4.4.2. The Concept of Geometric Vectors

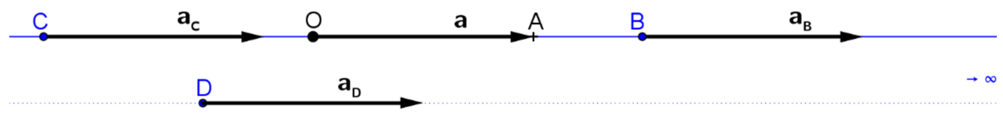
By physically indicating two different object points A and B (on a surface) in space we can set a ***direction*** from A to B. The line segment for this ***direction*** is called a spatial geometric vector $\overrightarrow{AB}$ object and given a designation with a small fat Latin Letter[195], e.g., $\mathbf{d} = \overrightarrow{AB}$.

The ***direction*** from a point O to another A is a ***pqg***-1 ***quality*** that provides the concept of a geometric vector (1-vector), here called $\mathbf{a} = \overrightarrow{OA}$, drawn

It has the magnitude $|OA|$, such that $|\mathbf{a}| = |\overrightarrow{OA}| \in \mathbb{R}_+$. The object of the geometrical vector is a ***quantity*** of the type $[\mathbb{R}^1_{+pqg\text{-}1}]$, wherein $\mathbb{R}_+$, stands for the magnitude represented by a positive real scalar $\mathbb{R}_+$ and ***pqg***-1 indicates a line ***direction*** in space. The square brackets $[\mathbb{R}^1_{+pqg\text{-}1}]$ show that ***quantity*** real scalar magnitude is relative to one unit as in (4.54)

The geometric vector corresponds here in the ***first grade*** to a map $\vec{a}: \ell_{OA} \to \ell_{OA}$ leading point O into the second point A over the distance $|\mathbf{a}| = |\overrightarrow{OA}|$, so that $A = \vec{a}(O)$ and $\vec{a}$ lead any arbitrary points over in new points

(4.55) $$\mathbf{a} = \overrightarrow{O\,\vec{a}(O)} = \overrightarrow{OA}, \qquad \mathbf{a} = \mathbf{a}_B = \overrightarrow{B\,\vec{a}(B)}, \qquad \mathbf{a} = \mathbf{a}_C = \overrightarrow{C\,\vec{a}(C)}, \qquad \mathbf{a} = \mathbf{a}_D = \overrightarrow{D\,\vec{a}(D)}$$



The four shown vector objects are all the same geometric vector subject $\mathbf{a} = \mathbf{a}_B = \mathbf{a}_C = \mathbf{a}_D$.
The bundle of all parallel oriented line segments $\underline{\overrightarrow{P\,\vec{a}(P)}}$ from all points $\forall P \in \mathfrak{G}$ in space represents the concept of a vector $\mathbf{a} = \overrightarrow{P\,\vec{a}\,(P)}$ from the intuition vector $\overrightarrow{OA}$, where the ordered representative $\underline{OA}$ is the generator of the vector $\mathbf{a} = \overrightarrow{OA}$ in the entire space $\mathfrak{G}$.

<u>The object representative</u> of $\mathbf{a} = \overrightarrow{P\,\vec{a}(P)}$ starting at any point P, we denote $\mathbf{a}_P \sim \underline{\overrightarrow{P\,\vec{a}(P)}}$.

The geometric vector ***direction*** given by $\mathbf{a}$ (also written $\mathbf{a} = \vec{\mathbf{a}} = \vec{a}$) are ideal straight linear, and all of the representing distances[196] from all $\forall P$, $|\mathbf{a}_P| = |\mathbf{a}_B| = |\mathbf{a}_C| = |\mathbf{a}_D| = |\mathbf{a}|$ are mutual equidistant, as one determent ***quantity***. Then we call the scalar $|\mathbf{a}|$ the magnitude of the vector $\mathbf{a}$. An arbitrary geometric vector $\mathbf{a}$ has a real scalar magnitude $|\mathbf{a}| \in \mathbb{R}$, constant, the same throughout the space $\mathfrak{G}$. This is the subject substance of a geometric 1-vector.

