuited as a Translation

In section 4.4.2.4 we described the addition of collinear 1-vectors and intuition with the addition of linear translations.
We can add linearly independent vectors and thus also translations, see Figure 4.5
We look at two linearly independent 1-vectors

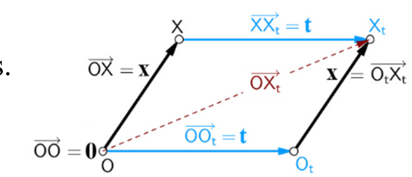


Figure 4.5 Addition of translations.

(4.66) $$\mathbf{x} = \overrightarrow{OX} = \overrightarrow{O\,\vec{x}(O)} \qquad \text{and} \qquad \mathbf{t} = \overrightarrow{OO_t} = \overrightarrow{O\,\vec{t}(O)}$$

both of which can be seen as translations $\vec{x}: O \to X$ and $\vec{t}: O \to O_t$.
The identical translation is defined as a zero vector $\mathbf{0}=\overrightarrow{OO} = \overrightarrow{PP}$, Which can represent all points.
The addition is defined (4.59) so that the resulting translation $O \to X_t$, (Figure 4.5)

(4.67) $$\overrightarrow{OX_t} = \mathbf{x}+\mathbf{t} = \overrightarrow{OX}+\overrightarrow{OO_t} = \overrightarrow{OX}+\overrightarrow{XX_t} = \overrightarrow{O\vec{x}(O)}+\overrightarrow{O\vec{t}(O)} = \overrightarrow{OX}+\overrightarrow{X\vec{t}(X)} = \overrightarrow{O\vec{t}(O)}+\overrightarrow{\vec{t}(O)\vec{x}(O_t)} = \overrightarrow{OO_t}+\overrightarrow{O_tX_t} = \mathbf{t}+\mathbf{x}.$$

From the starting point of the intuition, the origin O, we introduce the plot map of 1-vectors for points in space $\mathcal{P}_O: \mathbf{p} \to P$, so that $P = \mathcal{P}_O(\mathbf{p})$. We write object intuition in Figure 4.5 as

(4.68) $$O=\mathcal{P}_O(\mathbf{0}), \ X=\vec{x}(O)=\mathcal{P}_O(\mathbf{x}), \ O_t=\vec{t}(O)=\mathcal{P}_O(\mathbf{t}) \ \text{and} \ X_t=\mathcal{P}_O(\mathbf{x}+\mathbf{t}), \ \text{also} \ X_t=\mathcal{P}_X(\mathbf{t})$$

In general, for all points P we apply the map $\mathcal{P}_P$ of vectors into the space of points:

[199] As a co-linear bundle of parallel lines as (4.55) explained first at the start of section 4.4.2.

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