(3.232)
$$\begin{aligned} |\psi_{\Sigma\pm\odot\perp\hat{\vec{\omega}}}(t)\rangle &= A\left(N_+ a^\dagger_{\odot+\omega} + N_- a^\dagger_{\odot-\omega}\right)_{\perp\hat{\vec{\omega}}} |0,0\rangle \\ &\leftrightarrow \left[\left(\begin{matrix} A\sum_{n^+=1}^{N_+}\psi^{\odot}_{+\vec{\omega}}(\phi+\theta_{n^+}) \\ |\phi| \end{matrix}\right) + \left(\begin{matrix} A\sum_{n^-=1}^{N_-}\psi^{\odot}_{-\vec{\omega}}(\phi+\theta_{n^-}) \\ |\phi| \end{matrix}\right)\right]_{\hat{\vec{\omega}}} \\ &\leftrightarrow \left(\begin{matrix}\odot A2\tilde{r}(\rho)\left(\sum_{n^+=1}^{N_+} e^{+i(\omega t+\theta_{n^+})} + \sum_{n^-=1}^{N_-} e^{-i(\omega t+\theta_{n^-})}\right) \\ -ct \end{matrix}\right)_{\vec{\omega}} \quad \text{for } \begin{cases}\forall\rho\ge 0\\ \phi=\omega t\\ t\ge 0\end{cases} \end{aligned}$$

(3.232a)
$$\leftrightarrow \quad A(N_+ + N_-)\left(\sum_{n^+=1}^{N_+} e^{+i(\omega t+\theta_{n^+})} + \sum_{n^-=1}^{N_-} e^{-i(\omega t+\theta_{n^-})}\right) \circledcirc\perp ct\,\vec{\omega}/|\omega|$$

where the normalization gives $\sum\langle\psi|\psi\rangle = 1 \Rightarrow A = 1/(N_+ + N_-)$.

The normalization factor A ensures that the transversal cross-section $\circledcirc\perp\hat{\vec{\omega}}$ of the total monochromatic plane transversal wave is the same as for a single ***subton*** as in (3.220). the normalization does not affect the total wave propagates into the future given by $x_3 = -c|\phi|/|\omega| = -ct$, but only the transversal magnitude part. This is an example of, how to understand the 'mystery' of standards of normalization for wavefunctions. Here it is the ***subtons*** with their extension through there development space. The wavefunction magnitude of the wave is just not additive but rather joined in an indistinguishable unit community.

3.4.4.4. The Amplitude for a Monochromatic Transversal Plane Wave

The total intensity of the transversal wave is the sum of all the individual ***subtons*** intensity from (3.178) $\mathcal{I}(\rho,\theta)$ collected to $(N_+ + N_-)\mathcal{I}(\rho,\theta)$, to be integrated.

The power (quantum momentum energy flow) is then from (3.179) and (3.180) collected to $(N_+ + N_-)\hat{\vec{\omega}}$; this is seen in the autonomous picture of the wave.

By intuition we appoint the external field from one ***subton*** as

(3.233)
$$W^1_{\pm\odot\perp\vec{\omega}}(t) = \left(e^{\pm i\omega t}\right)_{\perp\vec{\omega}} \sim^{120} \sim e^{\pm i\omega t}\circledcirc\perp\vec{\omega}$$

and similar symbolised the field of several ***subtons*** $N_+ + N_-$ from (3.232) as

(3.234)
$$W^{N_+ + N_-}_{\Sigma\pm\odot\perp\vec{\omega}}(t) \sim |\psi_{\Sigma\pm\odot\perp\vec{\omega}}(t)\rangle.$$

The autonomous normed power (energy flow) for this field in the ***direction*** $\hat{\vec{\omega}}$ is then defined as

(3.235)
$$\left|W^{N_+ + N_-}_{\Sigma\pm\odot\perp\vec{\omega}}(t)\right|^2 \hat{\vec{\omega}} = (N_+ + N_-)\,\hat{\vec{\omega}},$$

Assuming that the **amplitude** of the field is proportional to $\sqrt{\text{power}}$, we see that the autonomous normed field amplitude is

(3.236)
$$\left|W^{N_+ + N_-}_{\Sigma\pm\odot\perp\vec{\omega}}(t)\right| \sim \sqrt{N_+ + N_-}$$

