The Platonic ideal of a plane has its physical reality in a surface on which we can draw geometric shapes, lines, and points. The plane which objective surface only has exists in physics because we can view it from the external outside.
It is impossible to see the plane through a line in the plane. –
The Platonic ideal (noumenon) is a line having its physical reality in a beam of light (phenomenon). When we draw a line on a surface or make an edge of a ruler or blade, it is the only way to establish rectitude, to aim along the 'line' using light. This is an a priori synthetic judgment that light in general is a priori for information coming as our enlightenment.
Light is a prerequisite for a universal space idea: – Esse extensa per se. (Existence in itself.)

4.2.2. The Dimensions and *Qualitative Grades* of Geometric Space

We in general acknowledge, that a geometric space for physics has several dimensions.
Normally we estimated 3 spatial extension dimensions for the natural space.
The origin is René Descartes' idea of extension consisting of Aristotelian ***length***, ***breadth***, ***depth***.
The principal main reason is Kepler and Newton's inverse square law of the distance, later confirmed by the Jacobian determinant ($dV = \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$) for the two angular spherical coordinates for the spherical 2D surface of the S^2 symmetry concerning the Cartesian 3D cubic coordinate frame. It is Immanuel Kant 1747, who first performed sustainable reasoning of this limitation of natural dimensions.[178]

> …"the number of lines that can be drawn from a point at right angles to one another, I have thought of adding the threefold dimension of extension from that point to show what one perceives with the powers of numbers. The first three powers of the same are quite simple and cannot be reduced to any other, but the fourth, as the square, is nothing but a repetition of the second power. As good as this property of numbers seemed to me to explain the threefold space dimension from it, it did not hold up in practice. The fourth power is absurd in everything that we can imagine of space through the imagination. In geometry one cannot multiply a square with itself, nor a cube with its root; hence the necessity of the threefold dimension rests never on the fact that, if several elements are placed, one would do nothing else than repeat the previous ones"… (author translation).

Later in this book we will see this confirmed by an area bivector square to a negative scalar and the fact that the product of two independent bivector areas multiplies to a new bivector joint with a scalar and a pseudoscalar so that $\mathcal{G}_3(\mathbb{R})$ performs a closed algebra for a 3D $\mathfrak{Z}$-space, see in 6.3, e.g. Figure 6.14.
Further as displayed on the title page i. the root of a cube as a 1-vector ***direction*** u multiplied by its triv-ector chiral pseudoscalar (cube) gives a transversal area bivector all inside the geometric algebra $\mathcal{G}_3(\mathbb{R})$.
By the fourth power, we achieve information by using the expanded geometric algebra $\mathcal{G}_{1,3}(\mathbb{R})$ chapter 7.

Anyway often, we gain an advantage by using extra dimensions up $n \in \mathbb{N}$. First, we remember a development dimension of information, and on top of that, myriads of dimensions for mutual interactions.

What is new is that the various levels of dimensions up to $n \in \mathbb{N}$ give rise to different ***primary***[179] ***qualities***, each of which we assign a ***grade***, referred to as the ***primary quality of*** $r^{'th}$ ***grade*** (***pqg***-r), where r take values from 0 to n, $r = 0, \dots n$.

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[178] Immanuel Kant AK 1: Von der Kraft der Körper überhaupt. p.23 §9 i.13-33: „Weil ich in dem Beweise, den Herr von Leibniz irgendwo in der Theodicee von der Anzahl der Linien hernimmt, die von einem Punkte winkelrecht gegen einander können gezogen werden, einen Zirkelschluß wahrnehme, so habe ich darauf gedacht, die dreifache Dimension der Ausdehnung aus demjenigen zu erweisen, was man bei den Potenzen der Zahlen wahrnimmt. Die drei ersten Potenzen derselben sind ganz einfach und lassen sich auf keine andere reduciren, allein die vierte, als das Quadratoquadrat, ist nichts als eine Wiederholung der zweiten Potenz. So gut mir diese Eigenschaft der Zahlen schien, die dreifache Raumes=Abmessung daraus zu erklären, so hielt sie in der Anwendung doch nicht Stich. Denn die vierte Potenz ist in allem demjenigen, was wir uns durch die Einbildungskraft vom Raume vorstellen können, ein Unding. Man kann in der Geometrie kein Quadrat mit sich selber, noch den Würfel mit seiner Wurzel multipliciren; daher beruht die Nothwendigkeit der dreifachen Abmessung nicht sowohl darauf, daß, wenn man mehrere setzte, man nichts anders thäte, als daß die vorigen wiederholt würden (so wie es mit den Potenzen der Zahlen beschaffen ist), sondern vielmehr auf einer gewissen andern Nothwendigkeit, die ich noch nicht zu erklären im Stande bin.“ https://korpora.zim.uni-duisburg-essen.de/kant/aa01/023.html .

