3.3.4.4. Polar Radial Distribution of the Angular Momentum Over the Circular Oscillator Plan

We note that the excited state has radial density distribution from (3.143) (3.144) as

(3.173) $\mathrm{pd}_1(\rho) = \quad 2\tilde{r}(\rho) = \quad 2\frac{1}{\sqrt[4]{\pi}}\rho e^{-\frac{1}{2}\rho^2}, \quad \text{for } \forall\rho \geq 0$

The excited state normalized as (3.150) $\int_0^\infty \psi_\pm^*(\rho)\psi_\pm(\rho)d\rho = \int_0^\infty \frac{4}{\sqrt{\pi}}\rho^2 e^{-\rho^2}d\rho = 1$

From here we infer that the action of the angular momentum is radially distributed as

(3.174) $|\psi_\pm(\rho)|^2 = \quad |\mathrm{pd}_1(\rho)|^2 = \quad \frac{4}{\sqrt{\pi}}\rho^2 e^{-\rho^2}$

integrated along the rings with radius ρ in the transversal plane as indicated in Figure 3.10.

3.3.4.5. The Area Distribution of the Action Over the Transversal Plane

The activity is spread across the transversal plane, radially, per area element we have

(3.175) $dA = \rho\, d\theta\, d\rho = \quad 2\pi\rho d\rho\,, \quad$ because $\int_0^{2\pi} d\theta = 2\pi$ of the angular symmetry.

A forward-flowing volume element at the radial coordinate ρ is then

(3.176) $dV = \rho\, d\theta\, d\rho\, d|\phi| = \quad \rho\, d\theta\, d\rho\, |\widehat{\vec{\omega}}|dt = \quad 2\pi\rho\, d\rho\, |\widehat{\vec{\omega}}|dt.$

Thus, the radial factor $1/2\pi\rho$, multiplied the radial distribution $\mathrm{pf}_1(\rho)$ getting the area probability density function

(3.177) $\mathrm{pd_A}(\rho,\theta) = \frac{pd_1(\rho)}{2\pi\rho} = \frac{2\tilde{r}(\rho)}{2\pi\rho} = \frac{1}{\pi\rho}\left(\frac{1}{\sqrt[4]{\pi}}\,\rho\, e^{-\frac{1}{2}\rho^2}\right) = \frac{1}{\pi}\frac{1}{\sqrt[4]{\pi}}\, e^{-\frac{1}{2}\rho^2} \quad \text{for } \forall\rho > 0$

and alternatively, the radial action $|\mathrm{pf}_1(\rho)|^2$ multiplied by $1/2\pi\rho$ to provide the *intensity* (the activity per unit area)

(3.178) $\mathcal{I}(\rho,\theta) = \frac{|pf_1(\rho)|^2}{2\pi\rho} = \quad \frac{1}{2\pi\rho}\frac{4}{\sqrt{\pi}}\rho^2 e^{-\rho^2} = \frac{2}{\pi}\frac{1}{\sqrt{\pi}}\rho e^{-\rho^2} \quad \text{for } \forall\rho > 0$

The radially dependent transversal distribution of the *intensity* is illustrated in Figure 3.11.

