3.3.4. The Possible Excitation of a Circular Oscillator with ± Signed Orientation.

The plane concept allows a circle rotation through the plane around a *locus situs* event point in the plane with a $U(1)$ symmetry, which requires an angular dualism $\theta \backslash \phi$ (3.139), from an eternal constancy $\frac{\partial}{\partial\phi}\theta = \frac{\partial}{\partial\phi}\odot = \frac{\partial}{\partial\phi}|0{,}0\rangle = 0$, for all angles $\forall\theta \in [0{,}2\pi[$ for the formulation $\odot^{\theta} = e^{i\theta}$, bearing in mind the symmetry as (3.137) $\odot = \{U_\theta\colon \theta \to e^{i\theta} \in U(1) \mid \forall\theta \in \mathbb{R}\}$.

One active creation at given angular frequency energy ω with the creation operator $a^{\dagger}_{\odot\pm\omega}$ from the ground state, where we presuppose $\rho > 0$, gives as in (3.148)-(3.149)

(3.163)
$$\psi^{\odot}_{\pm\omega} = |1,\pm1\rangle^{\odot}_{\omega} = a^{\dagger}_{\odot\pm\omega}|0{,}0\rangle_{\odot}$$
$$= e^{\pm i\phi}\left(\rho - \frac{\partial}{\partial\rho} \mp \frac{i}{\rho}\frac{\partial}{\partial\phi}\right)|0{,}0\rangle_{\odot} = e^{\pm i\phi}\left(\rho - \frac{\partial}{\partial\rho}\right)|0{,}0\rangle_{\odot} = 2\frac{1}{\sqrt[4]{\pi}}\rho e^{-\frac{1}{2}\rho^2}\odot e^{\pm i\omega t}$$

Here the angular phase coordinate ϕ of the state is translated into the energy angular frequency ω multiplied by the development parameter $t \in \mathbb{R}$, so that $\phi = \omega t$. The development goes from the past to the future as a ***direction*** dimension different from the circle oscillators plane.
We claim, that the plane of the circle oscillator is transversal to the development, and that this past to future ***direction*** will be governed by the angular momentum.
The excited ground state at ***quantum*** $\omega \in \mathbb{R}$ is an active revolving plane circular symmetric distribution of *frequency energy*

(3.164) $$|1,\pm1\rangle^{\odot}_{\omega} = a^{\dagger}_{\pm\omega}|0{,}0\rangle_{\odot} = \odot 2\tilde{r}(\rho)e^{\pm i\omega t}, \quad \text{for } \rho > 0.$$

We saw above that the resulting transversal radial distribution (3.144) is composed of parity inversion contradiction (3.151) $2\tilde{r}(\rho) = \tilde{r}(\rho) - \tilde{r}(-\rho)$. These are shown for intuition in Figure 3.8, where the normal vector $\vec{n}$ to the transversal plane points into the future.