[179] ***Primary quality*** is by Galileo, Descartes, Newton, Locke, etc. a property of the thing itself, while the *secondary quality* is the perceived characteristic.

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4.3. The Idea of a Point - in the Geometric Space 𝔊

0 dimensions and the Concept of a Point (*pqg*-0)

4.3.1.1. No Extension. The Euclidean Elements:

Quote [12] : " Euclid's Elements:
E I.De.1. A *point* is that which has no part. "[180] (is indivisible)

a. A point is a location in space without properties (no extension).
b. The idea a point is not an object in space. A point is a subject in the substance of space, where the location in our intuition is linked to an ***entity*** as an object.
c. A cross, a dot, the center of a small circle, or another small thing on a surface can specify a location as the object for our intuition of the concept of a geometric point.

4.3.1.2. The *Quality* of the Concept of Points

Points as locations in the space we designated by the upright capital letters A, B, C, ... etc. as names. For points it is meaningful to say:
– No point – one point – several points – all points in space.
For a set ***D*** of points, as ***quality*** it is possible to judge:[181]

1. affirmative, if a point $\mathrm{A} \in \boldsymbol{D}$; or
2. in the negative, if that point $\mathrm{B} \notin \boldsymbol{D}$; or
3. infinity, all points X, i.e., $\forall \mathrm{X} \in \boldsymbol{D}$, which then forms a whole.

Where we presume, ***D*** is part of the space or the entire space 𝔊, $\boldsymbol{D} \subseteq \mathfrak{G}$.

(i) The Concept of Primary Quality for Points

The fact that it is possible, to designate locations in space at points we call ***a primary quality of zero grade*** (***pqg***-0).

(ii) The Concept of Equality and Diversity

The meaning of[182] $\mathrm{A} = \mathrm{A}$ and $\mathrm{A} = \mathrm{B}$ of the two distinct designations A and B for the same substance necessitates the negation of the meaning $\mathrm{A} \neq \mathrm{B}$. It must be possible to provide an A and B, where A and B are substantially different.
Points designated A and B may be the same point $\mathrm{A} = \mathrm{B}$ or different points $\mathrm{A} \neq \mathrm{B}$. [183]

(iii) Indexing

It is possible in addition to names A, B, C, ... to assign points indexing numbers 1,2,3, ... so that the points of the P_i for $i \in \mathbb{Z}$ are distinguishable $\mathrm{P}_1 \neq \mathrm{P}_2 \neq \mathrm{P}_3 \neq \cdots$. Indexing with real or complex scalars λ can also be accepted P_λ, even indexing of points with a scalar in several dimensions $\mathrm{P}_{\lambda_1 \dots \lambda_n}$, $\lambda \leftrightarrow (\lambda_1 \dots \lambda_n)$ can be used, when we just accept that over-representation may occur $\exists \lambda \neq \nu : \mathrm{P}_\lambda = \mathrm{P}_\nu$.
The same point occurs repeatedly, see (*ii*).
The indexing is a contingent accident; it adds no ***primary quality***.
A numbering adds no ***quantity*** to the concept of points, cause – a point is indistinguishable, and that points we are indexing with scalars assign no ***quantity*** to the point itself. One, two, three or four specified points is not a ***quantity***, but as we shall see *a pure* ***primary quality***.

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[180] E I.De.1. – stands for Euclid book I. first definition. The text colour indicates quoted from [12].
[181] Kant's *tripartite* of a judgment on ***quality***.
[182] The equality $\mathrm{A} = \mathrm{A}$ has only significand importance if it is not the same reference on both sides, the statement says only that A is an eternal ***entity***. (The negation $\mathrm{A} \neq \mathrm{A}$ allows that A is not the same in all cases or modes.)
The judgment A=A indicates an eternal ontology of a being A.
When A is dependent on a development parameter $t_1 \neq t_2$, we understand that $\mathrm{A}(t_1) \neq \mathrm{A}(t_2)$ can be allowed.
[183] According to Kant's ***category*** schema possible relationships between points are a ***quality***. The fact that we can talk about several points has nothing with ***quantity*** to do. About the concept ***quantity*** see section 4.4 of one dimension.