4.1.1.9. The Abstract 1-Dimensional Vector Space for Real Numbers

The fundamental difference in the real scalars is the natural difference between 0 and 1.
If we look at the set of real numbers in the interval $[0,1] \subset \mathbb{R}$, and include the order in this interval $\overrightarrow{[0,1]}$ and call it the basic vector $\vec{1} = \overrightarrow{[0,1]} \in \mathbb{R}^1_{\overrightarrow{[0,1]}}$ for a linear space over the set of real intervals

(4.20) $\quad \overrightarrow{[0,\lambda]} = \lambda\overrightarrow{[0,1]} = \lambda\vec{1} \quad \in \mathbb{R}^1_{\overrightarrow{[0,1]}}$, generated from $\overrightarrow{[0,1]} \subset \mathbb{R}$, over the scalars $\lambda \in \mathbb{R}$.

For $\lambda < 0$ we interpret $\overrightarrow{[0,\lambda]}$ as $-\overrightarrow{[\lambda,0]}$, a negative orientation with the order from λ to 0.
By addition, we note that $\overrightarrow{[0,1]}+\overrightarrow{[0,1]} = 2\,\overrightarrow{[0,1]} = \overrightarrow{[0,2]}$, thus we have scalar multiplication.
In addition to this, we note the translative identity

(4.21) $\quad \overrightarrow{[0,1]} = \overrightarrow{[\alpha,\alpha+1]}$ and $\overrightarrow{[0,\lambda]} = \overrightarrow{[\alpha,\alpha+\lambda]}$ for all $\forall\lambda,\ \forall\alpha \in \mathbb{R}$.

The dimension of the linear vector space of ordered real intervals is $\dim\left(\mathbb{R}^1_{\overrightarrow{[0,1]}}\right) = 1$

(4.22) $\quad \mathbb{R}^1_{\overrightarrow{[0,1]}} = \mathrm{span}\left\{\overrightarrow{[0,1]}\right\}$

If we take the union of all the intervals $\bigcup_{\forall\lambda\in\mathbb{R}}\overrightarrow{[0,\lambda]} = \vec{\mathbb{R}}$, we just reach the order of all the real numbers. We know of course that the real numbers' order a priori is implicitly given in the real numbers' construction $\mathbb{R}$, so we conclude $\mathbb{R} \sim \vec{\mathbb{R}}$.
But we distinguish anyway between the dimension of real numbers as a vector space $\dim\left(\mathbb{R}^1_{\overrightarrow{[0,1]}}\right) = 1$ and the real numbers dimension $Dim(\mathbb{R}) = 0$ as scalars. The reasons are:
The individual real numbers as scalars $\alpha \in \mathbb{R}$ has no extension: $\overrightarrow{[\alpha,\alpha]} = \overrightarrow{[1,1]} = \overrightarrow{[0,0]} = 0$
The real scalars have no proper basis vector. This is formulated abstractly

(4.23) $\quad \{0\} = \mathrm{span}\{0\} = \mathrm{span}\left\{\overrightarrow{[0,0]}\right\} = \mathrm{span}\left\{\overrightarrow{[1,1]}\right\} = \mathrm{span}\left\{\overrightarrow{[\alpha,\alpha]}\right\} = \mathrm{span}\{\alpha\}$ for $\forall\alpha \in \mathbb{R}$.

therefore

(4.24) $\quad Dim(\mathbb{R}) = Dim(\mathrm{span}\{0\}) = Dim(\mathrm{span}\{1\}) = Dim(\mathrm{span}\{\alpha\}) = 0$ for $\forall\alpha\in\mathbb{R}$

Remember, you can never measure the extension of a single real number. Historically it has been usual not to distinguish between scalars $\mathbb{R}$ and a 1-dimensional linear vector space $\mathbb{R}^1$.
However, for an understanding of physics, it is a necessity.
Mandatory, we must distinguish the different ***qualities*** of all the ***quantities*** in physics.
That is in short, the basic idea of what Immanuel Kant considered in the concept:

(4.25) ***The Categorical Imperative***

YOU MUST decide the ***categories*** of all the ***qualities***, ***quantities***, dependencies, etc.
If you do not, you know nothing! After this, you can start measuring the specific ***quantities***.

One must know the ***quality*** of something!

4.1.2. The Real Spatial Linear Vector Space

We view by intuition a spatial vector space $\mathbb{R}^1_1$ by drawing the basis vector $\hat{1}_1 \in \mathbb{R}^1_1$ on a surface, e.g., on paper as in Figure 4.1, and form the number line generated by a linear form

(4.26) $\quad x = \lambda_1\hat{1}_1$, where $x \in \mathbb{R}^1_1$ and $\lambda_1 \in \mathbb{R}$. The number line is $\mathbb{R}^1_1 \sim \{x = \lambda_1\hat{1}_1 \mid \forall\lambda_1 \in \mathbb{R}\}$.

