[100] The reference of this ***quantity*** is the auto-norm ***direction*** $\widehat{\vec{\omega}}$ as a ***primary quality***.
The 'measure mechanics' of this substance is the 'by us' contrived helix ± 1 of the unitary circle group $\odot = \{\theta \to e^{i\theta} | \forall\theta \in \mathbb{R}\}$. The ***quantity*** of the subject can of course not be measured directly but is an autonomous capacity of the subject as an idea of the existence of the extension.
Comment: The idea for us of a subject ***entity*** ${}^{\mathrm{AB}}_{\circ}\Psi_{\pm\widehat{\vec{\omega}}}$ in physics that possess extension, is essential for the idea of natural space in physics. Due to Immanuel Kant: Space itself can never be an object for our intuition. Properly we can promote the idea ${}^{\mathrm{AB}}_{\circ}\Psi_{\pm\widehat{\vec{\omega}}}$ to some object for us (das Ding für uns) and make some measurements in space.

3.4.1.2. Annihilation of an Excited Circle Oscillator

The excited circle oscillator (3.163) is written without the 2 factor,[101] by including negative ρ, for $\forall\rho \in \mathbb{R}\backslash\{0\}$ we write (remembering $\phi = \omega t$)

$$\psi^{\circ}_{\pm\omega} = |1,\pm1\rangle^{\circ}_{\omega} = a^{\dagger}_{\odot\pm\omega}|0,0\rangle_{\odot} = e^{\pm i\phi}\left(\rho - \frac{\partial}{\partial\rho} \mp \frac{i}{\rho}\frac{\partial}{\partial\phi}\right)|0,0\rangle_{\odot} = \left[\frac{1}{\sqrt[4]{\pi}}\rho e^{-\frac{1}{2}\rho^2}\odot e^{\pm i\omega t}\right]_{\forall\rho\in\mathbb{R}} \tag{3.200}$$

The parity inversion problem we incorporated in the following by considering all proper real radial coordinates as a representation of the odd function dipolar opposition, see (3.120).
When the annihilation operator works on the first excitation, we get the ground state back

$$a_{\odot\pm\omega}a^{\dagger}_{\odot\pm\omega}|0,0\rangle = a_{\odot\pm\omega}|1,\pm1\rangle^{\odot}_{\omega} = \frac{e^{\mp i\phi}}{2}\left(\rho + \frac{\partial}{\partial\rho} \mp \frac{i}{\rho}\frac{\partial}{\partial\phi}\right)\frac{1}{\sqrt[4]{\pi}}\rho e^{-\frac{1}{2}\rho^2}\odot e^{\pm i\phi}$$
$$= \frac{1}{2}\frac{1}{\sqrt[4]{\pi}}(\rho^2-\rho^2+1+1)e^{-\frac{1}{2}\rho^2}\odot e^{\pm i\phi}e^{\mp i\phi} = 1\cdot\frac{1}{\sqrt[4]{\pi}}e^{-\frac{1}{2}\rho^2}\odot e^{\pm i\omega t}e^{\mp i\omega t} = \frac{1}{\sqrt[4]{\pi}}e^{-\frac{1}{2}\rho^2}\odot. \tag{3.201}$$

- Having an event A: The creation of excitation ${}^{\mathrm{A}}|1,\pm1\rangle_{\omega} = {}^{\mathrm{A}}a^{\dagger}_{\pm\omega}|0,0\rangle$ and
- subsequent event B: Annihilation ${}^{\mathrm{B}}a_{\pm\omega}{}^{\mathrm{A}}|1,\pm1\rangle_{\omega} = {}^{\mathrm{B}}a_{\pm\omega}{}^{\mathrm{A}}a^{\dagger}_{\pm\omega}|0,0\rangle = |0,0\rangle$ back to the ground state, we have produced a ***quantity***,

$$|\phi_{\mathrm{B}} - \phi_{\mathrm{A}}| = |\omega|(t_{\mathrm{B}} - t_{\mathrm{A}}), \tag{3.202}$$

which gives the difference between events A and B.
In the autonomous image with autonomous norm $|\widehat{\vec{\omega}}_{\mathrm{AB}}|=1$ we call the *phase angle development* $|\phi_{\mathrm{B}} - \phi_{\mathrm{A}}| = |\widehat{\vec{\omega}}_{\mathrm{AB}}|(t_{\mathrm{B}} - t_{\mathrm{A}})$ a measure of *the autonomous time* of the ***entity*** ${}^{\mathrm{AB}}_{\circ}\Psi_{\pm\widehat{\vec{\omega}}}$.
Here we recall that the angular frequency energy ω is given from an external norm $\widehat{\omega}$ as the reference standard. Then, the external extension from A to B is expressed as

$$z_{\mathrm{AB}} = -c(t_{\mathrm{B}} - t_{\mathrm{A}}) = \left(-c\,|\phi_{\mathrm{B}} - \phi_{\mathrm{A}}|/|\omega|\right)\ [c\widehat{\omega}^{-1}]. \tag{3.203}$$