4.4.2.2. Geometric Translation

The concept of geometric vectors illustrates in physics a conception of linear translation along a straight line. The translation $\mathbf{a} = \overrightarrow{OA}$ is a shift that is equidistant along the line, and we extend this to be equidistant above all with OA parallel lines.
We pass the a priori judgment; the map $\vec{a}: O \to A$ from (4.55) apply to all points $P \in \mathfrak{G}$ in space, so that the geometric vector $\mathbf{a} = \overrightarrow{OA} = \overrightarrow{O\vec{a}(O)} = \overrightarrow{P\vec{a}(P)}$ is the global representative of the translation, and thus as an object is a generator for translation.

4.4.2.3. The 1-vector Concept

It is common to count a vector as belonging to a vector space V_n of dimension $n \in \mathbb{N}$.
This does not imply that one vector has n ***directions***. – A general[197] vector v, which satisfies a linear additive algebra (4.1)-(4.11), may not necessarily possess any ***direction***. –

[195] We use the little **fat** upright Latin alphabet $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d} \ldots \mathbf{z}$ to denote a vector (1-vector) that has geometric reference to the natural space of up to 3 dimensions, 3D.
[196] Implicit the magnitude of a distance as a scalar is measured relative to a unit vector (e.g., itself), as described in (v).
[197] A general vector v without ***direction*** designated by a lowercase letter in italics, contrary to a regular geometric vector giving a ***direction*** is denoted by a small fat Latin letter $\mathbf{a}$. Another expression for a vector with ***direction*** is $\vec{a} = \vec{\mathbf{a}} = \mathbf{a}$.

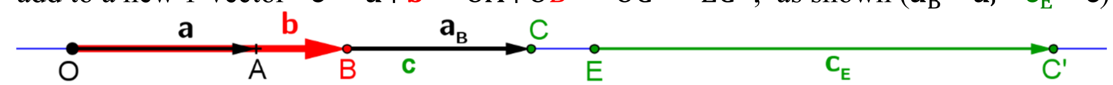
However, a regular geometric line vector has just one ***direction***, therefore called as a 1-vector.
The 1-vector subjects belong to a substance that has a ***primary quality of first grade*** (***pqg-1***).
The simple geometric vector is also often called 'a ***grade*** 1 vector' or 'a ***grade*** 1 blade'.
A geometric space has dimension $n \in \mathbb{N}$ if there is a set of precisely n finite basis 1-vectors $\{\mathbf{a}_1 \ldots \mathbf{a}_n\}$, (also called a maximal set) that are linearly independent, that is,

(4.56) $$\alpha_1\mathbf{a}_1 + \alpha_2\mathbf{a}_2 + \cdots \alpha_n\mathbf{a}_n = 0 \quad \Rightarrow \quad \forall \alpha_1 = \alpha_2 = \ldots \alpha_n = 0.$$

The geometric 1-vector space G_n is said to have n independent principal ***pqg-1 directions***.[198]
Two proper finite 1-vectors $\mathbf{a}$ and $\mathbf{e}$ are linearly dependent, if there exists a real scalar $\lambda \in \mathbb{R}$, so that $\mathbf{a} = \lambda\mathbf{e}$ for $\lambda \neq 0$, thus $\mathbf{a} + \alpha\mathbf{e} = 0 \Rightarrow \alpha = -\lambda \neq 0$.
In this case, we say, the 1-vectors $\mathbf{a}$ and $\mathbf{e}$ are co-linear (colinear).

4.4.2.4. Addition of Co-linear 1-vectors

The 1-vector $\mathbf{a} = \overrightarrow{OA}$ and the 1-vector $\mathbf{b} = \overrightarrow{OB}$ as objects on the same straight line ℓ_{OA} can add to a new 1-vector $\mathbf{c} = \mathbf{a} + \mathbf{b} = \overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC} = \overrightarrow{EC'}$, as shown ($\mathbf{a}_B = \mathbf{a}$, $\mathbf{c}_E = \mathbf{c}$)



