3.4. The Quantum Excited *Direction*

We have here in section 3.3.4 seen that a ***direction*** $\hat{\vec{\omega}}$ is linked to the transversal plane whose ***quality*** $\odot\perp\hat{\vec{\omega}}$ is associated with the rotational symmetry of the unitary rotation group $U(1)$
$\odot = \{U_\theta : \theta \to e^{i\theta} \in U(1) \mid \forall\theta \in \mathbb{R}\}$ (3.137)

3.4.1. The Direction of the First Excitation Described in Cylindrical Coordinates.

In cylindrical coordinates type (ρ, φ, z), we can write the angular frequency vector as
$\vec{\omega} = \omega\hat{\vec{\omega}} \leftrightarrow (0,0,\omega) = \omega(0,0,1)$, in which the first two coordinates $(0,0) \to (\rho,\theta) \sim (\rho_\omega, \theta)$ describe the transversal plane to the vector $\vec{\omega}$. Looking at the coordinates for the development of the excitation we have, as the new idea, the cylinder axis coordinate:

(3.189) $$z = -ct = -c\,|\phi|/|\omega|,$$

that we have chosen positive out in the ***direction*** $\hat{\vec{\omega}} \leftrightarrow (0,0,1)$, where the opposed development parameter t is defined positively as a manufactured past in the ***direction*** $-\hat{\vec{\omega}}$. The angular coordinate is written $\varphi = \theta \pm \omega t$. In particular, we have the active angle $\phi = \omega t$ and the start basic angle is θ for $t = 0$.
These two coordinates $(\varphi, z) \leftrightarrow (\phi, z)$ follow the information development parameter t.
The radial distribution ρ is independent of t. We note the rewriting $\rho = \omega\rho_\omega/c$,
therefore, I describe the following options for coordinate arguments for the wavefunction for the excitation of the circle oscillator development

(3.190) $$\psi_{\pm\vec{\omega}}\begin{pmatrix}\rho_\omega\\ \varphi\\ z\end{pmatrix} = \psi_{\pm\vec{\omega}}\begin{pmatrix}c\,\rho/|\omega|\\ \theta \pm |\omega|t\\ -ct\end{pmatrix} = \psi_{\pm\vec{\omega}}\begin{pmatrix}c\,\rho/|\omega|\\ \theta \pm \phi\\ -c|\phi|/|\omega|\end{pmatrix} \sim \frac{1c}{|\omega|}\psi^{(\theta)}_{\pm\omega\hat{\vec{\omega}}}\begin{pmatrix}\rho\\ \pm\phi\\ -|\phi|\end{pmatrix} \leftrightarrow \psi^{\odot}_{\pm\hat{\vec{\omega}}}\begin{pmatrix}\rho\\ \pm\phi\\ -|\phi|\end{pmatrix}.$$

As an example, I define the ***entity*** ${}^{AB}\Psi_{\pm\vec{\omega}}$ by creation in event A, to annihilation at B,

(3.191) $${}^{AB}\Psi_{\pm\omega\hat{\vec{\omega}}} = \left\{\psi_{\pm\omega}\begin{pmatrix}\rho\\ \theta \pm \omega t\\ -|\omega|t\end{pmatrix}_{\hat{\vec{\omega}}} \in \mathbb{C}\ \middle|\ \forall\rho \in [0,\infty[,\quad \forall\theta \in [0,2\pi[,\quad \forall t \in [t_A, t_B] \subset \mathbb{R}\right\}$$

I also by intuition consider the ***entity*** as a complex scalar field as a function of these cylindrical coordinates and maintain the angular coordinate in dualism for intuition; where $\phi = \omega t$ evolves and θ is an eternal constant distributed on $[0,2\pi[$ as a basic background for $\phi = \omega t$, joined in this duality by the resulting angular phase $\varphi = \theta + \omega t$ for a full description.
From the formulas (3.148)-(3.149) and (3.163) I write this as the complex scalar field in the excitation transversal plane $\odot\perp\vec{\omega}$

(3.192) $$\psi^{(\rho,\theta)}_{\pm\omega}(\phi) = a^\dagger_{\odot\pm\omega}|0,0\rangle = 2\tilde{r}(\rho)\odot e^{\pm i\phi} = \left(2\tfrac{1}{\sqrt[4]{\pi}}\rho e^{-\frac{1}{2}\rho^2}e^{i\theta}\right)e^{\pm i\phi} \in \mathbb{C},\ \text{for } \forall\rho \geq 0,\ \forall\theta \in [0,2\pi[,\ \forall\phi \in \mathbb{R},$$

and then expressed by an information development parameter $t = |\phi/\omega|$, with the transversal radial rewriting $\rho_\omega = c\rho/\omega$, we now for $\forall t \in \mathbb{R}$ getting