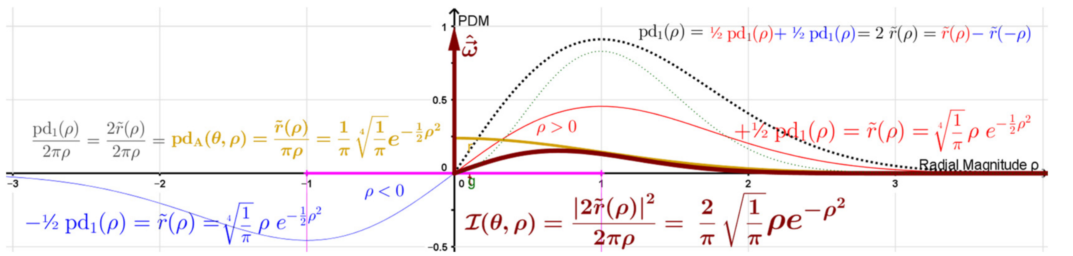

Figure 3.11 The intuition of the first excited state of the circle oscillator is based on the radial distribution function $\tilde{r}(\rho) = \frac{1}{\sqrt[4]{\pi}}\rho e^{-\frac{1}{2}\rho^2} \in \mathbb{R}$, for $\forall\rho \in \mathbb{R}$ with the odd balance $\tilde{r}(\rho) = -\,\tilde{r}(-\rho)$ for $\forall\rho \in \mathbb{R}$ and thus a parity inversion difference $\tilde{r}(\rho) - \tilde{r}(-\rho)$ which drives the excitation positive forward ***direction*** indicated by $\widehat{\vec{\omega}}$. The area density of this distributed state rotation symmetry over the transversal plane is then for $\forall\rho \geq 0$ $\mathrm{pd_A}(\rho,\theta) = \frac{\tilde{r}(\rho)}{\pi\rho} = \frac{1}{\pi}\frac{1}{\sqrt[4]{\pi}}e^{-\frac{1}{2}\rho^2}$. But the action of the excitation is seen as having an area *intensity* for $\forall\rho \geq 0$ $\mathcal{I}(\rho,\theta) = \frac{|\mathrm{pf}_1(\rho)|^2}{2\pi\rho} = \frac{2}{\pi}\frac{1}{\sqrt{\pi}}\rho e^{-\rho^2}$, (3.178).

The area distribution of the transversal intensity is shown in Figure 3.12.

Here the frequency energy is auto normalized $\omega = |\widehat{\vec{\omega}}| = |\widehat{\omega}| \equiv 1$, (its own reference).

When we integrate the action of the excited area intensity $\mathcal{I}(\rho,\theta)\widehat{\vec{\omega}}$ of the momentum of the transversal plane with the ***direction*** $\widehat{\vec{\omega}} = \vec{\mathbf{1}}$ we get the active power

(3.179) $\int_0^{2\pi} d\theta \int_0^\infty d\rho\, \mathcal{I}(\theta,\rho)\widehat{\vec{\omega}} = \quad 2\pi\widehat{\vec{\omega}} \int_0^\infty d\rho \frac{2}{\pi}\frac{1}{\sqrt{\pi}}\rho\, e^{-\rho^2} = \quad \widehat{\vec{\omega}}\int_0^\infty \frac{4}{\sqrt{\pi}}\rho^2 e^{-\rho^2}d\rho = \quad 1\widehat{\vec{\omega}} = \quad \widehat{\vec{\omega}}$

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As just ***one quantum*** $\vec{\mathbf{1}}$ of

- Angular momentum $\vec{L}_3^\pm = \pm 1\,\widehat{\vec{\omega}} \quad = \pm\vec{\mathbf{1}}$.
- Angular frequency energy $|\widehat{\vec{\omega}}|$ is ***one quantum*** of kinetic energy.
- Power as flowing energy $\widehat{\vec{\omega}}/|\widehat{\omega}|$ from the paste, measured per unit $|\widehat{\omega}| = |\phi|/t$ of the information development dimension, as *FORWARD* towards the future.
- Momentum as we shall see, can be interpreted as line ***direction*** momentum through space.