From this, we can specifically intuit the field as indicated by the formula

(3.237)
$$W^{N_+ + N_-}_{\Sigma\pm\odot\perp\vec{\omega}}(t) = \sqrt{N_+ + N_-}\left(\sum_{n^+=1}^{N_+} e^{+i(\omega t+\theta_{n^+})} + \sum_{n^-=1}^{N_-} e^{-i(\omega t+\theta_{n^-})}\right)_{\perp\vec{\omega}}$$

It is important to note, that the amplitude of the field is essentially different from the thickness of the transversal extension $\overset{\odot}{\tilde{r}}(\vec{\omega}) = \frac{c1}{|\omega|} = \frac{\lambda}{2\pi}$ of the beam in the ***direction*** of ***subtons***. More about the field amplitude below (3.251), (3.252), and in section 3.5.1 macroscopic modulation.

[120] The complex number $(e^{\pm i\omega t})_{\perp\vec{\omega}} \in \mathbb{C}$ with indices $\perp\vec{\omega}$ to remember that it is represented in the transversal plane $\odot$ or $\circledcirc$, which is omitted since an external phase for a macroscopic field must have an external angular reference $\theta_0 = 0$.

3.4.5. Linearly Polarization

3.4.5.1. Is the Idea of a Linearly Polarized 'Photon' an elementary particle?

When both $N_+ = 1$ and $N_- = 1$ we get as in (3.226) the superposition of two ±***subtons***

(3.238)
$$\begin{aligned} |\psi_{\Sigma^2\pm\odot\perp\hat{\vec{\omega}}}(t)\rangle &= \tfrac{1}{2}\left(a^\dagger_{\odot+\omega} + a^\dagger_{\odot-\omega}\right)_{\perp\hat{\vec{\omega}}}|0,0\rangle = \odot\tilde{r}(\rho)\left(e^{+i\phi} + e^{-i\phi}\right)_{\perp\hat{\vec{\omega}}} \\ &= \tilde{r}(\rho)\left(e^{+i(\omega t+\theta'+\theta)} + e^{-i(\omega t+\theta'-\theta)}\right)_{\hat{\vec{\omega}}} = \odot 2\tilde{r}(\rho)\cos(\omega t+\theta')_{\perp\hat{\vec{\omega}}},\ \text{for } \forall\rho\ge 0,\ \phi = \omega t+\theta'. \end{aligned}$$

Again, we have included the transversal rotational symmetry $\odot\perp\hat{\vec{\omega}}$ to the ***direction***.
The phase angle difference θ' exists as an angular ***quantity*** in the transversal plane.
By the simultaneous double creation of the two ±***subtons*** $\omega_+ = \omega_-$ it must therefore be possible to modulate the phase. I.e. it is possible to seat the information as a fixed angular ***quantity*** of the superposition related to the same unit vector $\hat{\vec{\omega}}$, which then flows out into the future $|\phi|\hat{\vec{\omega}}$ with the same angular parameter $|\phi| = |\omega|t$ for both, as a production of the past. The formal linking between the two opposed ±***subtons*** together into one ***entity***, the idea $\vec{L}^+_3 = +\hat{\vec{\omega}}$ joined with $\vec{L}^-_3 = -\hat{\vec{\omega}}$, ($\hbar = 1$). One way to form a proper vector superposition of this effect is $\vec{\omega}_{superposition} = \vec{L}^+_3 - \vec{L}^-_3 = 2\hat{\vec{\omega}}$.
As such, a ***direction*** of (3.226) and (3.238) into the future has meaning, and the idea of the transversal with $\odot\perp\hat{\vec{\omega}}$ is maintained. If we look at Hamilton's eigenvalue equation (3.166) for this double superposition state-mode $|\psi_{\Sigma^2\pm\odot\perp\hat{\vec{\omega}}}(t)\rangle$

