We conclude immediately that the intuitive vector space over $\mathbb{K}$, the object $\mathbb{K}^1 \sim (\mathbb{K}^1, \mathbb{K}) \sim \mathbb{K}_1$ has a recognizable dimension $Dim(\mathbb{K}^1) = \dim(\mathbb{K}^1) = 1$, because the generator $\hat{1} \in \mathbb{K}^1$ can be intuited as an object and thus form the basis of which the vector space $\mathbb{K}^1 = \mathrm{span}\{\hat{1}\}$ is an object for us.
This concept is first used in this book for the count and the past-to-future ***direction*** , .

4.1.1.5. Definition of One Dimension of First Grade

We say that the $\mathbb{K}^1$ vector space substance is inheriting its ***quality*** from the subject $\hat{1} \in \mathbb{K}^1$, which by necessity must be a recognisable object for our ethical intelligible intuition.

1. We have that scalars called $\mathbb{K}$ have no natural extension $Dim(\mathbb{K}) = 0$.
2. Immanuel Kant's **a priori transcendental** concept of ***direction***, ***a qualitative*** subject called $\hat{1}$, that we must represent as one for us selected object for intuition.[164]
3. The scalar $1 \in \mathbb{K}$ is multiplicative neutral to $\hat{1} = 1\hat{1}$. Obviously, $\hat{1}$ is not a scalar, but $\hat{1} \in \mathbb{K}^1$.
4. We form $\mathbb{K}^1 = \mathrm{span}\{\hat{1}\} = \{ a = \lambda\hat{1} \mid \lambda \in \mathbb{K} \}$. All the elements $\forall a \in \mathbb{K}^1$ are also called for ***elements of first grade***. The linear vector space dimension $\dim(\mathbb{K}^1) = \dim(\hat{1}) = 1$.

Alternatively, the concept $\hat{1}$ can also be called for a unit pseudo-element ***of the first grade***.
The pseudo-element concept of first ***grade*** is that, when orientation is reversed (from gain to loss) of the basis subject ***direction***, the entire $\mathrm{span}\{\hat{1}\}$ space reverses its ***direction*** to the opposite orientation. The conclusion is when the scalars $\mathbb{K}$ is seen as a 1-dimensional vector space $\mathbb{K}^1$ it becomes a pseudo-element space ***of first grade***. (In a one-dimensional world, no extra dimensions.)

4.1.1.6. The Simplest Multidimensional Space

The direct sum of the different $\mathbb{K}_i^1$ spaces can be written as $\underbrace{\mathbb{K}_1^1 \oplus \ldots \mathbb{K}_i^1 \oplus \ldots \mathbb{K}_n^1}_{n} = \mathbb{K}^n$.

It is traditional to write the direct sum of n vector spaces $\mathbb{K}_i^1$ as $\mathbb{K}^n$, although[165] the linear algebra is vector additive (4.1)-(4.4), and only multiplicative in terms of scalars (4.5)-(4.9).
We will write a linearly independent basis set for $\mathbb{K}^n$ as $\hat{1}_1, \ldots \hat{1}_n$, and then for any vector in the space $\mathbb{K}^n$ we write the linear form for $\lambda_i \in \mathbb{K}$, $i = 1, \ldots n$, as the combination

(4.17) $$a = \lambda_1 \hat{1}_1 + \ldots \lambda_n \hat{1}_n , \qquad \text{where } \hat{1}_i \in \mathbb{K}_i^1 , \quad \text{then } a \in \mathbb{K}^n, \quad \text{with} \quad \dim(\mathbb{K}^n) = n.$$

The basis set spans the vector space $\mathbb{K}^n = \mathrm{span}\{\hat{1}_1, \ldots \hat{1}_n\}$. Imagine $\hat{1}_i \in \mathbb{K}_i^1$ as physical objects.

4.1.1.7. The Real Numbers as a Vector Space

We assume the reader is familiar with the abstract nature of real numbers $\lambda \in \mathbb{R}$; from the counting of the natural numbers $\mathbb{N}$, the algebraic addition, subtraction (whole numbers $\mathbb{Z}$), multiplication and division (the rational numbers $\mathbb{Q}$) over the infinite series (and irrational geometric relationships) to the abstraction the real numbers $\mathbb{R}$.
The idea of the real numbers $\mathbb{R}$ is, in principle, an abstract scalar design of the thought, which according to Descartes's philosophy has no extension in the natural space of physics.
We look at the real numbers $\mathbb{R}$ and form the corresponding vector space $\mathbb{R}^1 \sim (\mathbb{R}^1, \mathbb{R})$ over the real scalars. As above, a natural basis for $\mathbb{R}$ the multiplicative neutral scalar $1 \in \mathbb{R}$.
The natural basis set $\{1\} \in \mathbb{R}^1$ and linear form $\lambda = \lambda 1$ produces simply the real scalars $\mathbb{R}$, as the individual scalar λ has no extension, in the natural space.[166]
Therefore, the a priori synthetic judgment: $\mathbb{R}$ has no recognisable dimension $Dim(\mathbb{R}) = 0$.

[164] The subject of the ***direction*** is here an internal ***quality***, but an external ***quality*** as an object for us is much more complicated. E.g., a simple object example is one able $\{\hat{1}\}$, then counting is the span, the amount is the ***direction***, and gain or loss is the orientation.
[165] $\mathbb{K}^n$, n appears as exponent factors, even the direct sum implies the sum of n linear independent vectors. Weird tradition!
[166] The lack of extension in natural space for scalars used in physics will be treated in further detail in later chapters. The lack of spatial extension of course also applies to other types of scalars $\mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$.