(3.193) $$\psi^{(\rho_\omega,\theta)}_{\pm\omega}(t) = a^\dagger_{\odot\pm\omega}|0,0\rangle = 2\tilde{r}\left(\tfrac{\omega\rho_\omega}{c}\right)\odot e^{\pm i\omega t} = \left(2\tfrac{1}{\sqrt[4]{\pi}}\tfrac{\omega\rho_\omega}{c}e^{-\frac{1}{2}\left(\frac{\omega\rho_\omega}{c}\right)^2}e^{i\theta}\right)e^{\pm i\omega t} \in \mathbb{C},$$

The parameter independent factor for this complex scalar field is symbolised by

(3.194) $$\circledcirc \leftrightarrow \left(2\tfrac{1}{\sqrt[4]{\pi}}\rho\, e^{-\frac{1}{2}\rho^2}e^{i\theta}\right) \leftrightarrow \left(2\tfrac{1}{\sqrt[4]{\pi}}\tfrac{\omega\rho_\omega}{c}e^{-\frac{1}{2}\left(\frac{\omega\rho_\omega}{c}\right)^2}e^{i\theta}\right) \leftrightarrow |\psi\rangle$$

Thus, hiding the plane coordinates (ρ,θ) in a classical transcendental substance $\circledcirc$.
What we a priori know about this subject, is that it is transversal $\circledcirc\perp\hat{\vec{\omega}}$.
Here it is noted that the idea is that the scale of the isotropic symmetric radial ρ in the transversal $\circledcirc$ refers to the autonomous norm $1 := |\hat{\vec{\omega}}|$.

The phase angle parameter dependent version can then be written (compare (2.62))

(3.195) $$e^{\pm i\phi}\circledcirc \mathrel{\tilde{\Rightarrow}} \psi^{\odot}_{\pm\hat{\vec{\omega}}}(\phi) \sim \psi^{\odot}_{\pm\omega}(\phi) = \psi^{\odot}_{\pm}(\omega t),\quad \text{or traditional written } |\psi(t)\rangle = e^{\pm i\hat{\omega}t}|\psi\rangle.$$

This is involved a 'new' kind of polar cylinder coordinate system $(\psi, z) \in \mathbb{C}, \mathbb{R}$, where
1. $\psi \in \mathbb{C}$ describe the complex transversal plane and
2. $z \in \mathbb{R}$ describes the linear real axis of the cylinder system

This makes me rewrite the idea in (3.190) to

(3.196) $$\begin{pmatrix}\psi\\ z\end{pmatrix} \in \begin{pmatrix}\mathbb{C}\\ \mathbb{R}\end{pmatrix} \mathrel{\tilde{\Rightarrow}} \begin{pmatrix}\psi^{\odot}_{\pm\omega}(\omega t)\\ -ct\end{pmatrix}_{\vec{\omega}} \leftrightarrow \begin{pmatrix}\psi^{\odot}_{\pm\hat{\vec{\omega}}}(\phi)\\ -|\phi|\end{pmatrix}_{\hat{\vec{\omega}}} \sim \begin{pmatrix}2\tilde{r}(\rho)\odot e^{\pm i\phi}\\ -|\phi|\end{pmatrix}_{\hat{\vec{\omega}}} \sim e^{\pm i\phi}\circledcirc\perp c|\phi|\hat{\vec{\omega}}$$

Here the autonomous normed angular frequency energy has a unit vector $\hat{\vec{\omega}} = \vec{\mathbf{1}} \sim \binom{0}{1} \in \binom{\mathbb{C}}{\mathbb{R}}$
for the ***direction*** of development, – a propagation – and the axis $\{z = c|\phi|\hat{\vec{\omega}} \mid \forall\phi \in \mathbb{R}\}$
of 'the cylinder'. From this, we write (3.191) seen from external in the form

(3.197) $${}^{AB}\Psi_{\pm\hat{\vec{\omega}}} = \left\{\circledcirc e^{\pm i\phi}\perp c|\phi|\hat{\vec{\omega}}\ \middle|\ \circledcirc\perp\hat{\vec{\omega}},\ \forall\phi \in [\phi_A, \phi_B] \subset \mathbb{R}\right\},\quad \text{existence from A to B.}$$

It is up to the reader to interpret the implications of the formulas (3.190)-(3.197) for physics.