This is measured by our external development parameter $t = |\phi|/|\omega|$ $[\widehat{\omega}^{-1}]$, in that the internal phase angle is counted with the external angular frequency $\omega[\widehat{\omega}]$ of the circle oscillator ${}^{\mathrm{AB}}_{\circ}\Psi_{\pm\widehat{\vec{\omega}}}$.

3.4.1.3. Change of Direction of an Excited Circle Oscillator

In another way, we have an excited state ${}^{\mathrm{AB}}|1,\pm1\rangle_{\omega}$ with a ***direction*** $\widehat{\vec{\omega}}_{\mathrm{AB}}$, and allows a combined sequential annihilation- and creation-operation event ${}^{\mathrm{B}}\widehat{N}_{\pm\omega} = {}^{\mathrm{B}}a^{\dagger}_{\pm\omega}{}^{\mathrm{B}}a_{\pm\omega}$ to act on the ***entity*** ${}^{\mathrm{AB}}\Psi_{\pm\omega}$ in penetration through the ground state at B back to a new excited state ${}^{\mathrm{BC}}|1,\pm1\rangle_{\omega}$ with a new ***direction*** $\widehat{\vec{\omega}}_{\mathrm{BC}}$. Assuming the same angular frequency energy ω measured by an external reference $\widehat{\omega}_0$ with the norm $|\widehat{\omega}_0| = 1_0$ as at (3.187)

$$\widehat{\vec{\omega}}_{\mathrm{AB}} \leftrightarrow \vec{\omega}_{\mathrm{AB}} = \omega_{\mathrm{AB}}\vec{1}_{\mathrm{AB}} = \omega_{\mathrm{AB}}\widehat{\omega}_0\vec{1}_{\mathrm{AB}} \quad \text{and} \quad \widehat{\vec{\omega}}_{\mathrm{BC}} \leftrightarrow \vec{\omega}_{\mathrm{BC}} = \omega_{\mathrm{BC}}\widehat{\omega}_0\vec{1}_{\mathrm{BC}} \tag{3.204}$$

An arbitrary ***directional*** basis vector $\vec{1}_{\mathrm{XY}}$ has a magnitude $|\vec{1}_{\mathrm{XY}}| = 1_0[\widehat{\omega}_0^{-1}]$ of the extension in relation to the angular frequency reference $\widehat{\omega}_0$, that is $|\vec{\widehat{\omega}}_0| = 1_0\,[\widehat{\omega}_0]$.
Assuming preservation of frequency energy in the event B, then, that $\omega_{\mathrm{BC}} = \omega_{\mathrm{AB}}$ and thus $|\vec{\omega}_{\mathrm{BC}}| = |\vec{\omega}_{\mathrm{AB}}|$. If here $\vec{\omega}_{\mathrm{BC}} \neq \vec{\omega}_{\mathrm{AB}}$, there is a change in B of the ***direction***[102] for the ***entity*** ${}^{\mathrm{ABC}}\Psi_{\pm\omega}$. On the other hand, if $\vec{\omega}_{\mathrm{BC}} = \vec{\omega}_{\mathrm{AB}}$, event B become irrelevant for the ***entity*** ${}^{\mathrm{AC}}\Psi_{\pm\vec{\omega}}$ as one and the same ***direction***.
As frequency energy $\omega_{\mathrm{BC}} = \omega_{\mathrm{AB}}$ can be conserved and the ***direction*** is changed in B, we conclude that the angular momentum change ***direction*** $\hbar\vec{L}^{\pm}_{3\mathrm{AB}} \neq \hbar\vec{L}^{\pm}_{3\mathrm{BC}} \Leftrightarrow \pm1\widehat{\vec{\omega}}_{\mathrm{AB}} \neq \pm1\widehat{\vec{\omega}}_{\mathrm{BC}}$.
We see that the ***direction*** is embedded in the angular momentum as a result of the idea of an angular frequency, producing rotation in a plane whereby we get a circle oscillator, which is transversal to the ***direction*** with orientation + for forward, and − for the past.
This is combined with + for progressive, and − for retrograde rotation.
This bipolar orientation of angular momentum defines the transversal plane with ***direction***.
We have the concept of a difference between A and B with two orientations AB and BA.
The claim is that for the possibility to distinguish two events A and B we shall be able to recognize a transversal plane between them. Yes, it should similarly be possible by intuition to conceive an ***entity*** ${}^{\mathrm{AB}}_{\circ}\Psi_{\pm\widehat{\vec{\omega}}}$ as illustrated in Figure 3.13, which can be used to measure the difference between A and B. – If the subject can be promoted to an object, we achieve extension.