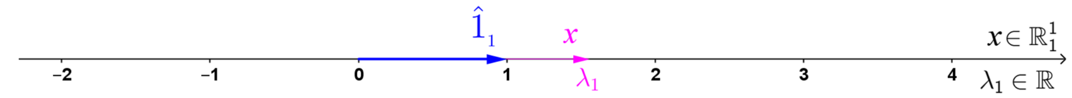

Figure 4.1 Number line for the real scalars is produced as a vector by the linear form $x = \lambda_1\hat{1}_1$.

The number line is a spatial representation of abstract real numbers. the geometric vectors are an analogy that are objects along the number line and thus the object for intuition.
The object, the number line we can also consider, as a timeline (4.19) given by $t = \tau\hat{1}_0$.
The scalar subjects τ is in this way mapped into space consisting of the number line on the surface in Figure 4.1, as the functionalities: $\tau \rightarrow t = \tau\hat{1}_0 \rightarrow x = \lambda_1\hat{1}_1$.

4.1.2.2. Multiple Linear Spatial Dimensions

We form other real dimensions of $\mathbb{R}^1_i$, which we together by the direct sum of the different $\mathbb{R}^1_i$ spaces write $\mathbb{R}^n = \underbrace{\mathbb{R}^1_1 \oplus \ldots \mathbb{R}^1_i \oplus \ldots \mathbb{R}^1_n}_{n}$.
The most well-known concept of a multi-dimensional vector space is the Cartesian[169] coordinate system on the paper in two dimensions $\mathbb{R}^2_{xy} = \mathbb{R}^1_x \oplus \mathbb{R}^1_y$, here indicated by x and y. We have the two number lines $\mathbb{R}^1_x \sim \{x = \lambda_x\hat{1}_x\}$ and $\mathbb{R}^1_y \sim \{y = \lambda_y\hat{1}_y\}$, that intersects each other at one point O. We call them coordinate axes, respectively $x \in x$-axis and $y \in y$-axis, see Figure 4.2. The requirement for a Cartesian coordinate system is geometrically perpendicular coordinate axes from orthogonal basis vectors $\hat{1}_x \perp \hat{1}_y$.
The two together form a basis vector set $\{\hat{1}_x, \hat{1}_y\}$ of the vector space $\mathbb{R}^2_{xy} = \mathbb{R}^1_x \oplus \mathbb{R}^1_y$.
The two scalars $\lambda_x \in \mathbb{R}$ and $\lambda_y \in \mathbb{R}$ generates from the real scalar substance by the two linearly independent generator basis vectors $\hat{1}_x$ and $\hat{1}_y$ set the two vectors $x = \lambda_x\hat{1}_x$ and $y = \lambda_y\hat{1}_y$.
From axes intersection O the two vectors x and y are pointing at points on each axis.
The sum of these two vectors $a = x + y = \lambda_x\hat{1}_x + \lambda_y\hat{1}_y$ point out from O a point A in the coordinate plane γ_{xy}. – We call the number set (λ_x, λ_y) the coordinates of the point A ← (λ_x, λ_y).

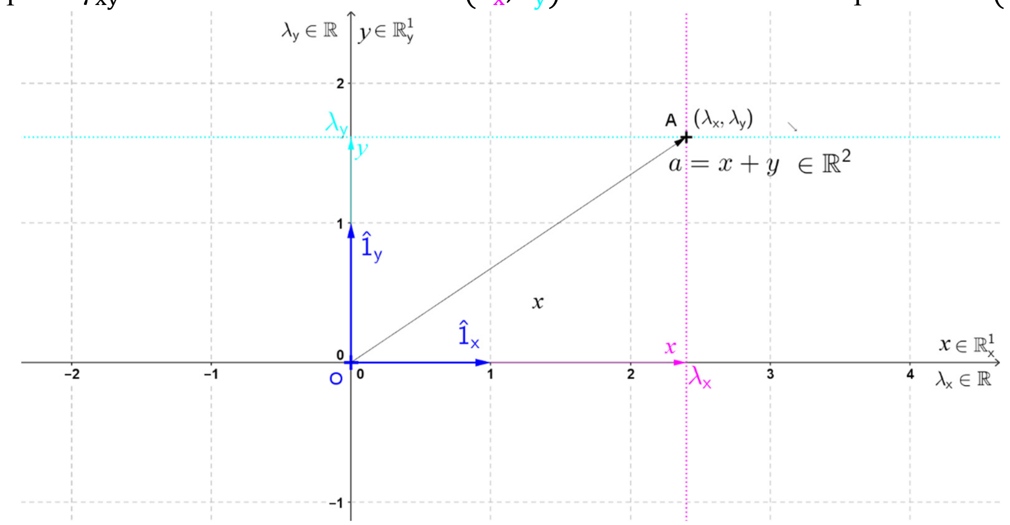

Figure 4.2 Cartesian coordinate system produces the plane from two real scalars through a vector addition.

[169] The term 'Cartesian' refers to Descartes, who invented the coordinate points in the plane with a pair of numbers (x, y) in a rectangular grid of squares or rectangles.