The light measures the depth of the space as the 'time' the development parameter is underway. The angular phase development in the light provides in its substance the depth into the past from the light emitted (created) to the received (annihilated). In principle, the ***subton*** counts the numbers of oscillations $|\phi| = |\phi_A - \phi_B|$ backward in the past and it is this ***quantity***, that is *invariant against a Lorentz gauge transformation*. The rationale is that annihilation is autonomous and therefore always *immediate local* in measuring the angular phase development joint to the creation of the ***subton***. (The internal development is a *null* curve interpreted by the external, see chapter III. 7.)

Here the ***subton*** ${}^{AB}\psi_{\pm\widehat{\overline{\omega}}} \leftrightarrow {}^{B}a_{\odot\pm\widehat{\overline{\omega}}}\ {}^{A}a^{\dagger}_{\odot\pm\widehat{\overline{\omega}}}|0{,}0\rangle$ defines the invariant quantity $|\phi| = |\phi_A - \phi_B|$

and measurement at event B can by filtration determine the ***direction*** $\widehat{\overline{\omega}} \sim \omega\widehat{\overline{\omega}}$ and frequency $|\omega|$. This ω depends on the observer reference frequency $[\widehat{\omega}]$, (and this reference can depend on a Lorentz transformation) just like the extension measurement

$$ {}^{AB}x_3 = c(t_A - t_B) = c|\phi_A - \phi_B|/|\omega| \tag{3.216}$$

Now, we have examined the possibility of the idea of the quantum harmonic oscillator with angular momentum (the circle oscillator) to create a *FORWARD* ***direction*** in space-time A through the transversal plane ***direction***.

Before we proceed to the description of a possible interpretation of space, we will elaborate more on the quantum development phase angle and parameter that is important for the information of excited state as a ***subton*** idea.

3.4.3. The Phase Angle and the Parameter Dependent States

For a parameter, the independent eigenstate $|\psi_n\rangle$ applies the eigenvalue equation as (3.31) $\widehat{N}|\psi_n\rangle = n\,|\psi_n\rangle$, where the counting number operator $\widehat{N} = a^{\dagger}a$ is defined as first to annihilate (lower\remove) $a|\psi\rangle$ and then to create (increase\restore) $a^{\dagger}|\psi\rangle$ the state.

In the transversal plane oscillating state, we have seen double degeneration (3.101) $\widehat{N}_{\pm} = a^{\dagger}_{\pm}a_{\pm}$, where the angular momentum operator is synthesised from the anti-symmetry (3.103)

$$\widehat{L}_3 = \left(a^{\dagger}_{+}a_{+} - a^{\dagger}_{-}a_{-}\right)\hbar = \left(\widehat{N}_{+} - \widehat{N}_{-}\right)\hbar. \tag{3.217}$$

Because of the parity inversion symmetry discussed in section 3.3.1 only single excitations $n=1$ in a transversal plane are considered. Therefore, we write the parameter independent stationary Schrödinger equation as $\widehat{H}|\psi_1\rangle \doteq E_1\,|\psi_1\rangle$. We reintroduce the given ***quantity*** ω for the harmonic oscillator as in (3.166) $\widehat{H}_{\overline{\omega}}|\psi_{\pm 1}\rangle \doteq E_{\omega,\pm 1}\,|\psi_{\pm 1}\rangle$ this energy eigenvalue is given by (3.169), $E_{\omega,1} = (1+1)\hbar\omega$, as we remember from (3.102) and (3.103) that

$$\widehat{H}_{\omega}|\psi\rangle = \hbar\omega\left(a^{\dagger}_{+}a_{+} + a^{\dagger}_{-}a_{-} + 1\right)|\psi\rangle = \hbar\omega\left(\widehat{N}_{+} + \widehat{N}_{-} + 1\right)|\psi\rangle \doteq \hbar\omega(n_{+} + n_{-} + 1)|\psi\rangle, \tag{3.218}$$

$$\widehat{L}_3|\psi\rangle = \hbar\left(a^{\dagger}_{+}a_{+} - a^{\dagger}_{-}a_{-}\right)|\psi\rangle = \hbar\left(\widehat{N}_{+} - \widehat{N}_{-}\right)|\psi\rangle \doteq \hbar(n_{+} - n_{-})|\psi\rangle, \tag{3.219}$$

wherein $n_{+} + n_{-} = 1$ (or $=0$). We say ideologically that the operator $\widehat{N}_{\pm}$ tests the state $|\psi\rangle$ and find the quantum number $n_{\pm}$ either for + or − for a circle oscillator rotation with angular momentum $n_{+} - n_{-} = m = \pm 1$ (or non).

