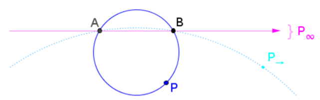
Given two distinct points $A \neq E \notin \ell_{AB}$, the two lines ℓ_{AB} and ℓ_{EF} can only be parallel $\ell_{AB} \parallel \ell_{EF}$ if they meet only in one point P_∞ at infinity.

g. The idea of parallel lines is linked to the concept of one ***direction*** in space. The bundle of parallel lines, in the ***direction*** of a point P_∞ at infinity, is called a *linear **direction***.

h. we shall determine two objects representing points A and B to determine one straight line[190] ℓ_{AB} and any resulting parallel lines will all point in the same ***direction*** P_∞.

i. One *linear **direction*** consists of a bundle of parallel lines through a point at infinity P_∞. The a priori transcendental common point at infinity P_∞ cannot be determined or be the object of experience but only an intuition of a subject as a ***quality*** of the space 𝔊 substance. – It is not the abstraction, one point at indefinitely P_∞ without any location,[191] but the two concrete object points A and B, which determine the ***direction*** in space.

j. A circle ○ through three points A, B, P is in contrast to the straight line ℓ_{AB}. The idea is the straight line, if the circular third point P is moved into the infinite $P \rightarrow P_\infty$.



We need two selected points to determine *one straight line* and thus only *one **direction***.
The straight-line-segment AB between two points A and B is a subject in space 𝔊 substance.
For intuition, the line segment we can objectify draw by an example, as shown in Figure 4.4

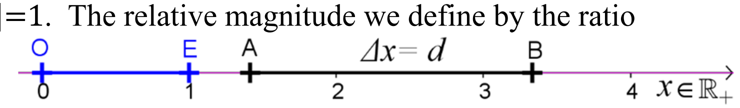
(iv) The concept of difference

There is no difference in space simpler than a line segment AB.
The proper finite line segment is the basic difference between two different points.
Here is the assumed a priori judgment (4.53) on the possible ***quality*** $A \neq B$ for $A, B \in \ell_{AB}$.
A difference as ***quality*** is a general concept of space 𝔊 that apply everywhere. *(ii)*[183]
Individual differences can be subdivided into several differences.
Several differences can be added together into one difference $AB \in \mathfrak{L}$

(v) The concept of quantity $d = |AB|$

A difference $AB \in \mathfrak{L}$ in space has a *distance* magnitude ***quantity.***
Given two different points $A \neq B$ make a line segment as an object for our intuition.
The magnitude of the line segment AB can endow the positive real number
$d = |AB| \in \mathbb{R}_+$, here after called the magnitude $|AB|$ for the line segment AB.
This is the simplest form of ***quantified*** extension.[186]
The numerical scalar ***quantity*** $d = d_{|OE|} = \Delta x = |AB| \geq 0$ is a relative term, when there exist points $\exists\, O, E \in \ell_{AB}$, so that $|OE| = 1$. The relative magnitude we define by the ratio

(4.54) $$\Delta x = \frac{|AB|}{|OE|} \Rightarrow \qquad d = |AB| \in \mathbb{R}_+$$



From the scalar concept of the real numbers ℝ, where $d = \Delta x \in \mathbb{R}_+$ of a line segment AB, we can as we know form the concept of a number line that arranges the numbers along the line in order. A size order of magnitude of the real numbers ℝ is a ***quantitative*** system.

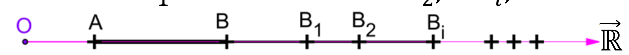
[190] The concept of straight for a line requires that there be points which are not on the straight line ℓ_{AB}. A line segment $d \sim d_{AB} \subset \ell_{AB}$ between points A and B can be discriminated straight in relation to a second curved line k between A and B in that there is at least one point K on k, $K \in k$, which is not on ℓ_{AB}, $K \notin \ell_{AB}$. The point K may thus not be on the one straight line through A and B, but K can belong to a parallel straight line $\ell_K \parallel \ell_{AB}$, $K \in \ell_K$, that thus belongs to a bundle of lines parallel to AB.

[191] Even when you point out (***direction***) at a point in the celestial sky, it is not an object you point to, but a subject P_∞. One point P_∞ is everywhere *out there*, thus it is infinity! In my opinion, this also applies to the concept of the *Big Bang*, which in its idea is finite? – or – only a platonic idea?