3.4.1.4. The Fundamental Substance for an *Entity* and the Extensive Difference

By intuition, we look at the difference between the two events A and B and write the symbolic expression for the creation of an ***entity*** *subject* for this AB difference

$${}^{\mathrm{AB}}\Psi \leftrightarrow {}^{\mathrm{B}}a_{\odot\pm\omega}{}^{\mathrm{A}}a^{\dagger}_{\odot\pm\omega}|0,0\rangle. \tag{3.205}$$

The causality for this intuition of the difference presupposes the possibility of excitation of a harmonic circle oscillation transversal to the difference AB with frequency energy $\hbar\omega$ and angular momentum $\pm\hbar\widehat{\vec{\omega}}_{\mathrm{AB}}$. A measurement of such an extensive difference as the number of oscillations will depend on $\omega[\widehat{\omega}]$ as a reference and the measured extensive ***quantity*** will then be expressed by (3.203) $z_{\mathrm{AB}} = -c\,|\phi_{\mathrm{B}} - \phi_{\mathrm{A}}|/|\omega|$ $[c\widehat{\omega}^{-1}]$.

3.4.1.5. The Substance of the Concept of a Photon

The expression ${}^{\mathrm{AB}}\Psi_{\pm\omega} \leftrightarrow {}^{\mathrm{B}}a_{\odot\pm\omega}{}^{\mathrm{A}}a^{\dagger}_{\odot\pm\omega}|0,0\rangle$ may appear to be associated with a photon which *subject* is created and transmitted from event A with the speed of light and is received and annihilate in B. However, above, we have described a more substantial structure for a difference AB illustrated by Figure 3.13.
Therefore, I will introduce the term a ***subton*** for this space-time *subject*.
Note that, the conventional space-time 'light-cone'[103] is wound up as a spiral helix in a cylinder along $|\phi| = -x_3$ in Figure 3.13. Thereby it is possible to understand by graphical intuition how simple quantum mechanics performs in space-time.
The ***subton*** is one quantum excitation count by autonomy its own internal phase angle ***direction*** driven external, by its external angular frequency energy $\omega[\widehat{\omega}]$ as its given ***quantity***.
This internal counting measure of the external extension ***quantity*** is translated to a space-time measure expressed by the formula (3.203).
Annihilation B of a ***subton*** under the premise of a prior creation A gives the concept of phase angle plane ***direction*** coherent locality transversal propagation extension in space-time.[104]

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[100] A depth is dependent on the transversal (length and breadth) $\odot$, as essential for the concept of extension by Descartes.
[101] The parity factor 2 occurs when the polar radius coordinate is restricted to positive values $\rho > 0$. (See Section 3.3.1).
[102] The change of ***direction*** in B requires interaction ${}^{\mathrm{B}}\widehat{N}_{\pm\omega} = {}^{\mathrm{B}}a^{\dagger}_{\pm\omega}{}^{\mathrm{B}}a_{\pm\omega}$ with the environment of ***entity*** ${}^{\mathrm{ABC}}\Psi_{\pm\omega}$.
[103] The space-time 'light cone' (Minkowski) is usually defined as $\{(t,x_1,x_2,x_3) \mid c^2t^2 = x_1^2+x_2^2+x_3^2\}$ in Cartesian coordinates, but here in the polar cylinder coordinates is it $\{(t,\rho,\varphi,x_3) \mid c^2t^2 = \rho^2+x_3^2\}$ where $\rho^2 = x_1^2+x_2^2$ with $\langle\rho\rangle=1$. As we view the unit cylinder surface $\overline{\odot}$, we get $\{(t,1,\phi+\theta,x_3) \mid c^2t^2 = x_3^2 = (\phi/\omega)^2,\ \varphi = \phi+\theta\}$ where we have removed $\rho=1$ on the unit cylinder surface as shown in Figure 3.13. As seen later in Figure 5.55 and page 334 that this cylinder it is a *null helix* curve.
[104] This is the idea of something real material: extensa (Descartes), in nature (Spinoza) or in physics for us. This is built on the a priori idea of ***direction*** by Immanuel Kant. Now it is an issue for *phase information* in a received signal from a transmission.