Hence only with progressive or retrograde helicity from developing transversal rotation.

The parameter independent factor in the transversal excited state, we described (3.194)-(3.196) as

$$|\psi\rangle \leftrightarrow \circledcirc \leftrightarrow \begin{pmatrix}\tilde{r}(\rho)\odot \\ 0\end{pmatrix} = \begin{pmatrix}\tilde{r}(\rho)e^{i\theta} \\ 0\end{pmatrix} \in \begin{pmatrix}\mathbb{C} \\ \mathbb{R}\end{pmatrix} \quad \text{for} \quad \forall\rho \in \mathbb{R},\ \ \forall\theta \in [0,2\pi[, \tag{3.220}$$

but the creation operator $a^{\dagger}_{\pm}$ from (3.128) and (3.129) also contain multiply an active information development factor $e^{\pm i\phi}$ to the ***direction*** $\widehat{\overline{\omega}}$, then

$$|\psi_{\pm\widehat{\overline{\omega}}}(\phi)\rangle = e^{\pm i\phi}|\psi\rangle \leftrightarrow \begin{pmatrix}\psi^{\odot}_{\pm\overline{\omega}}(\phi) \\ -|\phi|\end{pmatrix} \sim \begin{pmatrix}\tilde{r}(\rho)\odot e^{\pm i\phi} \\ -|\phi|\end{pmatrix} \leftrightarrow e^{\pm i\phi}\circledcirc\perp c|\phi|\,\widehat{\overline{\omega}}, \text{ where } |\phi| = |\omega t|. \tag{3.221}$$

As argued in section 3.3.4 the two-dimensional circle oscillator rotation is transmitted by this transversal idea through the third dimension of information into the development space.

It is this transmitting single state we call a ***subton***, for which we above introduced the concept of a ***direction*** $\vec{n} = \widehat{\overline{\omega}}$ perpendicular to the concept of a unitary rotation $\odot = \left\{U_{\theta}\colon \theta \to e^{i\theta} \in U(1) \,\middle|\, \forall\theta \in [0,2\pi[\,\subset\mathbb{R}\right\}$, determined transversely to the ***direction*** $\widehat{\overline{\omega}}\perp\odot$.

We remember that $\widehat{\overline{\omega}}$ is autonomous normed $|\widehat{\overline{\omega}}| \equiv 1$ ***direction*** also for the external given angular frequency energy ***quantity*** $\omega[\widehat{\omega}]$. This external measured frequency energy ω for the ***subton direction*** $\vec{\omega}$ has the following parallel ∥ relationship with the autonomous ***direction***

$$\widehat{\overline{\omega}} = \vec{\mathbf{1}} \leftrightarrow \vec{\omega} = \omega\widehat{\overline{\omega}} = \omega\widehat{\omega}\mathbf{e}_3 = \omega\mathbf{e}_3 \parallel \widehat{\overline{\omega}} \parallel \vec{\omega} \parallel \mathbf{e}_3, \quad \text{where } |\vec{\omega}| = |\widehat{\omega}| = \widehat{\omega} = 1[\widehat{\omega}] \text{ and } c=1. \tag{3.222}$$

$\widehat{\omega}$ is a selected external frequency standard as the angular frequency measurement reference.[114]

For one ***subton*** we imagine a single angular frequency ω and a single ***direction*** $\widehat{\overline{\omega}} \parallel \mathbf{e}_3$.

When we claim one single ***direction*** there must be found other ***directions*** $\widehat{\overline{\omega}}_m \neq \widehat{\overline{\omega}}$ or $\widehat{\overline{\omega}}_m \nparallel \widehat{\overline{\omega}}$, with $\vec{\omega}_m \nparallel \vec{\omega}$, but still $|\vec{\omega}_m| = |\vec{\omega}| = |\widehat{\omega}|$ for the same reference $[\widehat{\omega}]$. (Remember $|\widehat{\overline{\omega}}_m| = |\widehat{\overline{\omega}}| \equiv 1$).

In the following, we confine ourselves to one ***direction*** $\widehat{\overline{\omega}} \parallel \mathbf{e}_3$, and will examine several ***subtons***

[114] The unit vector $\mathbf{e}_3$ indicates the ***direction*** with length magnitude $|\mathbf{e}_3| = 1[\widehat{\omega}^{-1}_{c=1}]$, while $|\widehat{\overline{\omega}}| \equiv 1$[angular radian], and of cause the reference $\widehat{\omega} \equiv 1[\widehat{\omega}]$. – Then the given measured angular frequency energy ***quantity*** can be $\omega[\widehat{\omega}]$.