The system of points along a straight line is also a ***qualitative*** system, with the concept of |AB| for the real numbers as a ***quantity***.
In this way, the concept of numbers ℝ relate to the line concept.[192]
The real numbers ℝ, which are subject to addition, subtraction, multiplication, and division, with a commutative and distributive algebra, is assumed familiar to the reader.[160]
The real numbers with such an algebra constitute what we call a scalar field.[161]
With the concept of '*field*' we mean just that algebra for any ***category*** (scalar-, vector-, etc.) that can be applied everywhere in the space 𝔊 for physics. – A physical natural *field*.[193]

(vi) The concept of primary quality of first grade (pqg-1)

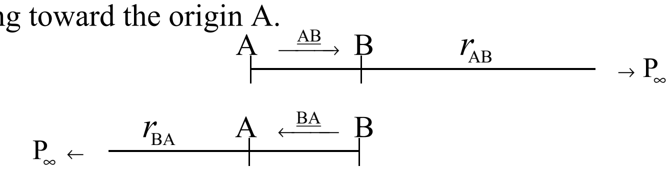
In that the difference between two points A and B can be assigned a ***quantity*** magnitude |AB| called a distance, all the points on a straight line ℓ_{AB} arranges in order as $|AB| < |AB_1| < |AB_2| < \cdots < |AB_i| < \cdots$ and hence the points $A, B, B_1, B_2, \ldots B_i, \ldots$ are located in a line on the straight line ℓ_{AB}. This order scheme indicates a ***direction*** from A out over B to B_1 and further on to $B_2, \ldots B_i, \ldots$. This ***direction*** we express as $\underline{AB}$.



According to *(iv)* we are able to express it $\underline{AB}_i = \underline{AB} + \underline{BB}_i$,
and when it is all done in a straight line it is $|\underline{AB}_i| = |\underline{AB}| + |\underline{BB}_i|$ addition of real numbers.
The fact, that it is possible to make a linear ordering of points by distances from one point A is called a ***primary quality of first grade*** (***pqg***-1), a concept of an ordered line, called $\vec{\mathfrak{L}}$, $\underline{AB} \in \vec{\mathfrak{L}}$. This ***pqg***-1 ***quality*** defines a ***direction*** concept to line concept 𝔏, as a part substance or a sub-substance to the 𝔊 space as a total substance.
We see that a ***pqg***-1 in space 𝔊 enables the concept of a ***quantity*** $[\mathbb{R}^1_{pqg\text{-}1}] \sim \vec{\mathbb{R}}$ in space.
The line concept is thus linked to the real numbers ℝ, a ***quantitative*** order $\vec{\mathbb{R}} \rightarrow \vec{\mathfrak{L}} \rightarrow \mathfrak{L}$, and thus *linear **direction*** is a ***primary quality of first grade*** (***pqg***-1).[194]

(vii) The concept of direction as quality of first grade

The real numbers progressive ordering of points as above $O, A, B, B_1, B_2, B_3, \ldots$ enables the concept of a ray $r_{OA} \subset \ell_{OA}$, a half line, that starts at a point O, goes through A and successively through $B, B_1, B_2, B_3, \ldots \in r_{OA} \subset \ell_{OA}$. To a line segment is added line segment etc...
For each point $P_i \in r_{OA}$ we attach a positive real number $x_i \in \vec{\mathbb{R}}_+$ with the mapping $\mathfrak{p}: \mathbb{R}_+ \rightarrow r_{OA}$, so that $P_i = \mathfrak{p}(x_i) \in r_{OA}$, where $O = \mathfrak{p}(0)$ is the start point.
We can use continuous indexing of the points, we write $P_x = \mathfrak{p}(x)$.
Just as two points indicate a ***direction*** $\underline{OA}$ the half-line ray $r_{OA} \subset \mathfrak{G}$ make a ***direction** outwards* away from O, which we call *positive*. In the order from O to A we write $\underline{OA}$, the *positive **direction*** away from O, has an *inverse* orientation (reverse order) from the A to O written $\underline{AO}$, which is the negative orientated ***direction***, pointing toward O, ***inwards*** to an origin O, the *inverse of **outwards***.
Further, we have the system from A to a point B written $\underline{AB}$ as a positive orientated ***direction*** pointing away from A, we have the *inverse* orientation reversed from B to A written $\underline{BA}$, as the negative orientated ***direction*** pointing toward the origin A.



[192] For the question of all real numbers related to the number line, please refer to the historical mathematical literature.
[193] It is important to distinguish from the *ideal pure mathematical* field concept in German *Zahl-Körper* (scalar fields): $\mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$.
[194] Here's the review of *(v)* and *(vi)* are reversed on the historical causality, as the real numbers are invented to calculate distances in space geometry. It is easier here to briefly explain the ***qualitative*** possibility for an ordering scheme of points along a line with the real numbers ***quantitative*** order system.

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