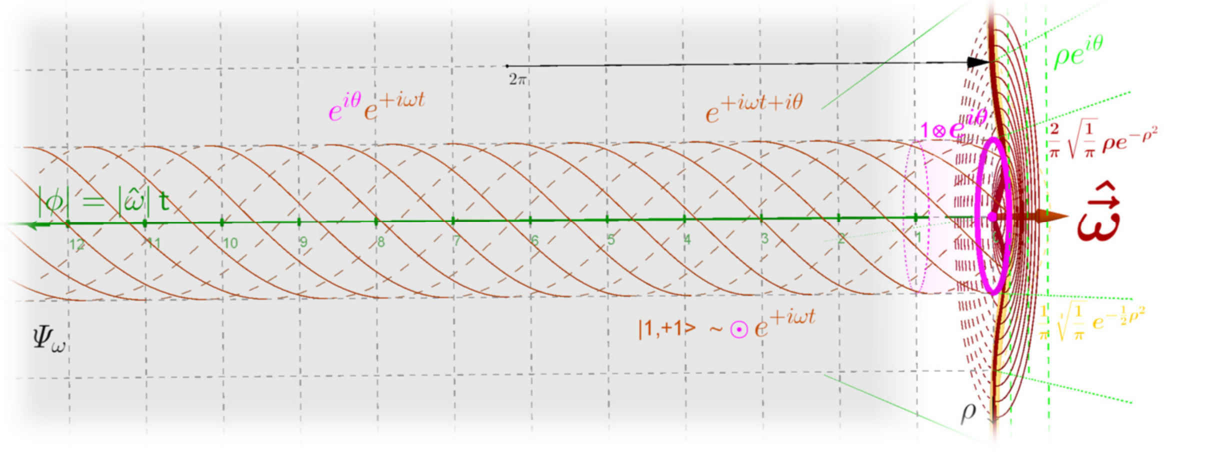

Figure 3.12 Here the intuition of the transversal area intensity $\mathcal{I}(\theta,\rho)$ of the progressive circle oscillating rotation induced by the unitary symmetry $\odot = \{U_\theta: \theta \to e^{i\theta} \in U(1)\,|\,\forall\theta \in \mathbb{R}\}$ in the transversal plane is designed as a magenta circle ring $\odot$ determining the symmetry of the excitation $\widehat{\vec{\omega}} \perp \odot$. The excitation is thus a creation from the ***direction*** $\widehat{\vec{\omega}}$ for the unitary rotational symmetry $\odot_{\widehat{\vec{\omega}}} \approx e^{i\theta}e^{\pm i|\widehat{\vec{\omega}}|t}$ as active form the transversal plane to $\widehat{\vec{\omega}}$. The rotation oscillation is driven by the ***quantity*** ω with the ***qualitative direction*** $\widehat{\vec{\omega}}$ through the transversal plane $\rho e^{i\theta}$. Here in this figure $\omega = |\widehat{\omega}| \equiv 1$, with a progressive excitation $m = +1$ illustrated as a unitary (ρ=1) helix cylinder stretching out in the past $|\phi| = |\widehat{\omega}|t$, with a positive development parameter $t = |\phi|/|\widehat{\vec{\omega}}|$. Both the progressive $\vec{L}_3^+ = \widehat{\vec{\omega}}$ as here, or the retrograde $\vec{L}_3^- = -\widehat{\vec{\omega}}$ excited circle oscillation are intuited as *FORWARD* flowing energy with an area distribution of intensity $\mathcal{I}(\theta,\rho) = \frac{2}{\pi}\frac{1}{\sqrt{\pi}}\rho e^{-\rho^2}$, only with radial dependence and angular symmetry. The excited circle oscillator around $\widehat{\vec{\omega}}$ draws an intensity with the transversal plane. – Comment: The reason for the drawing of the relative flatness of the intensity is in the normed scaling ($1 = |\widehat{\vec{\omega}}| = |\widehat{\omega}| \equiv 1 = \hbar = c^2 = 1$). Hereby $\widehat{\vec{\omega}}$ represents the flowing activity as both energy, power as well as momentum 'motored' by the ***quantum*** angular momentum $\vec{L}_3^\pm = \pm 1\,\widehat{\vec{\omega}}$. Overall, this is called the state momentum ***one quantum*** $\hbar\widehat{\vec{\omega}} = \hbar\vec{\mathbf{1}}$ for the creation $a_{+\widehat{\omega}}^\dagger$ of the circle oscillation ***direction quality*** $\vec{\mathbf{1}} \sim \widehat{\vec{\mathbf{1}}}$.

3.3.4.6. The Energy Intensity Momentum

The ***quality*** energy eigenvalue ω in the eigenvalue equation (3.166) is interwoven with the angular frequency ***direction*** $\widehat{\vec{\omega}} = \vec{L}_3^+$ given from the eigenvalue equation (3.167) dictates that the energy of a frequency has a flow ***direction*** perpendicular to the angular rotation plane, a future ***direction***. This current of energy has an intensity flux, spread over the area of the transversal plane as (3.178) above, of which an area element develops into a volume element (3.176). The forward flowing area element through a development unit radian phase angle gives an effective volume element, and the integration of the intensity with (3.179) we get the forward flowing kinetic energy

(3.180) $\int_0^1 |\widehat{\omega}|dt \int_0^{2\pi} d\theta \int_0^\infty d\rho\, \mathcal{I}(\rho,\theta)\,\widehat{\vec{\omega}} = \quad |\widehat{\omega}|\, 2\pi \int_0^\infty d\rho\, \mathcal{I}(\rho,\theta)\widehat{\vec{\omega}} = \quad |\widehat{\omega}|\,\widehat{\vec{\omega}} = \quad \widehat{\vec{\omega}}$

by or through the transversal plane as illustrated in Figure 3.12

We see that a ***direction*** $\widehat{\vec{\omega}}$ is intimately linked to the transversal plane whose ***quality*** $\odot\perp\widehat{\vec{\omega}}$ is associated with the rotational symmetry of the rotation group $U(1)$,

(3.137) $\odot = \{U_\theta: \theta \to e^{i\theta} \in U(1)\,|\,\forall\theta \in \mathbb{R}\}$.