In the ***primary quality of first grade*** (***pqg-1***) the co-linear vectors addition is equivalent to a real scalar addition. Here in the example is $|\overrightarrow{OA}| + |\overrightarrow{OB}| = |\overrightarrow{OC}| = |\overrightarrow{EC'}|$ along ℓ_{OA}.
The addition of oriented line segments is shown to illustrate the extension in space along a straight line, similar to the size of the real number is increased when the same sign number is added. – Of course, subtraction is also allowed. (consult ref. [10]p5→)
If we intuit 1-vector as a translation, we have

(4.57) $$\mathbf{a} = \overrightarrow{OA} = \overrightarrow{O\vec{a}(O)} = \overrightarrow{B\vec{a}(B)} \quad \text{and} \quad \mathbf{b} = \overrightarrow{OB} = \overrightarrow{O\vec{b}(O)} \quad \text{for the shown objects, and thus}$$

(4.58) $$\mathbf{c} = \mathbf{b} + \mathbf{a} = \overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{OA} = \overrightarrow{O\vec{b}(O)} + \overrightarrow{O\vec{a}(O)} = \overrightarrow{O\vec{b}(O)} + \overrightarrow{B\vec{a}(B)} = \overrightarrow{OC} = \overrightarrow{O\vec{c}(O)} = \overrightarrow{E\vec{c}(E)} = \overrightarrow{EC'} = \mathbf{c}_E = \mathbf{c}.$$

We add translations and have a combined translational represented by a 1-vector addition.
We introduce the additive neutral vector $\mathbf{0}$, such that $\mathbf{a} + \mathbf{0} = \mathbf{a} \sim \overrightarrow{OA} + \overrightarrow{PP} = \overrightarrow{OA}$.
This vector $\mathbf{0}$ is assigned to all points $P \cong \overrightarrow{PP}$. For the real numbers assigned to 1-vectors applies $|\overrightarrow{OA}| + |\overrightarrow{PP}| = |\overrightarrow{OA}| \sim |\mathbf{a}| + |\mathbf{0}| = |\mathbf{a}|$, therefore we concluded $|\mathbf{0}| = 0$, and $|\overrightarrow{PP}| = |P| = 0$.

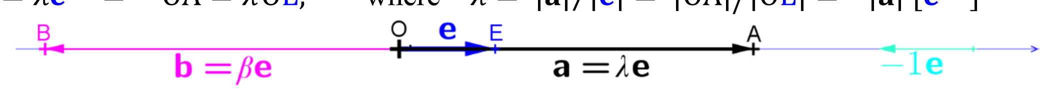
(4.59) $$\text{The identical translation } \vec{0}: P \to P \quad \text{is represented by} \quad \mathbf{0} = \overrightarrow{P\vec{0}(P)} = \overrightarrow{PP}$$

Since the $\mathbf{0}$ vector is without extension and therefore without ***direction***, this (4.59) applies everywhere in $\mathfrak{G}$ space.
All points in space are represented by the Null-vector $\mathbf{0}$, i.e., the points are even identical.

4.4.2.5. Multiplication of 1-vector with a Real Scalar

Multiplication with a real scalar $\lambda \in \mathbb{R}$ to a geometric 1-vector is the continuum concept in the straight number line as an expansion on the concept of addition with the same 1-vector a whole number of times.
We choose a 1-vector object $\mathbf{e} = \overrightarrow{OE}$ as the unit of the number line and get all the 1-vectors

(4.60) $$\mathbf{a} = \lambda\mathbf{e} \quad = \quad \overrightarrow{OA} = \lambda\overrightarrow{OE}, \qquad \text{where} \quad \lambda = |\mathbf{a}|/|\mathbf{e}| = |\overrightarrow{OA}|/|\overrightarrow{OE}| = \quad |\mathbf{a}|\,[\mathbf{e}^{-1}]$$



The object $\mathbf{a} = \overrightarrow{OA}$ point away from O in the ***direction*** $\mathbf{e} = \overrightarrow{OE}$, along the ray $r_{OE} \subset \ell_{OA}$.

[198] It has since Leibniz been recognized that the physical location-space is 3-dimensional, therefore we only use bold characters up to three dimensions, so we can only find three linearly independent fat-denoted basis vectors $\alpha_1\mathbf{a}_1 + \alpha_2\mathbf{a}_2 + \alpha_3\mathbf{a}_3 = 0 \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0$. See footnote[178] for a confirming reason.