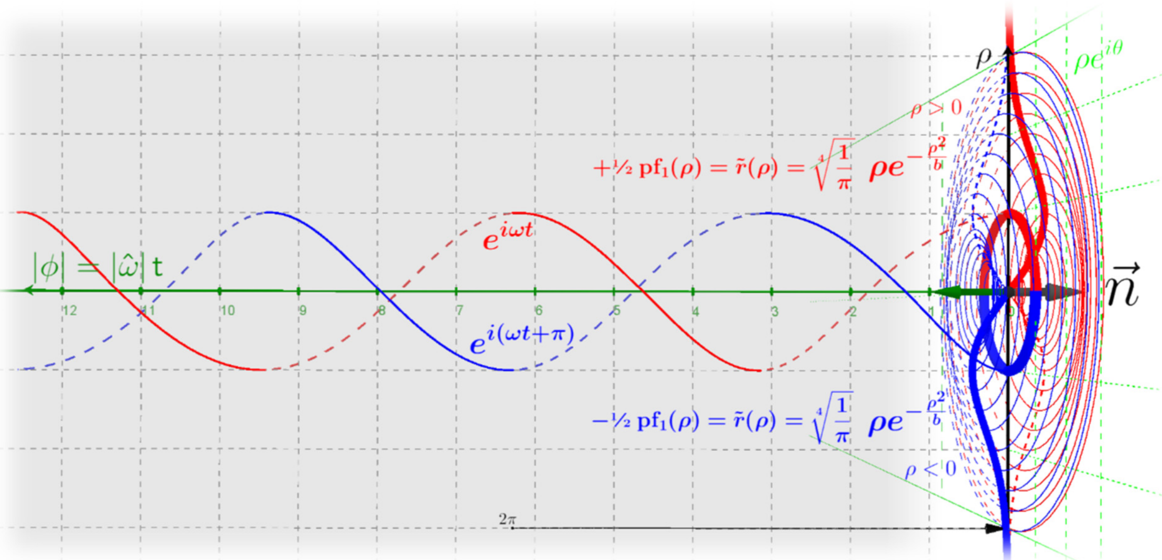

Figure 3.8 The intuition that the ground state is excited to a harmonic circle oscillator with the angular momentum $\vec{L}_3$ in which the density magnitude is shown as a parity inversion contradiction (red-blue). The transversal plane is represented by the polar coordinates (ρ,θ) is equivalent to the complex number $\rho e^{i\theta}$. The excitation is an active rotation by the factor $e^{\pm i\omega t}$ producing the past $|\phi| = |\hat{\omega}|t$, rearward from the rotation in the transversal plane, consistent with its momentum along the normal vector $\vec{n}$, $|\vec{n}|$=1. The rotation has an angular momentum $\vec{L}_3 = \pm1\hbar\vec{n}$ corresponding to the two possible state orientations of the rotation. Note here; the rotation $e^{+i\omega t}$ is shown by $\vec{L}^{+}_3 = +\hbar\vec{n}$.
The development parameter t is drawn backwards along with the angular development phase ϕ in the rotation $|\phi| = t/|\hat{\omega}|$.
Since $\hat{\omega}$ is the measure norm $|\hat{\omega}| \equiv 1$ (helix pitch 1). The drawing of the density magnitudes $\tilde{r}(\rho)$ and $\tilde{r}(-\rho)$ represents its own dimension (and has to be interpreted totally differently from the development parameter dimension) and is displayed as a rotation symmetric disc (red & blue circles) generated by the $U(1)$ rotational symmetry $\odot \sim e^{i\theta}$ through the plane $\rho e^{i\theta}$. The density is intuited as the thickness of the disc $\mathrm{pd}_1(\rho) = +½\,\mathrm{pd}_1(\rho) - ½\,\mathrm{pd}_1(-\rho) = \tilde{r}(\rho) - \tilde{r}(-\rho) = 2\tilde{r}(\rho)$.

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The circular symmetry plane as a background is here expressed by a complex number $\rho e^{i\theta} \in \mathbb{C}$ from the polar coordinates (ρ,θ), which are in correspondence with the symmetry of the unitary rotation group $U(1)$ of unitary operators

(3.165) $$U_\theta\colon \theta \sim \theta \to \odot^{\theta} = e^{i\theta} \sim e^{i\theta} \in U(1).$$

ρ is a dilation factor from a unitary number $e^{i\theta}$, that defines the plane by coordinates (ρ,θ).
The two opposite distribution functions are shown as red opposite blue. These distributions are not only rotation $U(1)$ symmetric in the plane, but information development rotates also by the factor $e^{\pm i\omega t}$. In this intuition, we see them as a distribution of angular momentum in the plane. This is shown in Figure 3.8 as circles of rotation over the transversal plane. The development dimension produces a ***direction*** into the past (along the green line to the left in Figure 3.8), wherein the circle oscillator produces a cylindrical *helical* structure, of which helicity has pitch ±1, relative to the development norm $|\phi| = |\omega|t$. (Figure 3.8 only display helicity +1)
We remember the eigenvalue equations for the rotating circle oscillator excitation for the Hamilton operator (3.113) and the angular momentum operator (3.114) are written

(3.166) $$\hat{H}_\omega|1,\pm1\rangle \doteq (1\hbar\omega + \hbar\omega)|1,\pm1\rangle_\omega$$

(3.167) $$\hat{L}_3\,|1,\pm1\rangle \doteq \pm1\hbar|1,\pm1\rangle$$

We note that when energy is measured in the same unit as the angular frequency, $\hbar$=1.
In (3.167) the eigenvalue of the angular momentum is $\pm1\hbar$. This means that the angular momentum[98] has a ***direction*** represented by the two oppositely oriented vectors

(3.168) $$\vec{L}^{-}_3 = -\vec{L}^{+}_3, \qquad \text{where } |\vec{L}^{\pm}_3| = \hbar = 1$$

Here, the progressive rotation is represented by, $\vec{L}^{+}_3 = \hbar\vec{n}$, as shown in Figure 3.8, and the retrograde by $\vec{L}^{-}_3 = -\hbar\vec{n}$, where $\vec{n}$ is the normal vector to the rotation plane.

3.3.4.2. The Oscillation Freedom from Portable Energy as the concept of Rest Mass

The circle oscillator excitation gives an eigenvalue for the Hamilton function $\hat{H}_\omega$ that is

(3.169) $$E_{\omega,1} = 2\hbar\omega = (\hbar\omega + \hbar\omega) = {}^{1}E_\omega + E_{\omega,0} = T_\omega + V_\omega$$

Here we compare with the classic formulation (2.92) $H_\omega = T_\omega + V_\omega$, where the kinetic energy then is $T_\omega = {}^{1}E_\omega = \hbar\omega$ and the potential energy is $V_\omega = E_{\omega,0} = \hbar\omega$, which is binding to the ubiquitous ground state that possesses the 'highest' symmetry of a plane. This potential energy is what I call the internal binding energy of the circle oscillator to its own parity inversion contradiction through a *locus situs* circular centrum in its plane. This internal coupling to the ground state is an intrinsic property of excitation to an ***entity*** $\Psi_{\pm\omega}$.
Just as (2.96) we have the formulation $H_\omega = T_\omega + V_\omega = 2T_\omega$ and further the function $L_\omega = T_\omega - V_\omega = 0$, which represents portable energy. A physical ***entity*** $\Psi_{\pm\omega}$ of the excited plane harmonic circle oscillator, therefore, has no interaction with the surroundings and moves freely through its surroundings (without any external forces). But moves with a kinetic energy

(3.170) $$T_\omega = \hbar\omega \sim m_\omega c^2,$$

where an abstract internal relativistic ***quantity*** m_ω is an inertia factor (internal 'mass')[99], with speed c, and proportional with ω. We see, that $\omega \in \mathbb{R}$ is the real ***quantity***, that defines a ***quantum excitation*** of the **circle oscillator** which just is a scalar eigenvalue in (3.166).
The fact that the kinetic and potential energies balance $L_\omega = T_\omega - V_\omega = 0$ means that the excited plane quantum harmonic circle oscillator is free from portable energy and thereby does not carry any rest mass along a path produced as a development parameter.

98 In the classical world, we describe the angular momentum by a vector $\vec{L} = \vec{r} \times \vec{p}$ with a ***direction*** in space (or a bivector **L=r∧p**).
99 We know from 20th-century physics that a classical external rest mass has no meaning in this, $L_\omega = T_\omega - V_\omega = 0$.