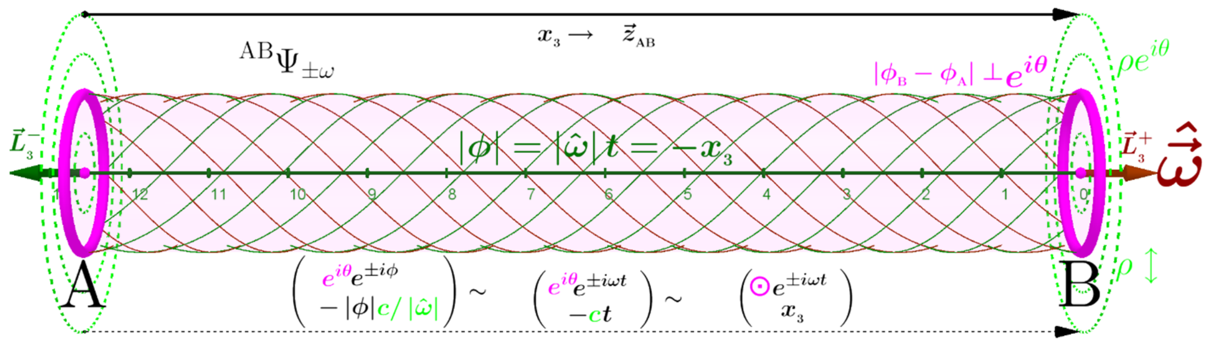

Figure 3.13
The intuition of ***causality*** as a ***primary quality***: The in A created ***entity*** ${}^{AB}\Psi_{\pm\hat{\vec{\omega}}}$, is ***causally*** annihilated in B.
The development ***direction*** $\overrightarrow{AB} = \vec{z}_{AB} = -|\phi_{AB}|\hat{\vec{\omega}}$ is indicated in the reading direction, while the development parameter $t \equiv |\phi|/\hat{\omega}$ has the opposite orientation. The autonomous wavelength is $\hat{\lambda} = 2\pi$, when $|\hat{\vec{\omega}}| \equiv 1$.
(Here is illustrated 2 full periods $|\phi_B - \phi_A| = 2\cdot 2\pi$).

It is well known from Einstein's theory of relativity that the time parameter t has a relativistic measure, therefore I prefer besides lights-peed-norming c=1 and angular frequency to energy standard ratio $\hbar$=1 to auto-norm the angular frequency $|\omega| = |\hat{\vec{\omega}}| = |\hat{\omega}| = 1$, so that the development parameter just give the evolution of the angular phase $|\phi| = |\hat{\omega}|t$ of the excited state.
In short, we set $\boxed{t := |\phi|}$ in the auto-norm picture, hence $x_3 = -|\phi|$ from the extension idea

(3.198) $$x_3 := z = -t = -ct\,,\quad \text{when}\quad c=1\,,\ \hbar=1,\quad \text{and}\quad |\omega| = 1.$$

These aspects of the ***quality*** a ***direction*** from (3.197) are illustrated Figure 3.13, where the intuition is confined to the unitary circle group $\odot = \{U_\theta : \theta \to e^{i\theta} \in U(1) \mid \forall\theta \in \mathbb{R}\}$, and we disregard the radial distribution $\circledcirc$ and concentrate our idea on a rotating unit helix (cylinder).

(3.199) $${}^{AB}\Psi^{\odot}_{\pm\hat{\vec{\omega}}} = \left\{\begin{pmatrix}e^{i\theta}e^{\pm i\phi}\\ -|\phi|\end{pmatrix} \sim \odot\begin{pmatrix}e^{\pm i\phi}\\ -|\phi|\end{pmatrix} \in \begin{pmatrix}\mathbb{C}\\ \mathbb{R}\end{pmatrix}\ \middle|\ \odot\perp\hat{\vec{\omega}},\ \forall\phi \in [\phi_A, \phi_B] \subset \mathbb{R}\right\}.$$

If we take a classic point of view, we must of causal reasons establish the reference system at the annihilation event B, as, what we call a measurement only can appear here. What can be measured or counted is in principle the number of oscillations $\frac{1}{2\pi}|\phi_B - \phi_A|$. In a traditional interpretation, the real measure has been accepted. Here, the autonomous norm measure is into the past from B, back to the creation A, defined as the extension in a reality from B back to A.[100]

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