(3.239)
$$\hat{H}_\omega|\psi_{\Sigma^2\pm\odot\perp\hat{\vec{\omega}}}(t)\rangle = \hbar\omega\left(a^\dagger_+ a_+ + a^\dagger_- a_- + 1\right)|\psi_{\Sigma^2\pm\odot\perp\hat{\vec{\omega}}}(t)\rangle \doteq \hbar\omega(1+1+1)|\psi_{\Sigma^2\pm\odot\perp\hat{\vec{\omega}}}(t)\rangle,$$

we get a double occupied energy eigenvalue $\hbar\omega(1+1) = 2\hbar\omega$, while the ground state virtual holds the potential energy $\hbar\omega$ of the angular frequency ω as an idea. In this way, we see that the idea of linear polarization of frequency energy having a ***direction*** into the future, consists of a pair of ***subtons*** of opposite rotational orientation.
Putting the energy eigenvalue of $|\psi_{\Sigma^2\pm\odot\perp\hat{\vec{\omega}}}(t)\rangle$ to $\hbar\omega_2 = \hbar\omega_{\text{superposition}} = \hbar 2\omega$ with the measurement reference $|\hat{\vec{\omega}}| = |\hat{\omega}| = 1$ and the wave ***direction*** $\mathbf{e}_3$ of the linear polarized superposition (3.226) or (3.238). This ***direction*** comes out from (3.222)

(3.240)
$$\vec{\omega}_2 = \omega_2\mathbf{e}_3 = 2\omega\hat{\omega}\mathbf{e}_3 \quad\leftrightarrow\quad 2\hat{\vec{\omega}} = 2\vec{1} = \vec{2} \qquad \text{auto-norm measured, and } \hat{\vec{\omega}}\parallel\mathbf{e}_3$$

Writing $|\psi_{\Sigma^2\pm\odot\perp\hat{\vec{\omega}}}(t)\rangle \simeq \left|\psi^{\vec{2}}_{\odot_{\theta'}\perp\omega\mathbf{e}_3}(t)\right\rangle = 2\tilde{r}(\rho)\cos(\omega t+\theta')_{\mathbf{e}_3}$, where $\theta' \leftrightarrow \odot_{\theta'}$ is a modulating relative ***direction***[121] in the transversal plane perpendicular to the ***direction*** $\mathbf{e}_3\perp\odot_{\theta'}$ into the future, where $\mathbf{e}_3\parallel\hat{\vec{\omega}}$.
We compare with Hamiltonian eigenvalue equation from (3.15) for the *linear*[122] harmonic oscillator

(3.241)
$$\boxed{\hat{H}_\omega|\psi\rangle = \hbar\omega_2\left(a^\dagger a + \tfrac{1}{2}\right)|\psi\rangle \quad\doteq\quad E_\omega|\psi\rangle = \hbar\omega_2\left(1+\tfrac{1}{2}\right)|\psi\rangle}$$

and this compared with (3.239) is

(3.242)
$$\hat{H}\left|\psi^{\vec{2}}_{\odot_{\theta'}\perp\mathbf{e}_3}(t)\right\rangle = E_\omega\left|\psi^{\vec{2}}_{\odot_{\theta'}\perp\mathbf{e}_3}(t)\right\rangle \doteq \hbar\omega_2\left(1+\tfrac{1}{2}\right)\left|\psi^{\vec{2}}_{\odot_{\theta'}\perp\mathbf{e}_3}(t)\right\rangle = \hbar\omega(1+1+1)\left|\psi^{\vec{2}}_{\odot_{\theta'}\perp\mathbf{e}_3}(t)\right\rangle.$$

Thus, we see a linearly polarized ***entity*** $\Psi^{\text{linear}}_\omega$ in physics with the angular frequency energy ω is ***quantised*** exited with the energy quantum $\hbar 2\omega$, compare with (3.152)

(3.243)
$$2^1 E_\omega = 2E_{\omega,1} - E_{\omega,0} = \hbar 2\omega$$

[121] As we shall see in a later chapter, a rotation will be expressed as a multi-vector given by two unit-vectors $\mathbf{e}_{\theta=0}$ and $\mathbf{e}_{\theta'}$ of a plane with an angle $\theta' = \angle(\mathbf{e}_0, \mathbf{e}_{\theta'})$, namely a geometric vector product $\mathbf{e}_0\mathbf{e}_{\theta'} = e^{i\theta'}$ exponential function of a bi-vector $i\theta'$. – Modulating means that something determines the relative angle.
[122] Along a geometric straight line, or just one dimension in a ***quantity*** space. – What ***quality*** has such a space?