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When in contrast to this we construct an object (e.g. as later (4.26) shown in Figure 4.1), which is recognisable by intuition, here referred to $\hat{1}$ as the basis vector for the vector space $\mathbb{R}^1$, then we can for every vector $x \in \mathbb{R}^1$ in the vector space write

(4.18) $$x = \lambda\hat{1},$$

where the scalar $\lambda \in \mathbb{R}$ is coordinate to the vector $x \in \mathbb{R}^1$ out from the generator $\hat{1} \in \mathbb{R}^1$.
We note that $\dim(\mathbb{R}^1) = 1$. Many write it as $\dim_{\mathbb{R}}(\mathbb{R}) = 1$, or confusing simply $\dim(\mathbb{R}) = 1$, where the real numbers are interpreted as a linear 'vector' space over the real scalars $\mathbb{R}$.
Interpreting the real numbers by intuition as a vector space, we write $\mathbb{R}^1$ (1-dimensional) and interpreting as real scalars, we simply write $\mathbb{R} \coloneqq \mathbb{R}^0$. Real scalars $\lambda \in \mathbb{R}$ have no extension in natural space! I would advocate[167] for the designation $Dim(\mathbb{R}) = Dim(\mathbb{R}^0) = 0$
The conclusion is that when the real numbers $\mathbb{R}$ is seen by intuition as a 1-dimensional vector space they become pseudoscalars $\mathbb{R}^1$, ***of first grade***, where the orientation switches with the ***direction quality*** $\hat{1}$ generator AB↔BA or illustrated for the events A and B as sequential counts.

4.1.1.8. The Reality of Time as a Vector Space

If we look at the real numbers' multiplicative neutral element $1 \in \mathbb{R}$ it has no extension in the natural space but it is part of the natural number order index by counting $1 \in \vec{\mathbb{N}}$.
The real numbers have a familiar order relationship, where the figures we consider represent a growing system order $\ldots \omega < 0 < \alpha < \beta < \gamma \ldots$ for the real numbers that we here represent by $\vec{\mathbb{R}}$.
We can e.g., use a rule regulation $0 < \delta_0 < 1 < \delta_1 < 2 < \cdots$ to synchronisation time counting with the natural numbers $\vec{\mathbb{N}}$.
The counting process is constantly adding the number 1 to a previous number λ by which we get $\lambda{+}1 \overset{+1}{\longleftarrow} \lambda$. We call $+1$ the counting operator. (alternative from the right: $\lambda \overset{+1}{\longrightarrow} \lambda{+}1$.) see .

We look at the ***entity*** that separates between two successive counts as a basis vector called $\hat{1}_0$ that we will associate with the actual interval $[0,1] = \{\forall\delta \in \mathbb{R} \mid 0 \leq \delta \leq 1\}$. This is then the real time interval, that elapses between each count. A unit of times $\hat{1}_0$ is viewed with intuition as generating a real number subject in the substance '**to count**'.
To count times is what the tradition calls time, this is *the founding concept of time*.
The foundation of time can never be anything else than to count some ***quantum***.
Through timing basis vector $\hat{1}_0$ the ordered real numbers $\vec{\mathbb{R}}$ achieves the character of a 1-dimensional linear vector space $\dim(\vec{\mathbb{R}}_0^1) = 1$. We, therefore, have some equivalence between. $\mathbb{R}^1 \sim \vec{\mathbb{R}}_0^1$, where the last represents the monotone raising continues development.
A linear time for the intuition is expressed by a linear form from a unit basis vector $\hat{1}_0 \in \vec{\mathbb{R}}_0^1$

(4.19) $$t = \tau\hat{1}_0 \quad \in \vec{\mathbb{R}}_0^1 \sim \mathbb{R}^1,$$

where $\tau \in \mathbb{R}$ is the real scalar for the time vector: $t \in \vec{\mathbb{R}}_0^1$.
The basis vector $\hat{1}_0$ generates the ***direction*** of time: $\vec{\mathbb{R}}_0^1 = \mathrm{span}\{\hat{1}_0\}$.
Index 0 in $\vec{\mathbb{R}}_0^1 \sim \mathbb{R}^1$ is just the ordinary conventional indices for a time dimension $t \sim x_0$.
We now understand that we can interpret time as a real vector space where the generating basis vector $\hat{1}_0$ is **to count**. In reality, we may only count *FORWARD* $(+1)$, therefor we prefer the term $\vec{\mathbb{R}}_0^1$ for the 1-dimensional ordered vector spaces for time.[168]

[167] Note the difference between the traditional dimension designation for a vector space $\dim(V)$ and the designation $Dim(A)$ for the dimension of the physical extension of an object for the abstract substance A.
[168] In this pure one-dimensional world time is a strange pseudo-element. Later when we understand geometric algebra it behaves both as a pseudovector and a pseudoscalar, when there is no interaction with other dimensions. One aspect is that it will have a opposite metric signature to the extension dimensions in a Minkowski space. This concept has made some literature use the consequence of this as $(\hat{1}_0)^2 = -1$, but we will use the metric standard $\gamma_0 \equiv \hat{1}_0$, $\gamma_0^2 = 1$ with three extension dimensions $\gamma_k^2 = -1$.